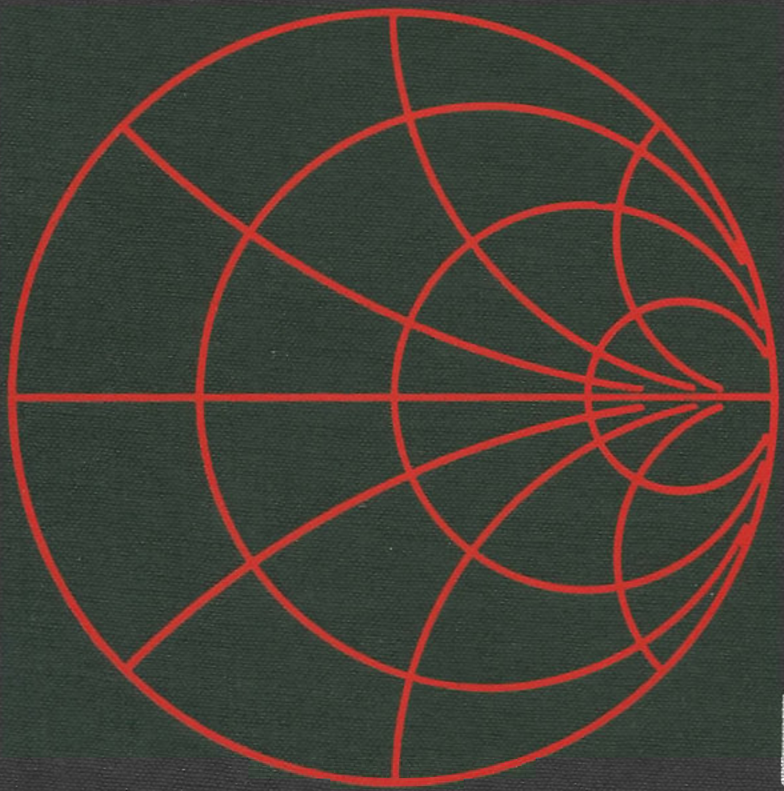

ELECTRONIC APPLICATIONS OF THE SMITH CHART

In Waveguide, Circuit, and Component Analysis

PHILLIP H. SMITH

Prepared under the sponsorship and direction of
KAY ELECTRIC COMPANY

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Preface

The purpose of this book is to provide the student, the laboratory technician, and the engineer with a comprehensive and practical source volume on SMITH CHARTS and their related overlays.

In general, the book describes the mechanics of these charts in relation to the guided-wave and circuit theory and, with examples, their practical uses in waveguide, circuit, and component applications. It also describes the construction of boundaries, loci, and forbidden regions, which reveal overall capabilities and limitations of proposed circuits and guided-wave systems.

The Introduction to this book relates some of the modifications of the basic SMITH CHART coordinates which have taken place since its inception in the early 1930s.

Qualitative concepts of the way in which electromagnetic waves are propagated along conductors are given in Chap. 1. This is followed in Chaps. 2 and 3 by an explanation of how these concepts are related to their quantitative representation on the "normalized" impedance coordinates of the SMITH CHART.

Chapters 4 and 5 describe the radial and peripheral scales of this chart, which show, respectively, the magnitudes and angles of various linear and complex parameters which are related to the impedance coordinates of the chart. In Chap. 6 an explanation is given of equivalent circuit representations of impedance and admittance on the chart coordinates.

Several uses of expanded portions of the chart coordinates are described in Chap. 7, including the graphical determination therefrom of bandwidth and Q of resonant and antiresonant line sections.

The complex transmission coefficients, their representations on the SMITH CHART, and their uses form the subject of Chap. 8. It is shown therein how voltage and current amplitude and phase (standing wave amplitude and wave position) are represented by these coefficients.

Impedance matching by means of single and double stubs, by single and double slugs, and by lumped L-circuits is described in Chaps. 9 and

10. Chapter 11 provides examples, illustrating how loci, boundaries, and forbidden areas are established and plotted.

The measurement of impedance by sampling voltage or current along the line at discrete positions, where a slotted line section would be excessively long, is described in Chap. 11.

The effect of negative resistance loads on transmission lines, and the construction and use of the negative SMITH CHART and its special radial scales, are described in Chap. 12. Stability criteria as determined from this chart are indicated for negative resistance devices such as reflection amplifiers.

Chapter 13 discusses, with examples, a number of typical applications of the chart.

Chapter 14 describes several instruments which incorporate SMITH CHARTS as a basic component, or which are used with SMITH CHARTS to assist in plotting data thereon or in interpreting data therefrom.

For the reader who may desire a more detailed discussion of any particular phase of the theory or application of the chart a bibliography is included to which references are made as appropriate throughout the text.

Fundamental mathematical relationships for the propagation of electromagnetic waves along transmission lines are given in Appendix A and details of the conformal transformation of the original rectangular to the circular SMITH CHART coordinates are included in Appendix B. A glossary of terms used in connection with SMITH CHARTS follows Chap. 14.

Four alternate constructions of the basic SMITH CHART coordinates, printed in red ink on translucent plastic, are supplied in an evelope in the back cover of the book. All of these are individually described in the text. By superimposing these translucent charts on the general-purpose complex waveguide and circuit parameter charts described throughout the book, with which they are dimensionally compatible, it is a simple matter to correlate them graphically therewith and to transfer data or other information from one such plot to the other.

The overlay plots of waveguide parameters used with these translucent SMITH CHARTS include the complex transmission and reflection coefficients for both positive and negative component coordinates, normalized voltage and current amplitude and phase relationships, normalized polar impedance coordinates, voltage and current phase and magnitude relationships, loci of current and voltage probe ratios, L-type matching circuit components, etc. These are generally referred to as "overlays" for the SMITH CHART because they were originally published as transparent loose sheets in bulletin form and because they were so used. However, as a practical matter it was found to be difficult to transfer the parameters or data depicted thereon to the SMITH CHART, which operation is more generally required. Accordingly, they are printed here on opaque bound pages and used as the background on which the translucent SMITH CHARTS in the back cover can be superimposed. The bound background charts are printed in black ink to facilitate visual separation of the families of curves which they portray from the red impedance and/or admittance curves on the loose translucent SMITH CHARTS. The latter charts have a matte finish which is erasable to allow pencil tracing of data or other information directly thereon.

Phillip H. Smith

Acknowledgments

The writer is indebted to many of his colleagues at Bell Telephone Laboratories for helpful discussions and comments, in particular, in the initial period of the development of the chart to the late Mr. E. J. Sterba for his help with transmission line theory, and to Messrs. E. B. Ferrell and the late J. W. McRae for their assistance in the area of conformal mapping. Credit is also due Mr. W. H. Doherty for suggesting the parallel impedance chart, and to Mr. B. Klyce for his suggested use of highly enlarge portions of the chart in determining bandwidth of resonant stubs. Mr. R. F. Tronbarulo's investigations were helpful in writing sections dealing with the negative resistance chart.

The early enthusiastic acceptance of the chart by staff members at MIT Radiation Laboratory stimulated further improvements in design of the chart itself.

Credit for publication of the book at this time is principally due to encouragement provided by Messrs. H. R. Foster and E. E. Crump of Kay Electric Company [14].

Phillip H. Smith

Introduction

I.1 GRAPHICAL VS. MATHEMATICAL REPRESENTATIONS

The physical laws governing natural phenomena can generally be represented either mathematically or graphically. Usually the more complex the law the more useful is its graphical representation. For example, a simple physical relationship such as that expressed by Ohm's law does not require a graphical representation for its comprehension or use, whereas laws of spherical geometry which must be applied in solving navigational problems may be sufficiently complicated to justify the use of charts for their more rapid evaluation. The ancient astrolabe, a Renaissance version of which is shown in Fig. I.1, provides an interesting example of a chart which was used by mariners and astronomers for over 20 centuries, even though the mathematics was well understood.

The laws governing the propagation of electromagnetic waves along transmission lines are basically simple; however, their mathematical representation and application involves hyperbolic and exponential functions (see Appendix A) which are not readily evaluated without the aid of charts or tables. Hence these physical phenomena lend themselves quite naturally to graphical representation.

Tables of hyperbolic functions published by A. E. Kennelly [3] in 1914 simplified the mathematical evaluation of problems relating to guided wave propagation in that period, but did not carry the solutions completely into the graphical realm.

I.2 THE RECTANGULAR TRANSMISSION LINE CHART

The progenitor of the circular transmission line chart was rectangular in shape. The original rectangular chart devised by the writer in 1931 is shown in Fig. I.2. This particular chart was intended only to assist in the solution of the mathematics which applied to transmission line problems inherent in the design of directional shortwave antennas for

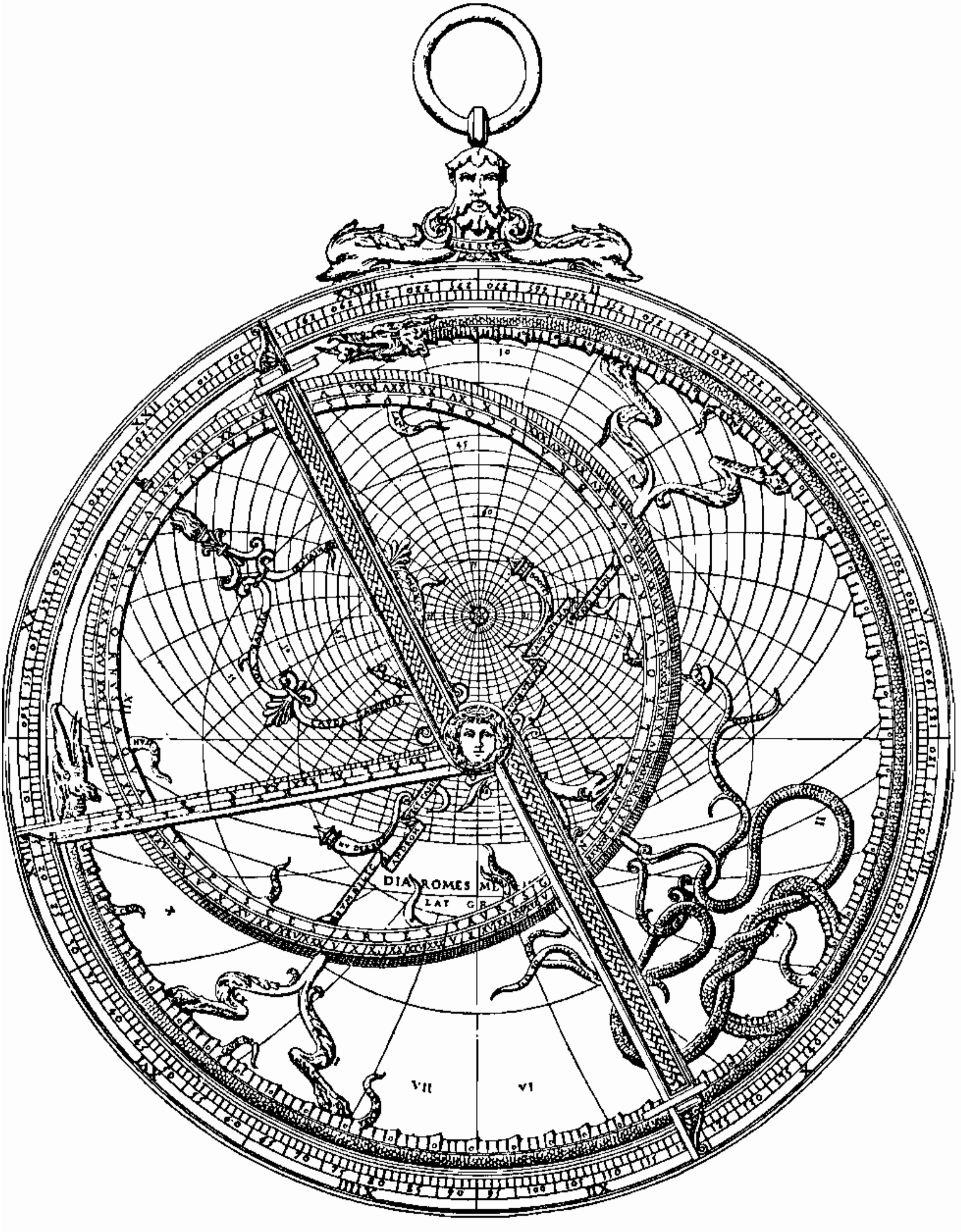


Fig. I.1. **A Renaissance version of the oldest scientific instrument in the world.** (*Danti des Renaldi, 1940.*)

Bell System applications of that period; its broader application was hardly envisioned at that time.

The chart in Fig. I.2 is a graphical plot of a modified form of J. A. Fleming's 1911 "telephone" equation [2], as given in Chap. 2 and in Appendix A, which expresses the impedance characteristics of high-frequency transmission lines in terms of measurable effects of electromagnetic waves propagating thereon, namely, the standing-wave amplitude ratio and wave position. Since this chart displays impedances whose complex components are "normalized," i.e., expressed as a fraction of the characteristic impedance of the transmission line under consideration, it is applicable to all types of waveguides, including open-wire and coaxial transmission lines, independent of their characteristic impedances. In fact, it is this impedance normalizing concept which makes such a general plot possible.

Although larger and more accurate rectangular charts have subsequently been drawn, their uses have been relatively limited because of the limited range of normalized impedance values and standing-wave amplitude ratios which can be represented thereon. This stimulated several attempts by the writer to transform the curves into a more useful arrangement, among them the chart shown in Fig. 7.7 which was constructed in 1936.

I.3 THE CIRCULAR TRANSMISSION LINE CHART

The initial clue to the fact that a conformal transformation of the circular orthogonal curves of Fig. I.2 might be possible was provided by the realization that these two families of circles correspond exactly to the lines of force and the equipotentials surrounding a pair of equal and opposite parallel line charges, as seen in Fig. I.1. It was then a simple matter to show that a bilinear conformal transformation [55,109] would, in fact, produce the desired results (see Appendix B), and the circular form of chart shown in Fig. I.3, which retained the normalizing feature of the rectangular chart of Fig. I.2, was subsequently devised and constructed. All possible impedance values are representable within the periphery of this later chart. An article describing the impedance chart of Fig. I.3 was published in January, 1939 [101].

During World War II at the Radiation Laboratory of the Massachusetts Institute of Technology, in the environment of a flourishing microwave development program, the chart first gained widespread acceptance and publicity, and first became generally referred to as the SMITH CHART.

Descriptive names have in a few instances been applied to the SMITH CHART (see glossary) by other writers; these include "Reflection Chart," "Circle Diagram (of Impedance)," "Immittance Chart," and "Z-plane Chart." However, none of these are in themselves sufficiently

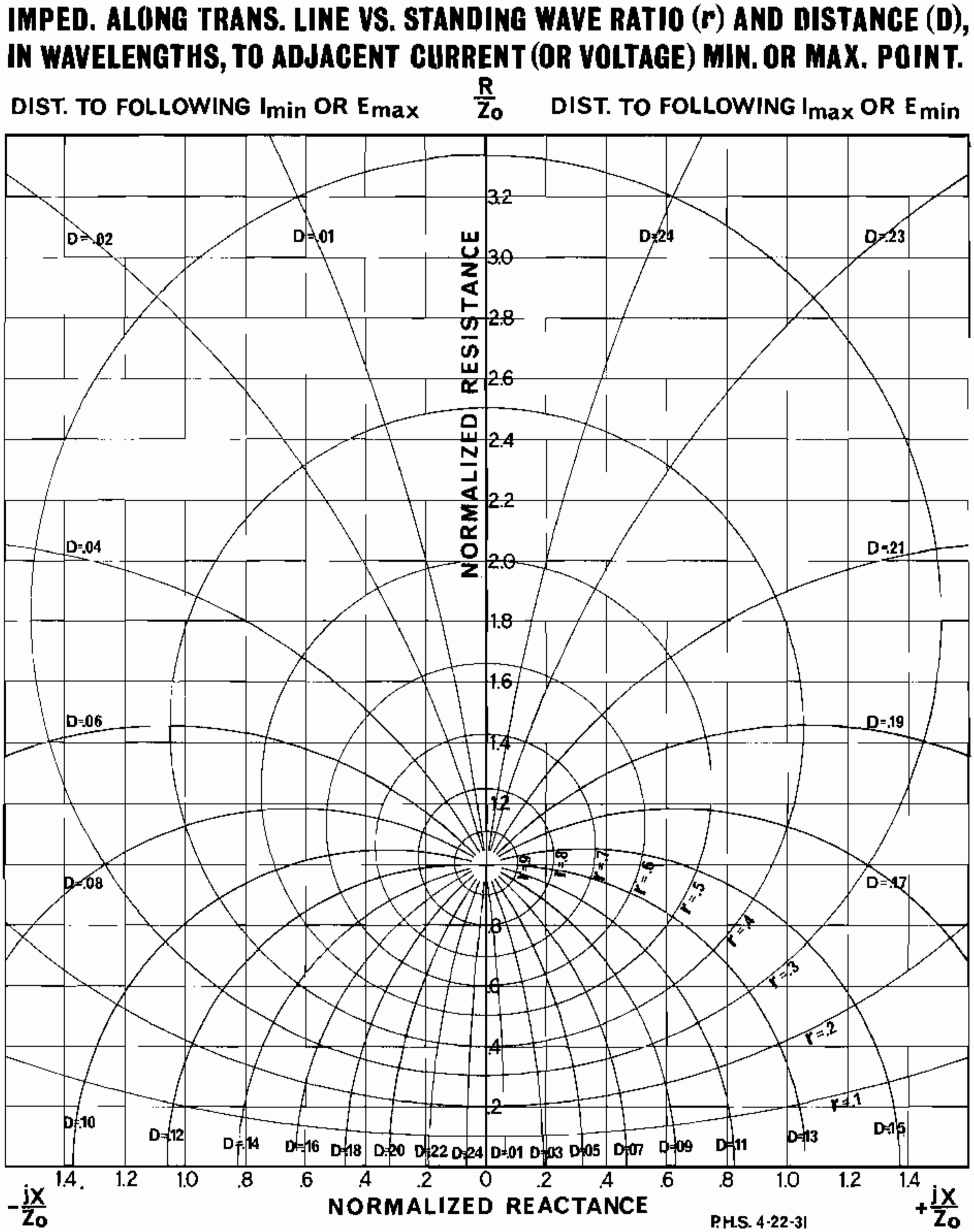


Fig. I.2. The original rectangular transmission line chart.

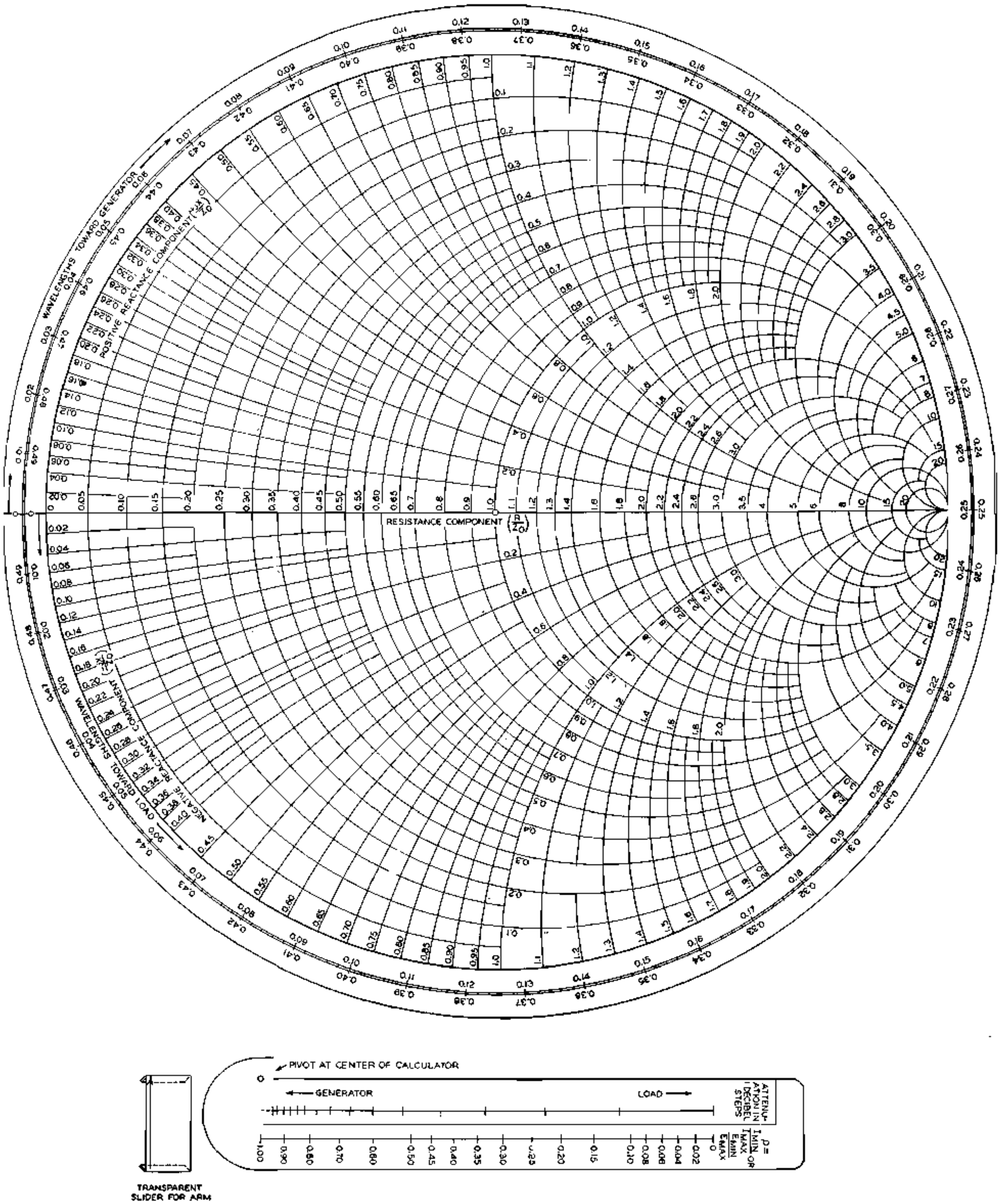


Fig. I.3. Transmission line calculator. (*Electronics, January, 1939.*)

definitive to be used unambiguously when comparing the SMITH CHART with similar charts or with its overlay charts as discussed in this text. For these reasons, without wishing to appear immodest, the writer has decided to use the more generally accepted name in the interest of both clarity and brevity.

Drafting refinements in the layout of the impedance coordinates were subsequently made and additional scales were added showing the relation of the reflection coefficient to the impedance coordinates, which increased the utility of the chart. These changes are shown in Fig. I.4. A second article published in 1944 incorporated these improvements [102]. This later article also described the dual use of the chart coordinates for impedances and/or admittances, and for converting series components of impedance to their equivalent parallel component values.

In 1949 the labeling of the chart impedance coordinates was changed so that the chart would display directly either normalized impedance or normalized admittance. This change is shown in the chart of Fig. 2.3. On this later chart the specific values assigned to each of the coordinate curves apply, optionally, to either the impedance or to the admittance notations.

In 1966 additional radial and peripheral scales were added to portray the fixed relationship of the complex transmission coefficients to the chart coordinates, as shown in Fig. 8.6.

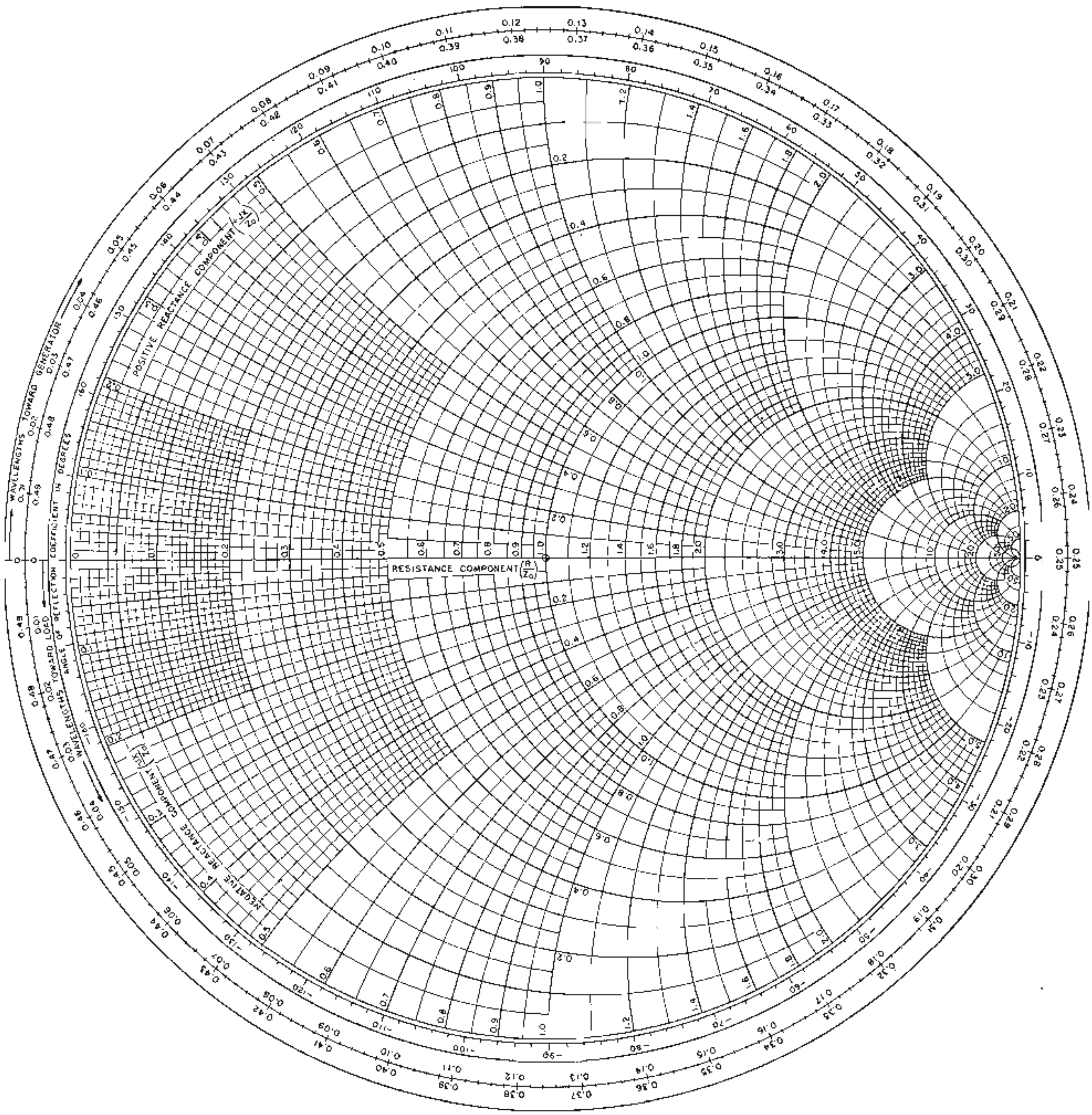
I.4 ORIENTATION OF IMPEDANCE COORDINATES

The charts in Figs. I.2 and I.3 as originally plotted have their resistance axes vertical. It became apparent shortly after publication of Fig. I.3, as thus oriented, that a horizontal representation of the resistance axis was preferable since this conformed to the accepted convention represented by the Argand diagram in which complex numbers ($x \pm iy$) are graphically represented with the real (x) component horizontal and the imaginary (y) component vertical.

Therefore, subsequently published SMITH CHARTS have generally been shown, and are shown throughout the remainder of this book, with the resistance (R) axis horizontal, and the reactance ($\pm jX$) axis vertical; inductive reactance ($+ jX$) is plotted above, and capacitive reactance ($- jX$) below the resistance axis.

I.5 OVERLAYS FOR THE SMITH CHART

Axially symmetric overlays for the SMITH CHART were inherent in the first chart, as represented by the peripheral and radial scales



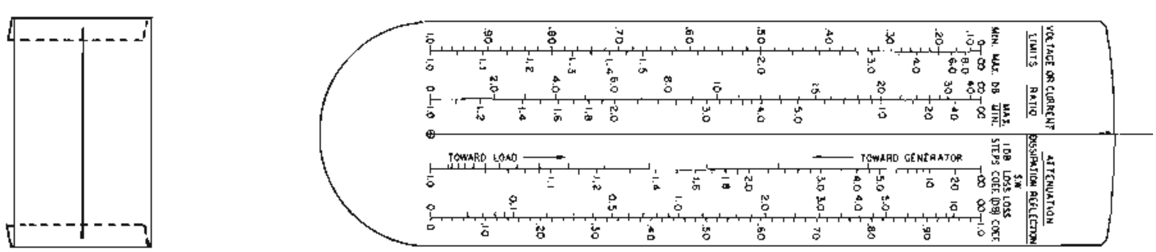


Fig. I.4. Improved transmission line calculator. (*Electronics, January, 1944.*)

for the chart coordinates. These overlays include position and amplitude ratio of the standing waves, and magnitude and phase angle of the reflection coefficients. Additionally, overlays showing attenuation and reflection functions were represented by radial scales alone (see Fig. I.4).

In the present text 26 additional general-purpose overlays (both symmetrical and asymmetrical) for which useful applications exist and which have been devised for the SMITH CHART are presented.

Contents

CHAPTER 1

Guided Wave Propagation

1.1 GRAPHICAL REPRESENTATION

The SMITH CHART is, fundamentally, a graphical representation of the interrelationships between electrical parameters of a uniform waveguide. Accordingly, its design and many of its applications can best be described in accordance with principles of guided wave propagation.

The qualitative descriptions of the electrical behavior of a waveguide, as presented in this chapter, will provide a background for better understanding the significance of various interrelated electrical parameters which are more quantitatively described in the following chapter. As will be seen, many of these parameters are represented directly by the coordinates and associated scales of a SMITH CHART, and their relationships are basic to its construction.

1.2 WAVEGUIDE STRUCTURES

The term *waveguide*, as used in this book, will be understood to include not only hollow cylindrical *uniconductor waveguides*, but all other physical structures used for guiding electromagnetic waves (except, of course, heat and light waves). Included are multi-wire transmission line, strip-line, coaxial line, triplate, etc. Waveguide terms as they first appear in the text will be italicized to indicate that their definitions are in accordance with definitions which have been standardized by the Institute of Electrical and Electronics Engineers [11], and are currently accepted by the United States of America Standards Institute (USASI). (Also, terms and phrases are sometimes italicized in lieu of underlining to provide emphasis or to indicate headings or subtitles.)

Although it may ultimately be of considerable importance in the solution of any practical waveguide problem, the particular waveguide structure is of interest in connection with the design or use of the SMITH CHART only to the extent that its configuration, cross-sectional dimensions in wavelengths, and *mode of propagation* establish two basic electrical constants of the waveguide, viz., the *propagation constant* and the *characteristic impedance*. Both of these constants are further discussed in Chap. 2.

1.3 WAVEGUIDE WAVES

Waveguide waves can propagate in numerous *modes*, the exact number depending upon the configuration and size of the conductor (or conductors) in wavelengths. Modes of propagation in a waveguide are generally described in terms of the electric and magnetic field pattern in the vicinity of the conductors, with which each possible mode is uniquely associated.

As is the case for the waveguide structure, the mode of propagation does not play a direct role in the design or use of the SMITH CHART. Its importance, however, lies in the fact that each mode is characterized by a different value for the propagation constant and characteristic impedance to which the variables of the problem must ultimately be related.

A specific waveguide structure may provide the means for many different modes of propagation although only one will generally be selected for operation. Field patterns for the more common dominant mode in a two-wire and in a coaxial transmission line are shown in Figs. 1.1 and 1.2, respectively. In this mode both electric and magnetic field components of the wave lie entirely in planes

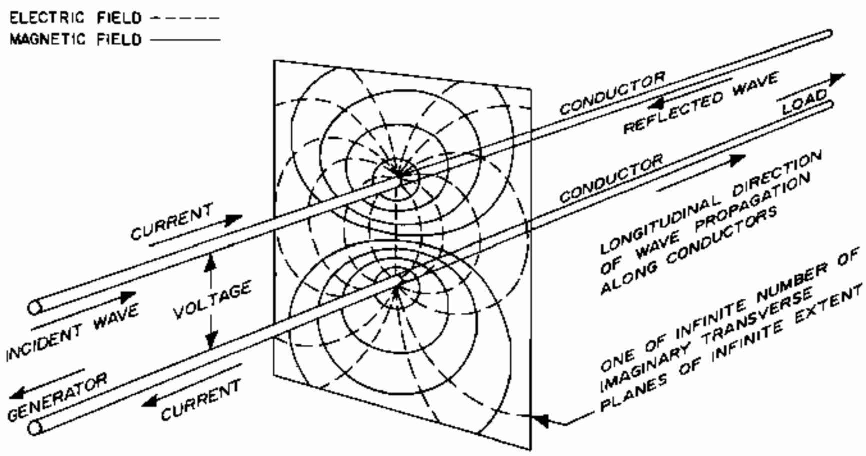


Fig. 1.1. Dominant mode field pattern on parallel wire transmission line.

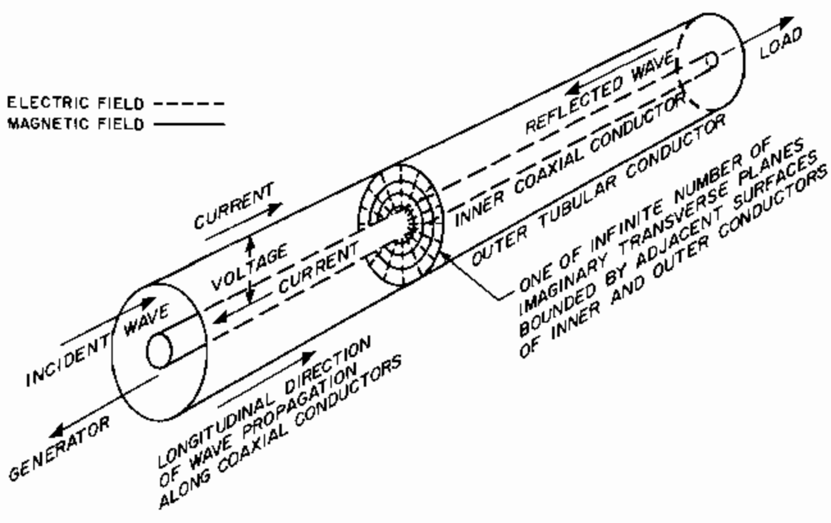


Fig. 1.2. Dominant mode field pattern on coaxial transmission line.

transverse to the *direction of propagation*, and the wave is, therefore, called a *transverse electromagnetic (TEM) wave.* There is no longitudinal component of the field in this mode.

When the mode of propagation is known in a particular uniconductor waveguide, and operation is at a particular frequency, the waveguide wavelength can readily be computed. The SMITH CHART can then be used in the same way in which it is used for problems involving the simple TEM wave in two-wire or coaxial transmission lines.

A further discussion of the subject of propagation modes and their associated field patterns will not be undertaken herein since the reader who may be interested will find adequate discussions of this subject in the literature [32,52].

1.4 TRAVELING WAVES

The propagation of electromagnetic wave energy along a waveguide can perhaps best be explained in terms of the component traveling waves thereon.

If a continuously alternating sinusoidal voltage is applied to the input terminals of a waveguide, a forward-traveling voltage wave will be instantly launched into the waveguide. This wave will propagate along the guide as a continuous wave train in the only direction possible, namely, toward the load, at the characteristic wave velocity of the waveguide. Simultaneously with the generation of a forward-traveling voltage wave, an accompanying forward-traveling current wave is engendered, which also propagates along the waveguide. The forward-traveling current wave is in phase with the forward-traveling voltage wave at all positions along a lossless waveguide. These two component waves make up the forward-traveling electromagnetic wave.

1.5 SURGE IMPEDANCE

The input impedance that the forward-traveling electromagnetic wave encounters as it propagates along a uniform waveguide is called the *surge impedance* or the *initial sending-end impedance.* In the case of a uniform waveguide it is called, specifically, the *characteristic impedance.* The input impedance has, initially, a constant value independent of position along the waveguide. Its magnitude is independent of the magnitude or of the phase angle of the load reflection coefficient, it being assumed that in this brief interval the forward-traveling electromagnetic wave has not yet arrived at the load terminals. At any given position along the waveguide the forward-traveling electromagnetic wave has a sinusoidal amplitude variation with time as the wave train passes this position.

1.6 ATTENUATION

As the forward-traveling voltage and current waves propagate along the waveguide toward the load, some power will be continuously dissipated along the waveguide. This is due to the distributed series resistance of the conductor (or conductors) and to the distributed shunt leakage resistance encountered, respectively, by the longitudinal currents and the transverse displacement currents in the dielectric medium between conductors. This dissipation of power is called *attenuation*, or one-way transmission loss. Attenuation does not change the initial input impedance of the waveguide as seen by the forward-traveling wave energy as it progresses along the waveguide, since it is uniformly distributed, but it nevertheless diminishes the wave power with advancing position along the waveguide. Attenuation should not be confused with the total dissipation of a waveguide measured under steady-state conditions, as will be explained later.

Upon arrival at the load, the forward-traveling voltage and current wave energy will encounter a load impedance which may or may not be of suitable value to "match" the characteristic impedance of the waveguide and, thereby, to absorb all of the energy which these waves carry with them.

1.7 REFLECTION

Assume for the moment that the load impedance matches* the characteristic impedance of the waveguide, and that the load, therefore, absorbs all of the wave energy impinging thereon. There will then be no reflection of energy from the load. In this case the forward-traveling wave will be the only wave along the waveguide. A voltmeter or ammeter placed across, or at any position along the waveguide, respectively, would then indicate an rms value in accordance with Ohm's law for alternating currents in an impedance which is equivalent to the characteristic impedance of the waveguide.

If, on the other hand, the load impedance does not match the characteristic impedance of the waveguide, and therefore does not absorb all of the incident electromagnetic wave energy available, there will be reflected wave energy from the mismatched load impedance back into the waveguide. This results in a rearward-traveling current and voltage wave. The complex ratio of the rearward-traveling voltage wave to the forward-traveling voltage wave at the load is called the *voltage reflection coefficient.* Likewise, the complex ratio of the rearward-traveling current wave to the forward-traveling current wave at the load is called the *current reflection coefficient.* If the waveguide is essentially lossless, the magnitude of the voltage reflection coefficient will be constant at all points from the load to the generator and will be equal to the voltage reflection coefficient magnitude at the load. If there is attenuation along the waveguide, the magnitude of the voltage reflection coefficient will gradually diminish with distance toward the generator, due to the additional attenuation encountered by the reflected-wave energy, as compared to that encountered by the incident-wave energy in its shorter path. Thus, there will appear to be less reflected energy at the input to such a waveguide than at the load.

The voltage reflection coefficient will also vary in phase, with position along the waveguide. The phase angle of the voltage reflection coefficient at any point along a waveguide is determined by the phase shift undergone by the reflected voltage wave in comparison to that of the incident voltage wave at the point under consideration, including any phase change at the load itself. If the reflection coefficient phase angle exceeds 180° (representing a path difference of more than one-half wavelength between reflected- and incident-wave energy), the relative phase angle of the voltage reflection coefficient only is usually of interest. This is obtainable by subtracting the largest possible integer multiple of 180° (corresponding to integer lengths of one-quarter wavelengths of waveguide) from the absolute or total phase angle of the voltage reflection coefficient, to yield a relative phase angle between plus and minus 180°.

The traveling current waves on a waveguide, which accompany traveling voltage waves, will likewise be reflected by a mismatched load impedance. The magnitude of the current reflection coefficient at all points along a waveguide is identical to that of the voltage

*If the waveguide is lossy its characteristic impedance will be complex, that is, $R_0 - jX_0$. For a conjugate match, the condition which provides maximum power transfer, the load impedance, should be $R_l + jX_l$, where $R_l = R_0$ and $X_l = X_0$.

reflection coefficient in all cases. The phase angle of the current reflection coefficient at any given point along the waveguide may, however, differ from that of the voltage reflection coefficient by as much as $\pm 180^\circ$, depending upon the relative phase changes undergone by the two traveling waves at the load.

An open-circuited load will, for example, cause complete reflection of both current and voltage incident traveling waves. The voltage and current reflection coefficient magnitude at the load will both be unity in this case. The phase angle of the voltage reflection coefficient at the load will be zero degrees under this condition, since the same voltage is reflected (as a wave) back into the waveguide at this point. However, the current reflection coefficient phase angle at the open-circuited load will be -180° since the incident current wave amplitude upon arriving at such a load will suddenly have to drop to zero (there being no finite value of load impedance) and will build up again in opposite polarity to be relaunched as a reflected wave into the waveguide.

The collapse of the magnetic field attending the incident current wave, when it encounters an open-circuited load, results in an accompanying buildup of the electric field and in a corresponding buildup of voltage at the open-circuited load to exactly twice the value of the voltage attending the incident voltage wave. This is because electromagnetic wave energy along a uniform waveguide is divided equally between the magnetic and the electric fields. The reflected wave voltage builds up in phase with the incident wave voltage and the voltage reflection coefficient phase angle is, accordingly, zero degrees at this point.

1.8 STANDING WAVES

At this point in the order of events, the load is in the process of engendering *standing waves* along the waveguide, for there suddenly appears a maximum voltage and a minimum current at the load terminals, when just prior to this, the voltage and current at all points along the line were constant (except for the diminishing effects of attenuation). The modified load voltage now launches back into the waveguide a rearward-traveling voltage and current wave in a manner similar to the initial launching of these waves at the generator end. These rearward-traveling waves combine with the respective forward-traveling waves and, because of their relative phase differences at various positions along the line (changing phase angle of voltage and current reflection coefficients), cause alternate reinforcement and cancellation of the voltage and current distribution along the line. This phenomenon results in what has previously been referred to as standing or stationary waves along the waveguide. The shape of these standing waves with position along a waveguide is shown in Fig. 1.3. It will be seen that their shape is sinusoidal only in the limiting case of complete reflection from the load, i.e., when the *standing wave ratio* of maximum to minimum is infinity. A graphical representation of the combination of the two traveling waves is shown in Fig. 1.4.

Upon arrival back at the generator terminals, the rearward-traveling voltage wave combines in amplitude and phase with the voltage being generated at the time, to produce a change in the generated voltage amplitude and phase. At this instant, the generator is first presented with a change in the waveguide input impedance and readjusts its output accordingly. This is the beginning in time sequence of a series of regenerated waves at each end of the waveguide, which after undergoing multiple reflections therefrom, eventually combine to produce steady-state conditions and become, in effect, a single forward-traveling and a single rearward-traveling current and voltage wave. Thus, in practice, it is not usually

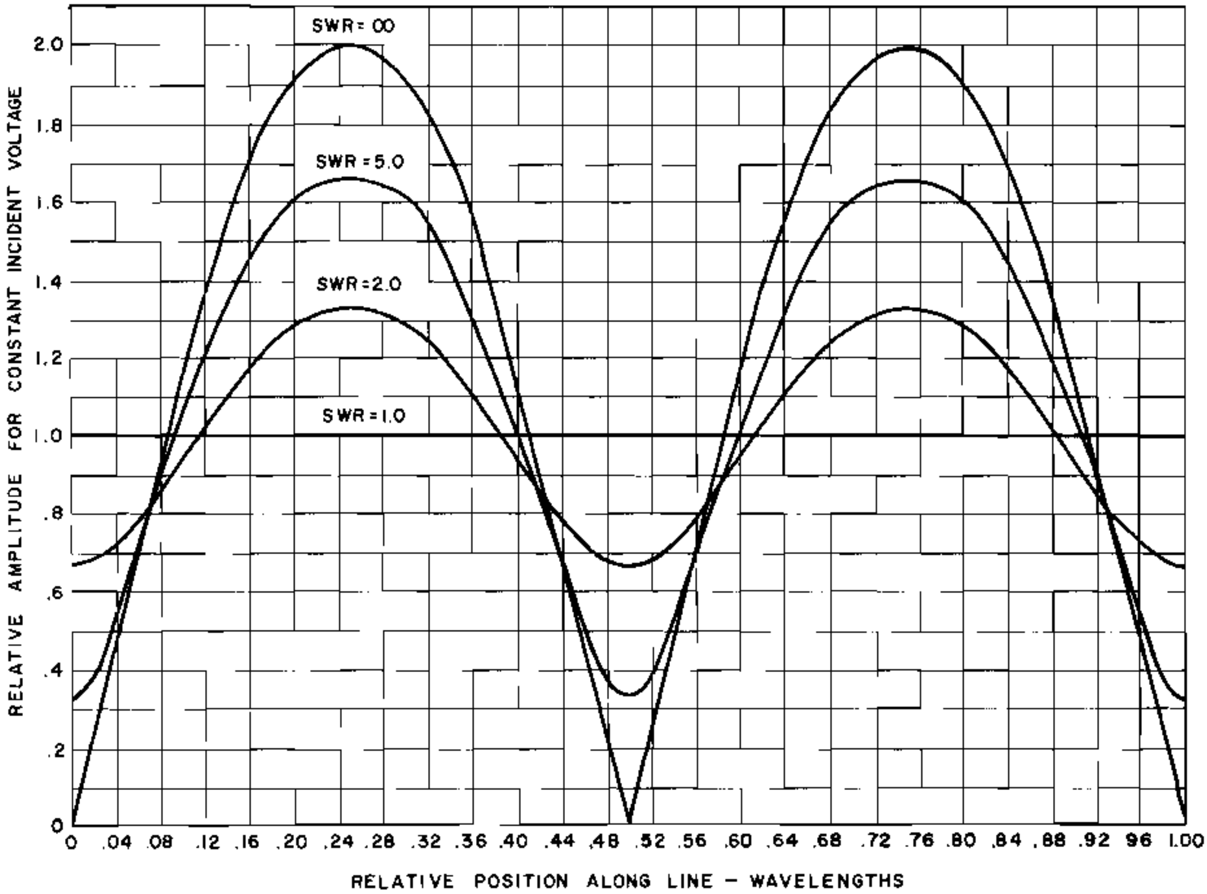


Fig. 1.3. Relative amplitudes and shapes of voltage or current standing waves along a lossless waveguide (constant input voltage).

necessary to consider transient effects of multiple reflections between generator and load beyond that of a single reflection at the load end of the waveguide.

The magnitude and phase angle of the voltage and current reflection coefficient bear a direct and inseparable relationship to the amplitude and position of the attending standing waves of voltage or current along the waveguide, as well as to the input impedance (or admittance) at all positions along the waveguide. Through suitable overlays this relationship can very simply be described on the SMITH CHART.

As a specific example of this relationship, the magnitude of the voltage reflection coefficient at the open-circuited load previously considered is unity. Its phase angle is zero degrees. The accompanying standing voltage wave has a maximum-to-minimum wave amplitude ratio of infinity. The position of the maximum point of the voltage standing wave is at the open-circuited load terminals of the waveguide, at which point the input impedance is infinity.

Expressed algebraically, in terms of the amplitude of the incident i and reflected r traveling waves, the standing wave ratio S is seen to be their sum divided by their difference, i.e.,

$$S = \frac{i + r}{i - r} \tag{1-1}$$

and when $i \to r$, $S \to \infty$. Also, if the incident voltage or current wave amplitude is held

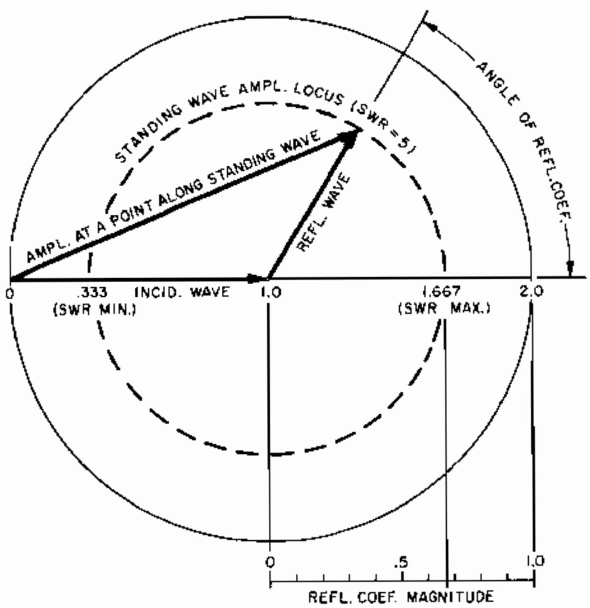


Fig. 1.4. Graphical representation of incident and reflected traveling waves of voltage or current forming standing waves.

constant at unity (the case of a well-"padded" oscillator), as the reflected wave amplitude r changes (accompanying changes in load reflections)

$$S = \frac{1 + r}{1 - r} \tag{1-2}$$

or

$$r = \frac{S - 1}{S + 1} \tag{1-3}$$

The voltage (or current) reflection coefficient magnitude ρ is, by definition, simply

$$\rho = \frac{r}{i} \tag{1-4}$$

PROBLEMS

1-1. Assuming constant incident voltage amplitude, and using the graphical construction in Fig. 1.4, plot the spatial shape of a voltage standing wave whose ratio S is 3.0, along a uniform, loss-free waveguide. Plot points at one-sixteenth-wavelength intervals toward load from a voltage minimum (current maximum) point.

Solution:

As shown in Fig. 1.5:

1. Draw a horizontal axis (0-2.0) of some convenient length, and divide into ten equal parts; label these 0, 0.2, 0.4, etc., to 2.0.

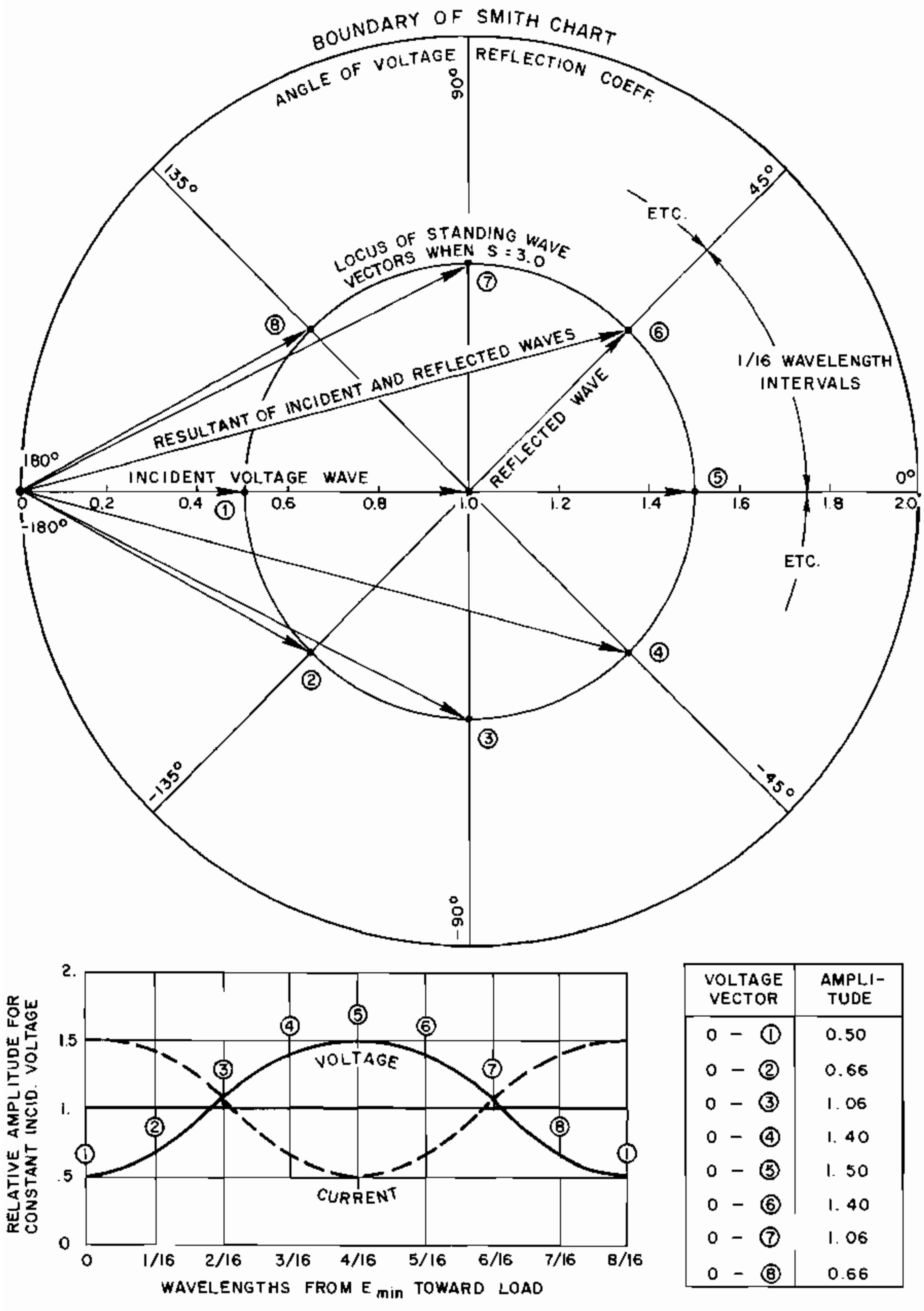
2. Draw a circle, in which the above horizontal axis is a diameter, representing the boundary of a SMITH CHART within which all possible waveguide voltage or current vectors can be represented.

3. Draw three straight lines intersecting the center point (1.0) of the horizontal axis (real axis of the SMITH CHART) at angles of 45, 90, and 135° from the horizontal, corresponding to one-sixteenth-wavelength intervals along a waveguide. (On the SMITH CHART 360° corresponds to one-half wavelengths.)

4. Draw a circle centered at 1.0 on the horizontal axis, representing the locus of the voltage standing wave vector along the waveguide, intersecting two values (1.5 and 0.5) on the horizontal axis, the ratio S of which will be equal to 3.0. (From Eq. (1-3), if $S = 3.0$, $r = 0.5$; and if $i = 1.0$, $i + r = 1.5$, and $i - r = 0.5$.)

5. Draw voltage vectors from each of the intersections of this circle (4) with the respective angle lines (3) to the origin (0) on the horizontal axis, and number as shown in Fig. 1.5. Measure with horizontal axis scale, and tabulate voltage vector lengths 0 - ①, 0 - ②, 0 - ③, etc. to 0 - ⑧.

6. Starting at ① (the voltage minimum point of the standing wave) plot the voltage standing wave on rectangular coordinates from the tabulated amplitudes at eight equally spaced positions (① through ⑧).



VOLTAGE VECTOR	AMPLITUDE
0 – ①	0.50
0 – ②	0.66
0 – ③	1.06
0 – ④	1.40
0 – ⑤	1.50
0 – ⑥	1.40
0 – ⑦	1.06
0 – ⑧	0.66

Fig. 1.5. Solution for Prob. 1-1, construction of standing wave shapes.

1-2. Plot the corresponding current standing wave shape.

Solution:
Construct the dashed curve in Fig. 1.5, exactly duplicating that for the voltage standing wave except with its null displaced one-quarter wavelength toward load, representing a 180° phase shift with respect to the voltage standing wave.

1-3. (a) What is the magnitude and angle of the voltage reflection coefficient at the minimum point (①) of the voltage standing wave?

Solution:
The ratio of the reflected to the incident traveling voltage waves, i.e.,

$$\rho_E = \frac{0.5\angle 180^\circ}{1.0} = 0.5\angle 180^\circ$$

(b) What is the magnitude and angle of the current reflection coefficient at the same point (maximum point (⑤) of the current standing wave)?

Solution:
The vector ratio of the reflected to the incident traveling current waves, i.e.,

$$\rho_I = \frac{0.5\angle 0^\circ}{1.0} = 0.5\angle 0^\circ$$

In summary, as oriented in Figs. 11.1 through 11.4 with respect to the overlay Charts A, B, or C, in the cover envelope, the intersection point of any pair of current ratio curves gives the impedance at P_ℓ, while the intersection point of any pair of voltage ratio curves gives the admittance at P_ℓ. When rotated 180° about their axes with respect to the coordinates of the overlay charts A, B, or C, the intersection point of any pair of current ratio curves gives the admittance at P_ℓ, while the intersection point of any pair of voltage ratio curves gives the impedance at P_ℓ.

11.3 CONSTRUCTION OF PROBE RATIO OVERLAYS

Information is given in Fig. 11.5 for plotting probe ratios which correspond to any desired probe separation. All such plots require only straight lines and circles for their construction. The outer boundary of such a construction corresponds to the boundary of the SMITH CHART coordinates.

As shown in Fig. 11.5 the separation of any two sampling points S/λ determines the angle α as measured from the horizontal R/Z_0 axis. This angle establishes the position of a straight line through the center of the construction which represents the locus of impedances at the probe position P_ℓ when the current standing wave ratio is varied from unity to infinity while the wave is maintained in such a position along the transmission line with respect to the position of the two sampling points that they always read alike, that is, that $P_g/P_\ell = 1.0$.

A construction line perpendicular to the locus $P_g/P_\ell = 1$ and passing through the infinite resistance point on the R/Z_0 axis will then lie along the center of all of the P_g/P_ℓ circles which it may be desired to plot.

The ratio of each of these circular arcs (R_1 and R_2) which corresponds to a particular current ratio, and the distance of their centers from the chart rim (D_1 and D_2), is given by the formulas in Fig. 11.5 as a function of the ratio of P_g to P_ℓ and the SMITH CHART radius R.

CHAPTER 2

Waveguide Electrical Parameters

2.1 FUNDAMENTAL CONSTANTS

Two fundamental waveguide constants, the *characteristic impedance* and the *propagation constant,* will next be discussed in terms of traveling voltage and current waves, as well as in terms of primary circuit elements. The relationship of these two waveguide constants to the *normalized input impedance* characteristics of a waveguide will be shown. This relationship is the basis for the coordinate arrangement of the SMITH CHART. Following this, the use of the SMITH CHART coordinates in converting from normalized impedances to normalized admittances will be described.

2.2 PRIMARY CIRCUIT ELEMENTS

It is well known that the impedance characteristics of any electrical circuit may be completely described in terms of four primary circuit elements: resistance R, inductance L, capacitance C, and conductance G.

Waveguides may be regarded as specific forms of electrical circuits composed, basically, of these four primary circuit elements. If the waveguide is of uniform configuration along its length the primary circuit elements are uniformly distributed and, what is equally if not more important, are always related one to the other as constant ratios, such as the ratio L/G, R/G, etc., per unit length of waveguide. One may develop directly from these primary circuit elements the two previously mentioned fundamental waveguide constants by any one of several methods. The results only will be given here.

To the same extent that four primary circuit elements are required and used for analysis of circuit impedance characteristics, the two fundamental waveguide constants, characteristic impedance and propagation constant, are required and used for analysis of waveguide impedance characteristics. The importance of these two waveguide constants cannot be overemphasized. They provide a means for completely expressing the impedance characteristics of any uniform waveguide in relation to its length and terminating impedance.

Each of the two fundamental waveguide constants is, in general, a complex quantity having a real and an imaginary component. It is possible, and from a graphical point of view useful, to attach a physical significance to these constants, as well as to their real and imaginary parts, as will be seen.

2.3 CHARACTERISTIC IMPEDANCE

The complex characteristic impedance is a waveguide constant which is equivalent to the initial input (or surge) impedance encountered by the forward-traveling electromagnetic wave along a uniform waveguide. (This was discussed in Chap. 1.) Stated another way, it is the ratio of the voltage in the forward-traveling voltage wave to the current in the forward-traveling current wave.

Although the characteristic impedance of a waveguide has the dimensions and some of the properties of an impedance, it is important to note that these properties are not physically equivalent to the impedance properties of a single-port circuit. For example, although it has a real and an imaginary part, its real part, *per se,* does not dissipate energy nor does its imaginary part store energy.

The characteristic impedance Z_0 of any uniform waveguide, in terms of the aforementioned distributed primary circuit constants, may be expressed as

$$Z_0 = \left(\frac{R + j\omega L}{G + j\omega C}\right)^{1/2} \tag{2-1}$$

where ω is 2π times the frequency f in Hz.

At all radio frequencies the resistance R per unit length of practical waveguides is generally negligible in comparison with the inductive reactance $j\omega L$ per this same unit length of waveguide. Likewise, the conductance G per unit length is negligible in relation to the capacitive susceptance $j\omega C$ per unit length. The radio frequency characteristic impedance of a uniform waveguide is, therefore, frequently expressed by the simpler relationship

$$Z_0 \cong \left(\frac{L}{C}\right)^{1/2} \tag{2-2}$$

This latter expression is seen to be independent of frequency. A small imaginary component, which would be contributed to the characteristic impedance by loss terms, may generally be neglected in computations involving the impedance characteristics of high-frequency waveguides. However, if more accuracy is required, Eq. (2-1) may be expanded to give the following expression for the high-frequency characteristic impedance of a waveguide in complex form [50]. This expression neglects only those terms above the second powers in R and G:

$$Z_0 \cong \left(\frac{L}{C}\right)^{1/2}\left[1 + \left(\frac{R^2}{8\omega^2 L^2} - \frac{3G^2}{8\omega^2 C^2} + \frac{RG}{4\omega^2 LC}\right) - j\left(\frac{R}{2\omega L} - \frac{G}{2\omega C}\right)\right] \tag{2-3}$$

Although the characteristic impedance of any waveguide is definable in terms of its electrical parameters, its real part can, particularly in the case of transmission line-type waveguides, also be expressed in terms of its physical configuration, and the dimensions of the conductors of which a given waveguide is composed. This is seen to be the case since such physical properties will establish the primary circuit element values.

Nomographs from which the characteristic impedance of coaxial and balanced-to-ground two-wire transmission lines may be obtained from the physical size and spacing of conductors are shown in Figs. 2.1 and 2.2, respectively. Similar monographs could, of course, be drawn for other conductor configurations. Formulas for less common configurations are available in handbooks [35].

The characteristic impedance of uniconductor waveguides is not as clearly defined as that of transmission line-types. Three approved methods exist for defining the characteristic impedance of lossless rectangular uniconductor waveguides in which the dominant (TE_{10}) wave is propagated. These methods yield slightly different results, all of

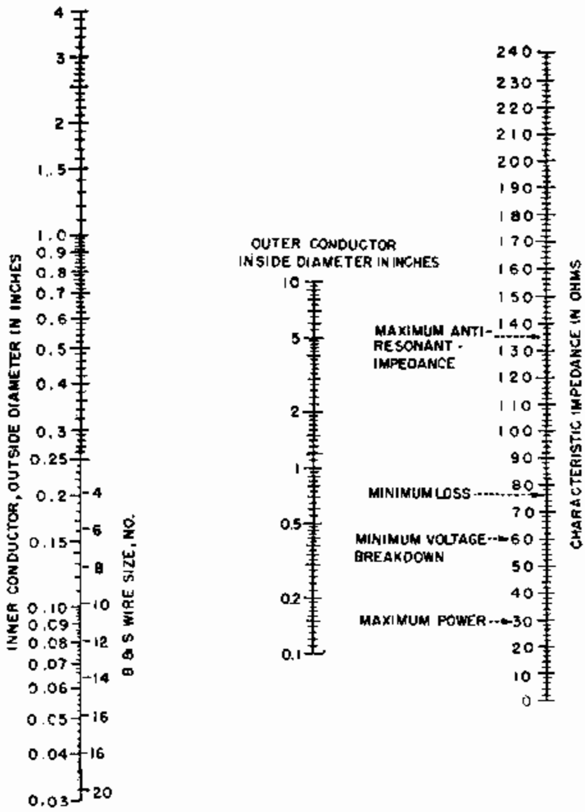


Fig. 2.2. Characteristic impedance of coaxial lines.

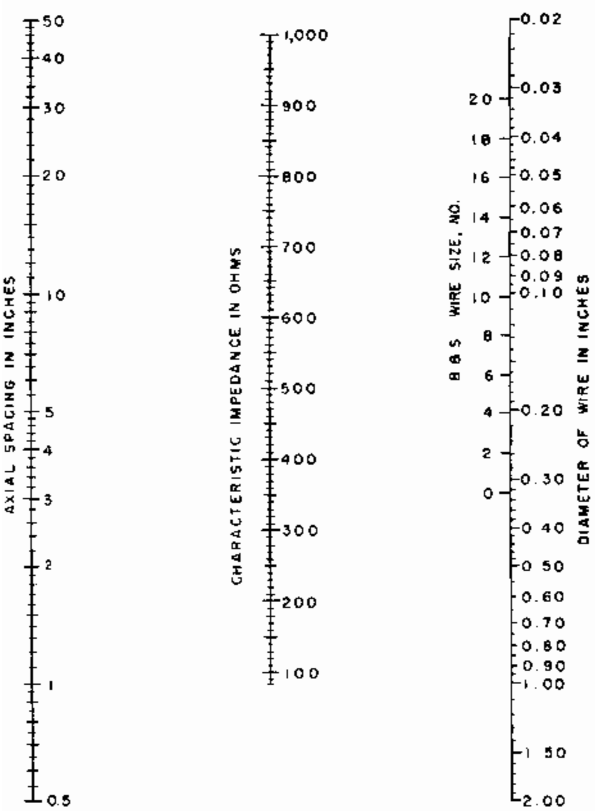


Fig. 2.1. Characteristic impedance of open-wire lines.

which are acceptable if properly used in relation to the electrical parameters from which they are derived.

The evaluation of a specific characteristic impedance for a uniconductor waveguide is further complicated by the fact that it varies with frequency.

Method 1 for evaluating the characteristic impedance $Z_{W,E}$ utilizes the total power W and the maximum rms voltage E:

$$Z_{W,E} = \frac{E^2}{W} \tag{2-4}$$

Method 2 utilizes the total power W and the total current I on a wide face of a rectangular waveguide in the longitudinal direction:

$$Z_{W,I} = \frac{W}{I^2} \tag{2-5}$$

Method 3 utilizes the maximum rms voltage E and the total current I on a wide face of a rectangular waveguide in the longitudinal direction:

$$Z_{E,I} = \frac{E}{I} \tag{2-6}$$

Method 3 yields a value of characteristic impedance which is the geometric mean between the values obtained by methods 1 and 2.

Although in most engineering applications it is important to have some knowledge of the approximate value of the characteristic impedance of the waveguide, in practice the need to determine accurately the characteristic impedance of a waveguide, particularly that of a uniconductor waveguide, is frequently avoided through a process called *normalization*. This is a very useful dodge, which makes it possible to essentially eliminate further consideration of this constant and to represent the impedance characteristics of all types of waveguides on a single set of normalized coordinates on the SMITH CHART. This will be described in more detail in the latter part of this chapter.

2.3.1 Characteristic Admittance

The characteristic admittance of any type of waveguide is the reciprocal of its characteristic impedance. It may therefore be used for normalizing the conductance and susceptance components of the input admittance of the waveguide in the same way that the characteristic impedance is used for normalizing the resistance and reactance components.

2.4 PROPAGATION CONSTANT

The propagation constant of a uniform waveguide is most simply stated as the natural logarithm of the ratio of the input to the output current in the forward-traveling wave, where the input and output terminals are separated by a unit length of the waveguide.

As its name implies, the propagation constant describes the propagation characteristics of electromagnetic waves which may be propagated along a waveguide. These propagation characteristics include the attenuation, and the current and voltage phase relationships.

In general, the propagation constant is a complex number. The real part, expressed in nepers per unit length, is called the *attenuation constant*. This constant determines the energy dissipated in the waveguide per unit length. The imaginary part is called the *phase constant*, or the *wavelength constant*, and is expressed in radians per unit length.

In terms of the primary circuit elements, the complex propagation constant P is

$$\begin{aligned} P &= [(R + j\omega L)(G + j\omega C)]^{1/2} \\ &= \alpha + j\beta \end{aligned} \tag{2-7}$$

The real and imaginary components of the propagation constant α and $j\beta$ are obtained by expanding Eq. (2-7) and separating the real and imaginary parts.

The attenuation constant, in terms of the primary circuit elements per unit length [35], is expressed as:

$$\alpha = \left(\frac{1}{2}\right)^{1/2} \left\{[(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)]^{1/2} + (RG - \omega^2 LC)\right\}^{1/2} \tag{2-8}$$

The attenuation constant at high frequencies (α_{HF}) can be adequately represented by the simpler relationship

$$\alpha_{HF} \longrightarrow \frac{R}{2}\left(\frac{C}{L}\right)^{1/2} + \frac{G}{2}\left(\frac{L}{C}\right)^{1/2} \tag{2-9}$$

When the leakage conductance G can be neglected, as in the case of a uniconductor waveguide operating in its dominant mode at microwave frequencies, the attenuation constant reduces to

$$\alpha \cong \frac{R}{2}\left(\frac{C}{L}\right)^{1/2} \cong \frac{R}{2Z_0} \tag{2-10}$$

The attenuation constant of a high-frequency type of waveguide, such as a two-wire or a coaxial transmission line composed of copper or other conductors of known resistivity, may also be expressed as a function of the conductor dimensions and spacing [10].

The phase constant β determines the wavelength in the waveguide and the velocity of propagation. Expressed in terms of the primary circuit elements per unit length, the phase constant is

$$\beta = \left(\frac{1}{2}\right)^{1/2}\left\{[(R^2 + \omega^2L^2)(G^2 + \omega^2C^2)]^{1/2} + (\omega^2LC - RG)\right\}^{1/2} \tag{2-11}$$

As the frequency is increased to the point where losses are small in relation to L and C, the phase constant expressed in Eq. (2-11) can be represented by the simpler relationship

$$\beta_{HF} \rightarrow \omega(LC)^{1/2} \tag{2-12}$$

A wavelength is defined as the length of line l such that $\beta l = 2\pi$. The length of line corresponding to one wavelength λ is, therefore, equal to $2\pi/\beta$, and β may be expressed as

$$\beta = \frac{2\pi}{\lambda} \tag{2-13}$$

The propagation constant of a waveguide continues to vary as the frequency is increased, and loss terms, which generally increase with frequency, cannot always be neglected (as was generally permissible for the loss terms in the characteristic impedance) if the effect of attenuation on input impedance is to be evaluated.

If losses cannot be neglected, a good approximation for the phase constant β may be obtained by expanding Eq. (2-7) and neglecting all imaginary terms above the second power, to give [50]

$$\beta \cong \omega(LC)^{1/2}\left[1 + \frac{1}{2}\left(\frac{R}{2\omega L} - \frac{G}{2\omega C}\right)^2\right] \tag{2-14}$$

2.5 PARAMETERS RELATED TO SMITH CHART COORDINATES

The coordinates of the SMITH CHART (Fig. 2.3) are basic to its construction. They comprise a set of normalized input resistance and reactance, and/or normalized conductance and susceptance curves of constant values arranged in a unique graphical relationship within a circular boundary. Upon this unique coordinate system all possible values of complex impedance and/or complex admittance may be represented. As will be seen, the relationship of the normalized impedances and admittances will satisfy the conditions encountered along any uniform waveguide.

Other important and useful waveguide parameters will also be seen to be graphically related to these basic coordinates as either peripheral or radial scales, or as asymmetrical overlays. These other parameters and their graphical representations on the SMITH CHART coordinates will be further discussed in subsequent chapters.

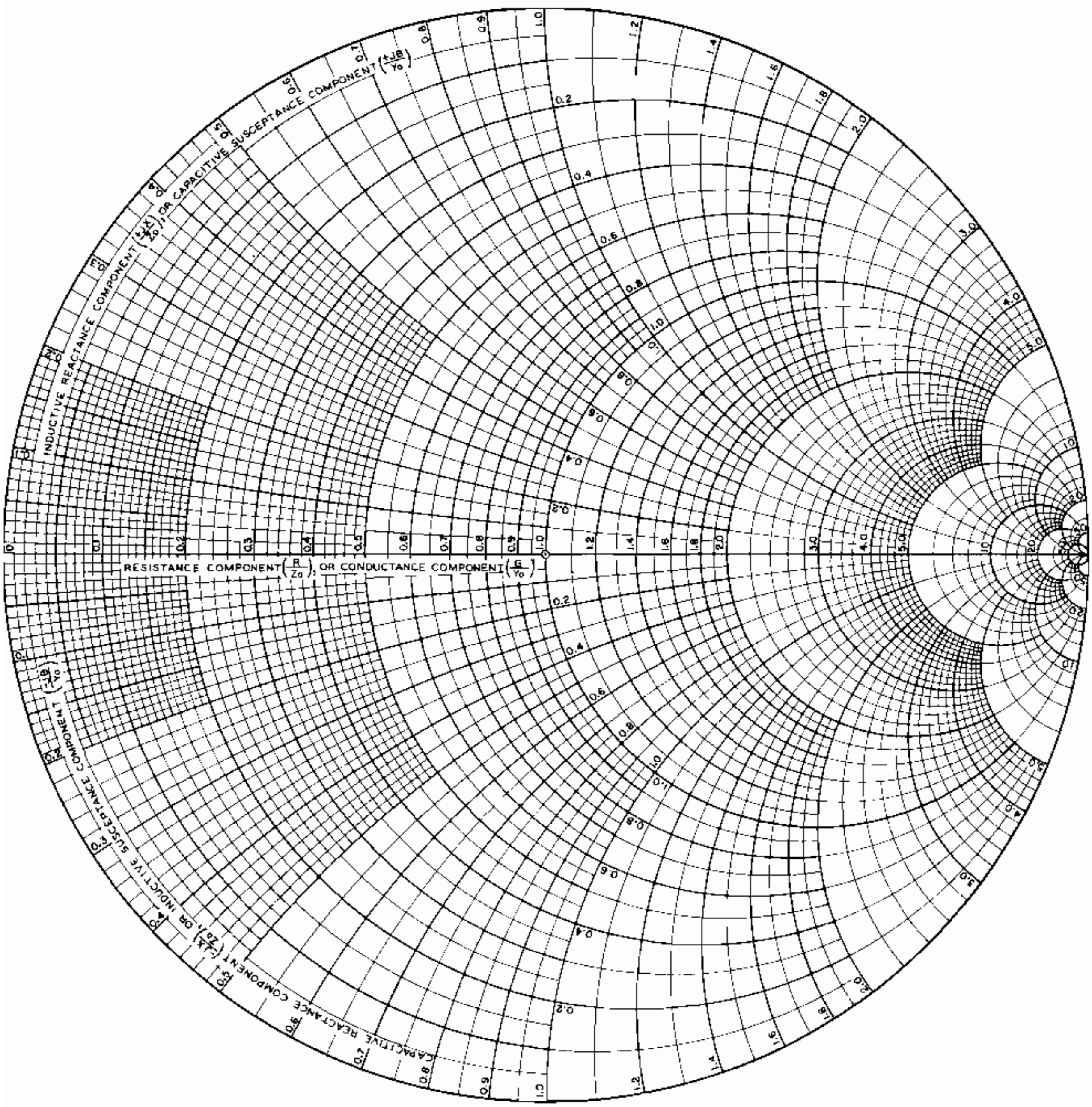


Fig. 2.3. SMITH CHART coordinates displaying rectangular components of equivalent series-circuit impedance (or of parallel-circuit admittance) (overlay for Charts B and C in cover envelope).

2.6 WAVEGUIDE INPUT IMPEDANCE

The input impedance of a waveguide is usually defined as the impedance between the input terminals with the generator disconnected. This is sometimes called the final sending-end impedance when it is desired to stress the fact that steady-state conditions have been established. The input terminals may be selected at any chosen position along the waveguide. Thus, a waveguide may be considered to have an infinite number of input impedances existing simultaneously, since there is an infinite number of positions along any finite length of waveguide.

Waveguide input impedance coordinates are depicted on the central circular area of the chart in normalized form, as described in Sec. 2.8 entitled "Normalization." These coordinates are arranged to portray the series components of the normalized input impedance with respect to a waveguide, as a function of the position of observation.

The input- or sending-end impedance Z_s of any waveguide may be completely expressed in terms of the two fundamental waveguide constants Z_0 and P, the complex load impedance Z_r, and the length l. The relationship [2] is as follows:

$$Z_s = Z_0 \frac{Z_r + Z_0 \tanh Pl}{Z_0 + Z_r \tanh Pl} \tag{2-15}$$

The input impedance Z_s and load impedance Z_r may be normalized to a common characteristic impedance Z_0 by dividing Eq. (2-15) by Z_0 to give

$$\frac{Z_s}{Z_0} = \frac{(Z_r/Z_0) + \tanh(\alpha l + j\beta l)}{1 + (Z_r/Z_0)\tanh(\alpha l + j\beta l)} \tag{2-16}$$

One may simplify this relationship by eliminating the effect of losses on the input impedance computations (except to the extent that these losses involve Z_0) by simply setting α equal to 0. The hyperbolic term ($\tanh j\beta l$) may then be expressed as its trigonometric equivalent ($j \tan\beta l$), and Eq. (2-16) reduces to

$$\frac{Z_s}{Z_0} = \frac{(Z_r/Z_0) + j\tan\beta l}{1 + j(Z_r/Z_0)\tan\beta l} = \frac{(Z_r/Z_0) + j\tan(2\pi l/\lambda)}{1 + j(Z_r/Z_0)\tan(2\pi l/\lambda)} \tag{2-17}$$

This latter relationship involves only three variables, viz., the normalized input impedance Z_s/Z_0, the normalized load impedance Z_r/Z_0, and the length l/λ, expressed as a fractional part of the wavelength. Two of these variables (Z_s/Z_0 and Z_r/Z_0) are complex and each of these is composed of two other variables, viz., the normalized resistance R/Z_0 and the normalized reactance $\pm jX/Z_0$. For any fixed load impedance $R_r/Z_0 \pm jX_r/Z_0$, one may plot on any two-variable coordinate system the locus of magnitudes for the variables R_s/Z_0 and $\pm jX_s/Z_0$ as l/λ is varied.

It will be observed that when l/λ is 0, or any integral number of one-half wavelengths, the input impedance equals the load impedance. It makes an excursion, controlled by the tangent function, and returns exactly to its initial value for every incremental half wavelength added to l/λ, regardless of its initial value. An infinite number of two-variable coordinate systems exists whereupon one could trace the locus of R/Z_0 and $\pm jX/Z_0$, but at this point it is sufficient to say that only one such coordinate system exists whereupon the loci of these variables, as the length changes, are concentric circles which close on themselves in one-half wavelength. These are the coordinates of the SMITH CHART (Fig. 2.3). The derivation will be further discussed in later chapters.

It will be evident from a consideration of the physics of wave propagation (as discussed in Chap. 1) that the input impedance of a waveguide cannot in any degree be changed or affected by the internal impedance of any generator which may be connected across the input terminals of the waveguide. This is so, even though there may be a serious mismatch of impedances between the waveguide characteristic impedance and the generator internal impedance. The generator impedance, therefore, does not have to be considered in a steady-state analysis of waveguide propagation characteristics, except as it affects the level of power available from the generator to the waveguide. This is a rather important point to bear in mind as it is quite commonly not fully appreciated.

The input impedance terminals of a uniconductor waveguide, like the terminals of the primary circuit elements of which the waveguide is composed, are somewhat undefinable. However, the input impedance concept is still a useful one. Where energy is propagated in the dominant mode (Fig. 2.4), the accuracy is sufficient for many engineering evaluations, if one regards the waveguide terminals as being located at the centers of the wide interior faces of the waveguide, between which the voltage is a maximum in any given transverse plane.

2.7 WAVEGUIDE INPUT ADMITTANCE

The final input admittance, or simply input admittance of a waveguide, is the reciprocal of

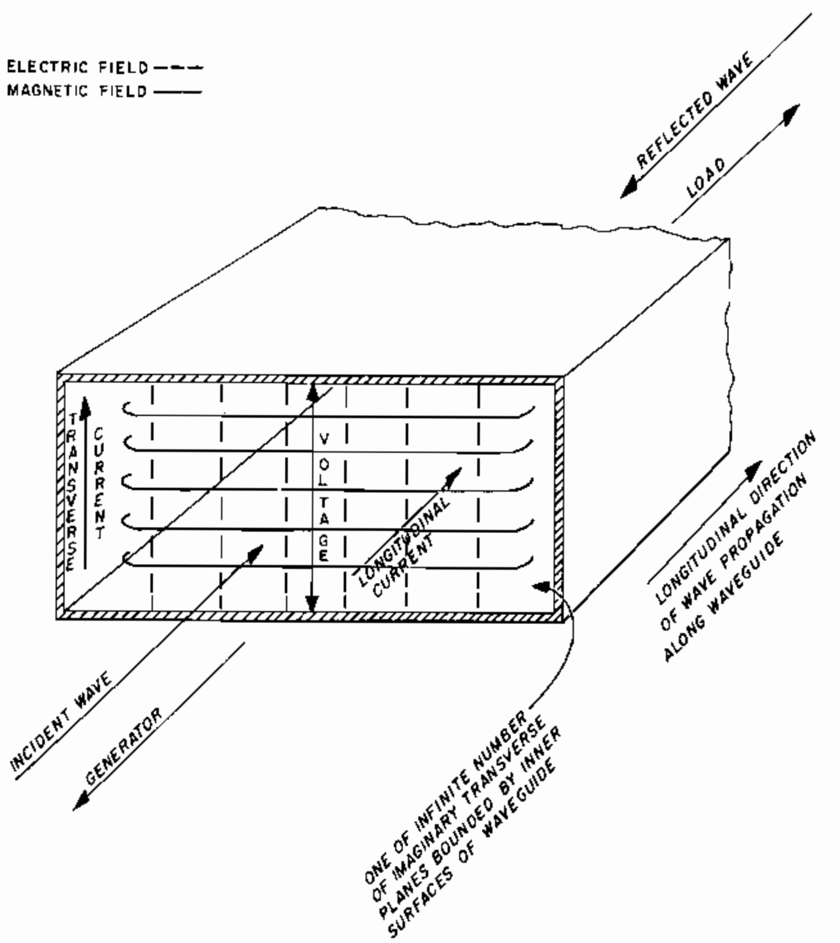


Fig. 2.4. Dominant mode field pattern in a rectangular uniconductor waveguide.

the input impedance. The coordinate component curves of the SMITH CHART (shown on Fig. 2.3) display both normalized input impedance and normalized input admittance, in order to broaden its application. As will be seen, this is possible because of the use of normalized values to designate the individual component curves, and because of the way in which the coordinate component curve families are labeled. Thus, the real coordinate component curve family is labeled "Resistance Component R/Z_0, or Conductance Component G/Y_0," and the imaginary coordinate component family is labeled "Inductive Reactance Component $+jX/Z_0$, or Capacitive Susceptance Component $+jB/Y_0$," and "Capacitive Reactance Component $-jX/Z_0$, or Inductive Susceptance Component $-jB/Y_0$."

The SMITH CHART is most conveniently used as an admittance chart when the effects of shunt elements on the waveguide are to be considered, and as an impedance chart when the effects of series elements are to be considered. One may also transfer the problem from the admittance coordinates to the impedance coordinates, and vice versa as occasion demands, when both shunt and series elements are involved. This is further discussed in Chap. 6.

2.8 NORMALIZATION

Normalized impedance (with respect to a waveguide) is defined as the actual impedance divided by the characteristic impedance of the waveguide. The input impedance coordinates on the SMITH CHART (Fig. 2.3) are, as previously stated, designated in terms of "normalized" values. Normalizing is done to make the chart applicable to waveguides of any and all possible values of characteristic impedance. In addition, as has been pointed out, this makes the coordinate component curves applicable to either impedances or admittances.

To further clarify the normalization process, consider the specific example of a coaxial transmission line having a characteristic impedance of $50 + j\,0$ ohms, terminated in a load impedance of $75 + j\,100$ ohms. This particular load impedance, when normalized with respect to this particular transmission line characteristic impedance, would be expressed as $1.5 + j\,2.0$ ohms, which would appear on the SMITH CHART coordinates (Fig. 2.3) at the intersection of the two families of impedance component curves, viz., where $R/Z_0 = 1.5$ and $+jX/Z_0 = 2.0$. On the other hand, this same load impedance ($75 + j\,100$ ohms), when normalized with respect to the characteristic impedance of a two-wire transmission line of, say, 500 ohms, would be expressed as $0.15 + j\,0.20$ ohms. This would appear on the chart impedance coordinates at a different position, viz., at the intersection of the impedance component curves where $R/Z_0 = 0.15$ and $+jX/Z_0 = 0.20$. The same chart is thus seen to be applicable to transmission lines of different characteristic impedances.

The characteristic impedance of 50 ohms is equivalent to a characteristic admittance of 1/50 or 0.020 mho, and a load impedance of $75 + j\,100$ ohms is equivalent to a load admittance of

$$\frac{1}{75 + j100} \quad \text{or} \quad 0.0048 - j0.0064 \text{ mho}$$

Normalized load admittance for this assumed transmission line is then, by definition,

$$\frac{0.0048 - j0.0064}{0.020} \text{ mho} \quad \text{or} \quad 0.24 - j0.32 \text{ mho}$$

which, as previously stated, is the reciprocal of the normalized complex load impedance, viz., $1.5 + j\,2.0$ ohms.

2.9 CONVERSION OF IMPEDANCE TO ADMITTANCE

The conversion from impedance to admittance, or vice versa, is readily accomplished on the SMITH CHART by simply moving from an initial impedance point on the normalized impedance coordinates of the chart to a point diametrically opposite and at equal distance from the center of the chart. Equivalent admittance values are then read on the normalized admittance coordinates. Any two such diametrically opposite points at the same chart radius give reciprocal normalized values of the coordinate components and, thereby, convert normalized impedances directly to normalized admittances and vice versa. Thus, in the example given for the normalization of impedance and admittance, the respective coordinate points, viz., $1.5 + j\ 2.0$ and $0.24 - j\ 0.32$, will be seen to lie diametrically opposite each other at equal distance from the center of the chart in Fig. 2.3.

CHAPTER 3

Smith Chart Construction

3.1 CONSTRUCTION OF COORDINATES

This chapter describes the construction of the basic coordinates of the SMITH CHART (Fig. 2.3) and then discusses some of the more important waveguide electrical parameters related thereto. All of these related parameters may be graphically portrayed as either radial or peripheral scales.

The construction of the basic series impedance or parallel admittance coordinates of the SMITH CHART is shown in Figs. 3.1 and 3.2. Figure 3.1 applies to the normalized resistance circles R/Z_0, while Fig. 3.2 applies to the superimposed normalized reactance circles $\pm jX/Z_0$. The same construction is applicable to the basic admittance coordinates, by substituting G/Y_0 for R/Z_0 in Fig. 3.1, and by substituting $+jB/Y_0$ for $+jX/Z_0$ (and $-jB/Y_0$ for $-jX/Z_0$) in Fig. 3.2.

3.2 PERIPHERAL SCALES

The impedance or admittance coordinates of the SMITH CHART would be of little use were it not for the accompanying related peripheral and radial scales which have general application to waveguide propagation problems, and which serve as the entry and exit to the chart coordinates. Peripheral scales generally have to do with chart coordinate quantities which vary with position along the waveguide, while radial scales have to do with chart coordinate quantities which vary with reflection and attenuation characteristics of the waveguide.

Two important linear peripheral scales are shown on the SMITH CHART in Fig. 3.3. The outermost of these is the *electrical length* scale, the other is the phase angle of the voltage reflection coefficient scale. A nonlinear peripheral scale showing the angle of the

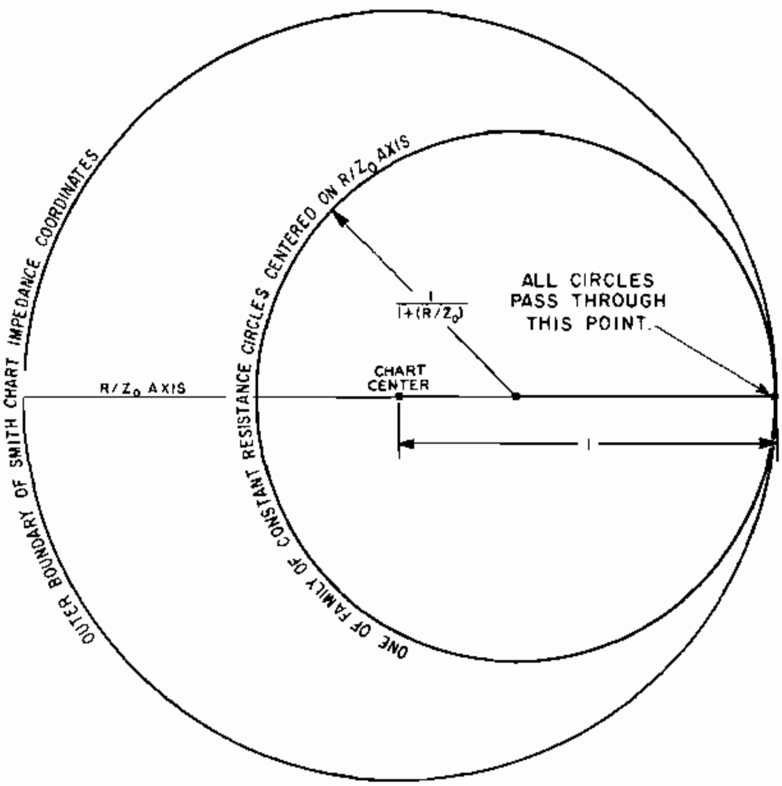


Fig. 3.1. Construction of normalized resistance circles for SMITH CHART of unit radius.

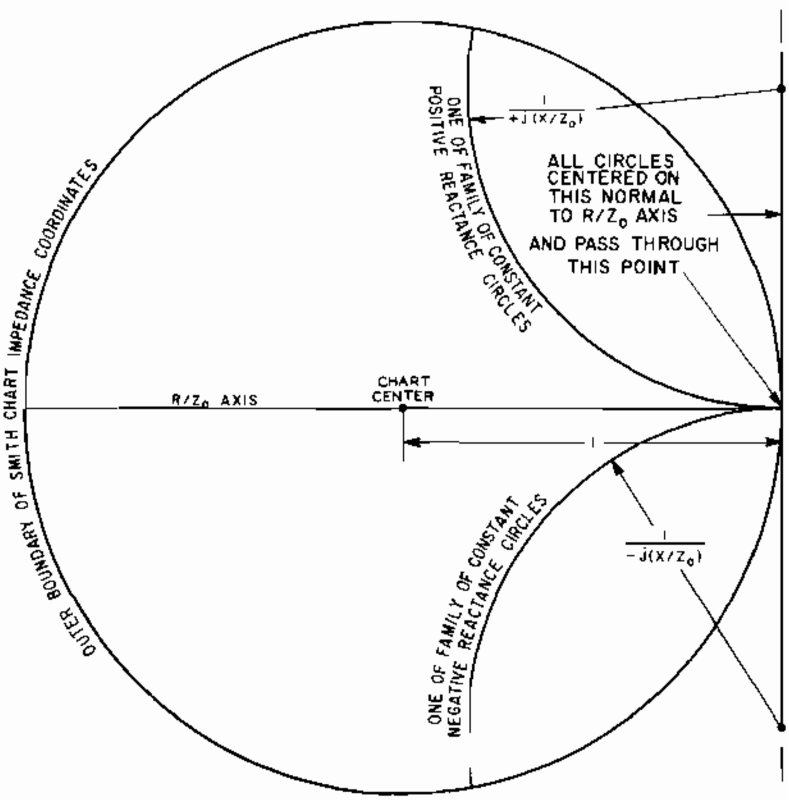


Fig. 3.2. Construction of normalized reactance circles for SMITH CHART of unit radius.

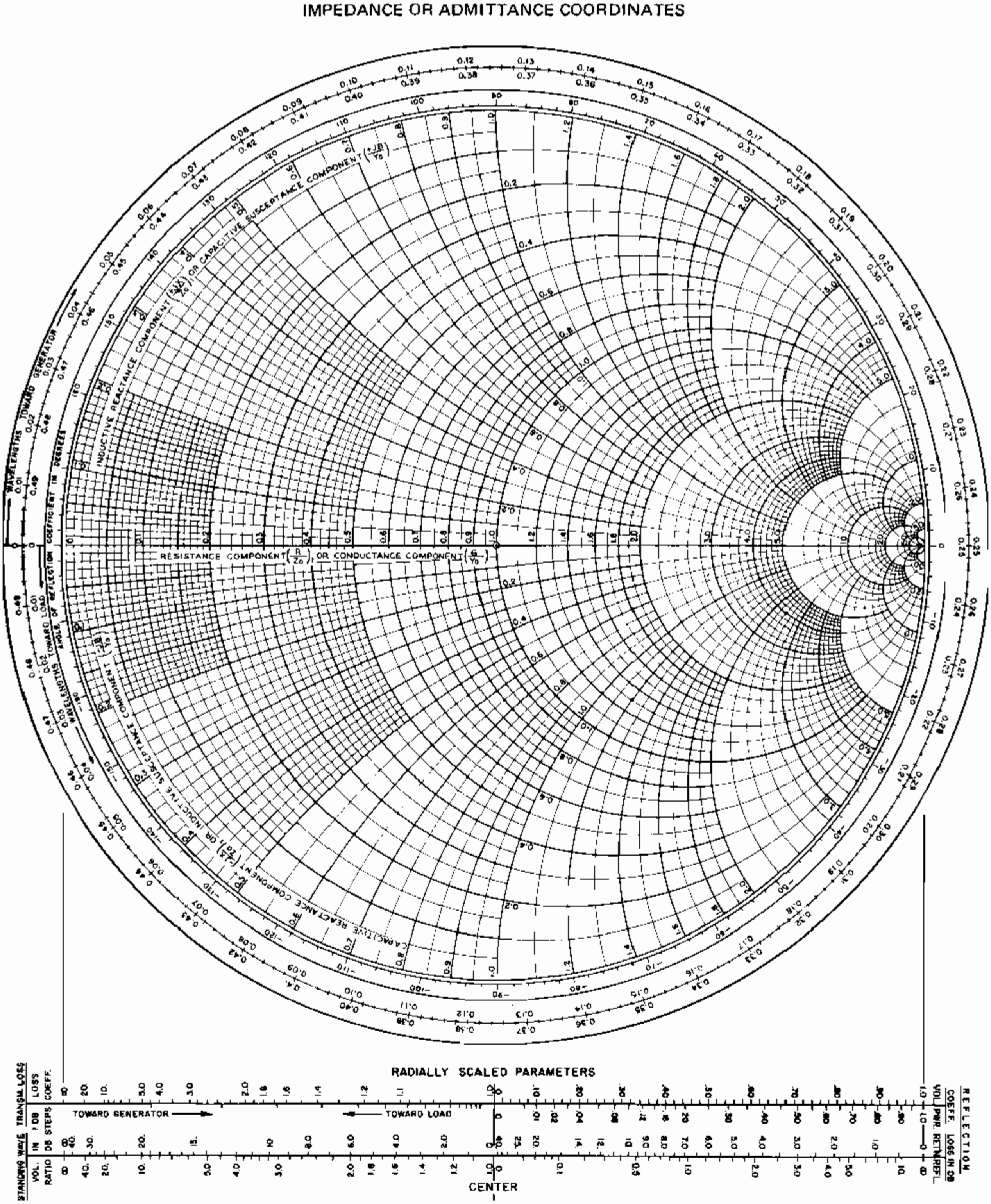


Fig. 3.3. SMITH CHART (1949) with coordinates shown in Fig. 2.3.

voltage transmission coefficient is described in Chap. 8.

3.2.1 Electrical Length

It was stated in Chap. 2 that the phase constant β of a waveguide determines the wavelength in the waveguide and the velocity of propagation. The velocity of propagation, as therein stated, more accurately refers to the velocity of phase propagation, or simply the wave velocity. The electrical length is derived from considerations of phase velocity and wavelength.

As is also stated in Chap. 2, a wavelength is defined as the length of waveguide l such that $\beta l = 2\pi$. The length of waveguide corresponding to one wavelength λ is, therefore, defined as

$$\lambda = \frac{2\pi}{\beta} \tag{3-1}$$

In the case of waveguides along which energy is propagated in the TEM mode (which excludes uniconductor waveguides), the wavelength λ, in meters, will be

$$\lambda = \frac{v}{f} \tag{3-2}$$

where f is the frequency in Hz, and v is the velocity of phase propagation in the waveguide in m/sec. For such waveguides, which include coaxial and open-wire transmission lines with air insulation, v is generally very close to 300,000,000 m/sec.

Linear length scales around the periphery of the SMITH CHART (Fig. 3.3) indicate values of electrical length of waveguide or transmission line, in wavelengths. Two scales are used to indicate distances in either direction from any selected point of entry of the chart, such as a point corresponding to the nodal point of a voltage standing wave, or the input terminals, or load terminals of the waveguide. These scales relate the position along the waveguide to the input impedances (or admittances) encountered at these points as read on the normalized input impedance (or admittance) coordinates. These two scales are labeled, respectively, "Wavelengths Toward Generator" and "Wavelengths Toward Load." For convenience, the zero points are oriented (Fig. 3.3) so that they are always aligned with a voltage standing wave minimum point. If a measurement of distance is to be made from some reference point other than a voltage minimum position, the scale value as read radially in line therewith must be interpolated and subtracted from subsequent scale value readings, since these length scales are not physically rotatable with respect to the coordinates on a single sheet of paper.

The length scales on the periphery of the SMITH CHART encompass only one-half wavelength (180 electrical degrees) of waveguide and this corresponds to a physical rotation of 360° in either direction around the chart periphery. However, a graphical representation of the conditions along one-half wavelength of waveguide is all that it is necessary to consider, since the input impedance values along any uniform waveguide or transmission line repeat cyclically at precisely this interval of distance, if one ignores the effect of attenuation, which effect may be taken into account in a manner to be described later. Any waveguide electrical length in excess of one-half wavelength may always be reduced to an equivalent length less than one-half wavelength, to bring it within the scale range of the chart, by subtracting the largest possible integral number of half wavelengths.

The electrical length of a uniform loss-less section of high-frequency open wire, or coaxial transmission line, having predominantly a gas dielectric, is only slightly longer than its physical length when the latter is expressed in terms of the wavelength in free space. However, the electrical length of practical low-loss coaxial transmission line with a solid dielectric insulating medium, such as polyethylene, is materially increased from its electrical length, in the absence of solid insulation, due to the slower velocity of propagation of electromagnetic waves in dielectric media, by a factor which is equivalent to the refractive index of the dielectric medium between conductors, or by the factor $\sqrt{\epsilon}$, where ϵ is the dielectric constant of the medium.

For uniconductor waveguides, the electrical length is expressed with reference to *waveguide wavelength,* which, in turn, depends upon the mode of propagation of the energy (field pattern) within the waveguide as well as the specific dimensions and configuration of the waveguide. For all hollow uniconductor waveguides, including those of rectangular or circular cross section, the waveguide wavelength is longer than the free space wavelength. Thus, a given section of rectangular or circular uniconductor waveguide is electrically shorter than its physical length.

The waveguide wavelength is directly measurable in uniconductor waveguides or transmission lines when standing waves are present. It is equal to twice the distance between adjacent voltage or current nodal points. In uniconductor waveguides the waveguide wavelength is also calculable from the physical dimensions of the waveguide at the frequency of interest [10].

3.2.2 Reflection Coefficient Phase Angle

The phase angle of the voltage reflection coefficient was described briefly under the heading "Reflection" in Chap. 1. This changes linearly with distance along a waveguide as does the absolute phase of the traveling wave. However, the former changes twice as rapidly with position as does the latter because of the fact that the reflected wave, which is the reference for the reflection coefficient (rather than a fixed point along the waveguide) has traveled twice as far as the incident wave in any given length of waveguide.

The phase angle of the voltage reflection coefficient is portrayed on the SMITH CHART (Fig. 3.3) as a linear peripheral scale ranging in value from 0, along the resistance axis in the direction of maximum resistances, to 180° along this same axis in the opposite direction. Progressing clockwise from the zero-degree point on the voltage reflection coefficient phase angle scale, which is the direction in which one moves on the chart when moving toward the generator, the voltage reflection coefficient phase angle increases negatively from 0 to -180°, indicating an increasing lag in the reflected voltage wave as compared to the incident wave. The reverse is, of course, true in the opposite direction.

3.3 RADIAL REFLECTION SCALES

The radial scales to be described in this chapter apply to all angular positions on the basic coordinates of the SMITH CHART. These are, therefore, equivalent to families of coaxial overlays, since the radial scales could be graphically represented as concentric circles about the center of the SMITH CHART. Each circle of each family of concentric circles could then be assigned a specific value corresponding to the value of the radially scaled parameter which it represented. However, if this were done the resulting large number of concentric circles would mask the basic chart coordinates and result in confusion. Radial scales avoid this.

Figure 3.4 is a plot of four radial reflection scales which will be discussed in this chapter. These include (1) the voltage reflection coefficient magnitude, (2) the power reflection coefficient, (3) the voltage standing wave ratio, and (4) the voltage standing wave ratio in dB. They are applicable to the coordinates of Fig. 2.3.

3.3.1 Voltage Reflection Coefficient Magnitude

Although one may correctly refer to the reflection coefficient in any transmission medium as the ratio of a chosen quantity associated with the reflected wave to the corresponding quantity in the incident wave, the usual quantity referred to, for an electromagnetic wave along a waveguide, is voltage. The voltage (and current) reflection coefficient was discussed in Chap. 1 in the section on "Waveguide Operation." The voltage reflection coefficient is defined as the complex ratio of the voltage of the reflected wave to that of the incident wave.

On the SMITH CHART, the magnitude of the voltage reflection coefficient is represented as a radial scale, starting with zero at the center and progressing linearly to unity at the outer boundary circle (see Fig. 3.4). Since this scale is linear it is a convenient mathematical reference for equating all radial scale parameters.

If the waveguide is uniform and lossless, the voltage reflection coefficient is constant in magnitude throughout its length. If, on the other hand, the waveguide has attenuation, the voltage reflection coefficient will be maximum at the load end and will diminish with distance towards the generator in accordance with the attenuation characteristics of the waveguide.

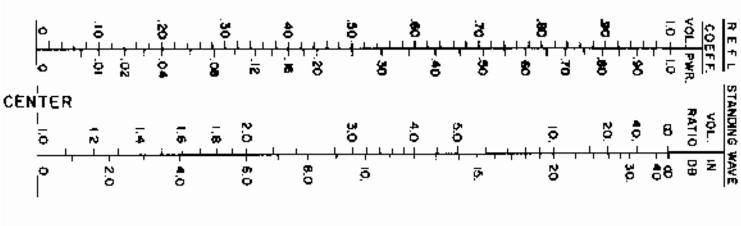


Fig. 3.4. Radial reflection scales for SMITH CHART coordinates in Fig. 2.3 (see radial loss scales in Fig. 4.1).

Figure 3.5 shows a family of constant magnitude circles and a superimposed family of constant phase lines (radial) graphically depicting the voltage reflection coefficient magnitude and phase angle, respectively, along a uniform waveguide.

The polar coordinates formed by these two families of circles may be superimposed on the basic SMITH CHART coordinates (Fig. 2.3, or Chart A, B, or C in the cover envelope) to obtain the relationship of the complex reflection coefficient at any point along a waveguide to the complex impedance or admittance.

3.3.2 Power Reflection Coefficient

The *power reflection coefficient* is defined simply as the ratio of the reflected to the incident power in the waveguide. Numerically it is equivalent to the square of the voltage reflection coefficient. However, unlike the voltage reflection coefficient, the power reflection coefficient has magnitude only, since "phase" as applied to power is meaningless. Like the voltage reflection coefficient, the power reflection coefficient is constant throughout the length of a uniform lossless waveguide, but in a waveguide with attenuation it diminishes with distance toward the generator from a maximum value at the load end.

The power reflection coefficient, expressed in dB, is called *return loss*. This will be discussed further in the next chapter.

3.3.3 Standing Wave Amplitude Ratio

The standing wave amplitude ratio of maximum to minimum voltage along the waveguide (symbolized as VSWR) is a measure, indirectly, of the degree of mismatch of the waveguide characteristic impedance and load impedance. Limiting values are unity for a matched

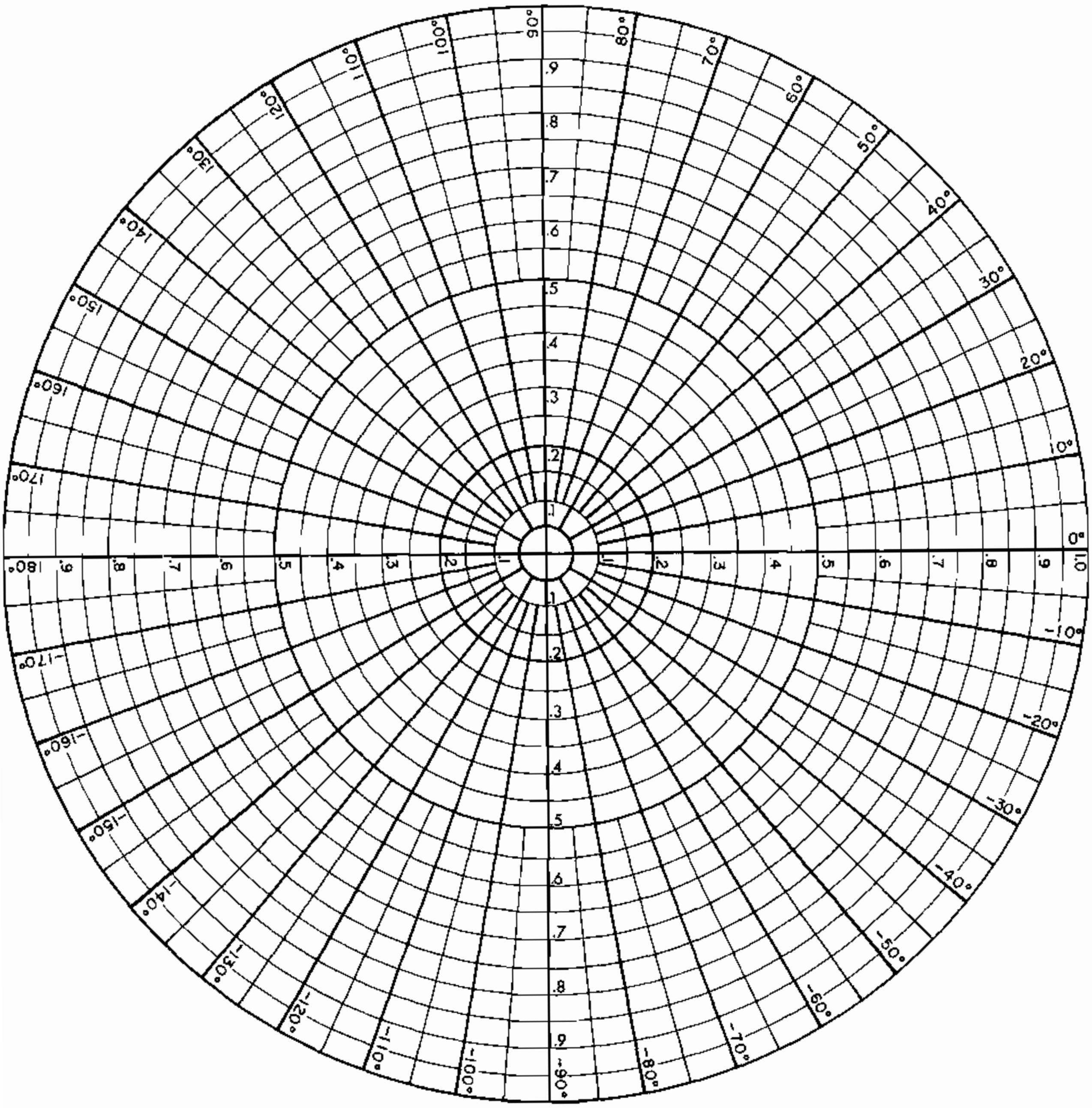


Fig. 3.5. Complex voltage or current reflection coefficient along a waveguide (overlay for Charts A, B, or C in cover envelope).

waveguide and infinity for a lossless open (or short-circuited) waveguide.

As discussed in the section on "Waveguide Operation" in Chap. 1, a standing (or stationary) wave may be thought of as the resultant of two traveling waves moving in opposite directions along the waveguide, i.e., the incident (or direct) and the reflected waves. A single reflection from the load only need be considered under steady-state conditions, since multiple load reflections due to several round trips of the wave between the load and the generator can, under steady-state conditions, be considered to add up to a single effective load reflection. Multiple generator reflections likewise can be considered to add up to a single effective generator output.

Standing waves are always accompanied by a change in input impedance as a function of the position of observation along the waveguide which would otherwise be constant and equal to the characteristic impedance. If the waveguide is lossless the locus of input impedances, for a given standing wave ratio, is represented on the SMITH CHART as a circle concentric to the center of the chart. The radius of the circle is a function of the standing wave ratio and consequently the standing wave ratio may be represented as a radial scale on the chart. This scale is shown in Fig. 3.4. Individual points on an impedance circle give the input impedance of the waveguide at corresponding individual positions.

It will be shown by reference to Eqs. (A-11) and (A-12) in Appendix A that the voltage and current wave shapes are identical along a lossless waveguide, for the sending-end voltage E_s and current I_s as a function of the length l, namely,

$$E_s = E_r \cos\beta l + jZ_0 I_r \sin\beta l \qquad (3\text{-}3)$$

and

$$I_s = I_r \cos\beta l + j\frac{E_r}{Z_0}\sin\beta l \qquad (3\text{-}4)$$

where

E_r = receiving-end voltage
I_r = receiving-end current
Z_0 = characteristic impedance

For comparative purposes E_r and Z_0 can be considered to be a constant equal to unity, in which case

$$E_s = \cos\beta l + jI_r \sin\beta l \qquad (3\text{-}5)$$

and

$$I_s = I_r \cos\beta l + j\sin\beta l \qquad (3\text{-}6)$$

Considering the current I_s' at a point one-quarter wavelength removed from the voltage E_s, that is, $\pi/2$ radians,

$$I_s' = I_r \cos\left(\beta l \pm \frac{\pi}{2}\right) + j\sin\left(\beta l \pm \frac{\pi}{2}\right) \qquad (3\text{-}7)$$

but

$$\cos\left(\beta l \pm \frac{\pi}{2}\right) = \mp \sin\beta l$$

and

$$\sin\left(\beta l \pm \frac{\pi}{2}\right) = \mp \cos\beta l$$

Therefore

$$I_s' = \mp I_r \sin\beta l \mp j\cos\beta l$$

$$= \mp j(\cos\beta l \pm jI_r \sin\beta l) \qquad (3\text{-}8)$$

A comparison of Eq. (3-8) with Eq. (3-5) shows that E_s and I_s' have the same shape and amplitude for unit E_r and that they are displaced 90°, or one-quarter wavelength. The current minimum point always occurs coincident in position with the voltage maximum point along the waveguide and vice versa.

The position of a voltage standing wave minimum point along a waveguide always coincides with the position of minimum normalized input impedance (or maximum normalized input admittance), whereas the position of a minimum current coincides with the position of maximum normalized input impedance (or minimum normalized input admittance). Except in the case of highly dissipative waveguides the input impedance (or admittance) is a pure resistance (or conductance) at both of these points. At all other points the input impedance (or admittance) is complex.

The shape of the voltage and current standing waves along a waveguide are plotted in Fig. 3.6 for standing waves of several amplitude ratios. On this plot each wave is transmitting the same *power* to the load. If the waveguide is lossless, the voltage at the maximum point of a standing wave whose ratio is infinity is also infinity. The waveguide would, of course, arc over long before this point could ever be reached.

A plot similar to that in Fig. 3.6 was shown in Fig. 1.3, wherein the relative amplitudes and shapes of several standing waves were plotted for the condition where the same *incident voltage* is applied to the waveguide. Under these latter conditions, which prevail when the generator is decoupled from the waveguide with sufficient attenuation, it was shown that the voltage at the maximum point

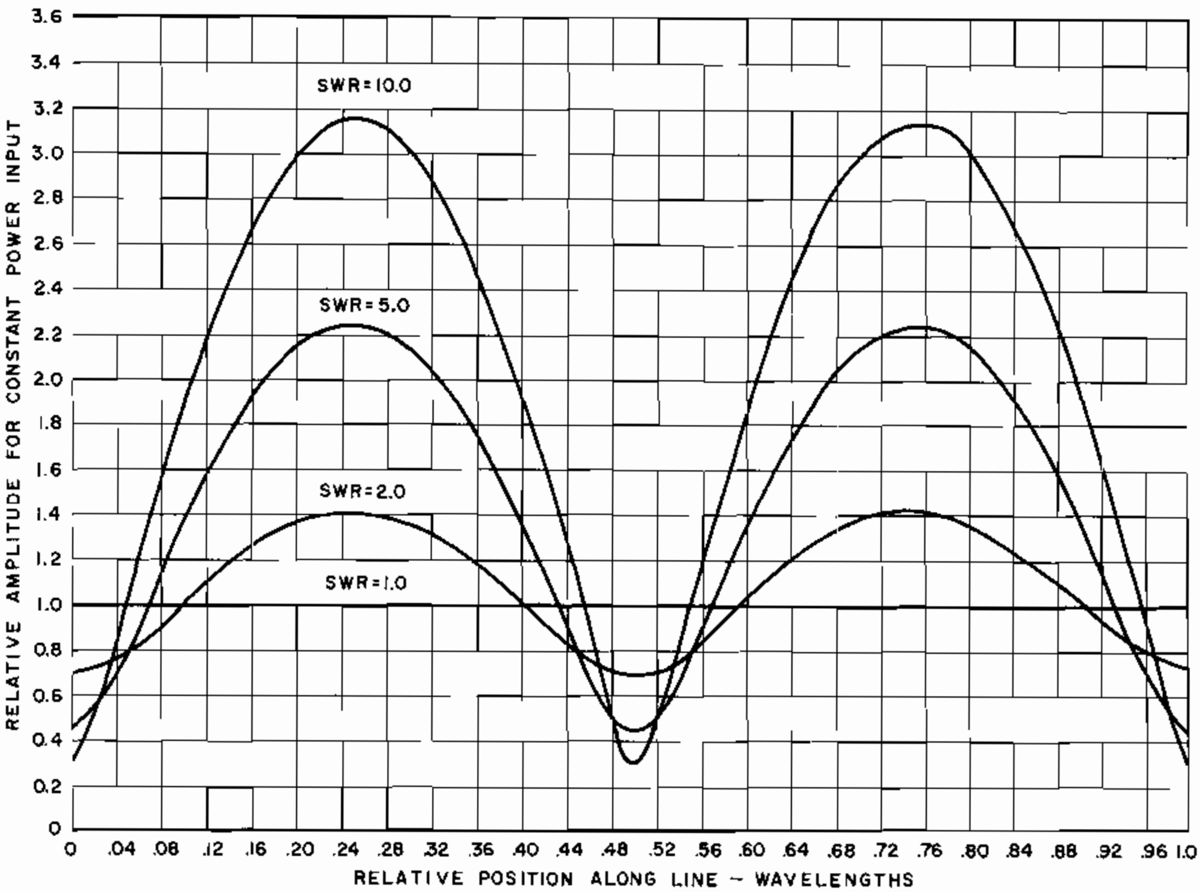


Fig. 3.6. Relative amplitudes and shapes of voltage or current standing waves along a lossless waveguide (constant input power).

of a standing wave, whose ratio is infinity, is exactly twice the incident voltage.

It will be observed from the curves in Figs. 1.3 and 3.6 that the shape of a standing wave is different for each standing wave ratio and that although traveling waves are sinusoidal in shape when observed as a function of time, standing waves are not sinusoidal in shape when their amplitude is plotted as a function of position along the waveguide, except in the limiting case when the standing wave amplitude ratio is infinite. When the standing wave ratio approaches unity the wave shape approaches that of a sine wave of twice the frequency, becoming finally a straight line when the standing wave ratio reaches the unity value. When the standing wave ratio approaches infinity the wave shape approaches that of a succession of half sine waves.

The standing wave shape in a short length, such as one-quarter wavelength, of high-frequency waveguide is unaffected to any appreciable extent by attenuation (one-way transmission loss), although this may be sufficient to affect the standing wave amplitude ratio.

The standing wave amplitude and wave position are readily measurable waveguide parameters using slotted sections of waveguide. This data is useful in providing an entry to the SMITH CHART. Like the input impedance, the standing wave ratio or position in a waveguide cannot be affected by the internal impedance of the generator. However, caution should be exercised to ensure that the conductance and susceptance of any measuring probe used to sample VSWR will not, as it moves along the waveguide, change the impedance presented to the generator, thereby changing the input power or frequency and causing what appears to be a distortion of the standing wave shape or ratio.

It may be of interest to note that amplitude values for the voltage standing wave ratio (VSWR), as scaled radially on the SMITH CHART coordinates, are exactly the same as amplitude values of the circles of constant normalized resistance (or normalized conductance) at the points where these cross the R/Z_0 (or G/Y_0) axis. This may be seen by comparing the VSWR scale in Fig. 3.3 with the R/Z_0 coordinate labeling.

The voltage reflection coefficient magnitude ρ and phase at a point along a waveguide are uniquely related to the voltage (or current) standing wave amplitude ratio S and standing wave position. A simple amplitude relationship exists between these parameters, namely,

$$S = \frac{1 + \rho}{1 - \rho} \tag{3-9}$$

The position of the voltage standing wave maximum point on a waveguide always corresponds to the position where the phase angle of the voltage reflection coefficient is zero. The adjacent voltage standing wave minimum position on the waveguide one-quarter wavelength (90 electrical degrees) removed from the maximum point on either side always corresponds to the position where the voltage reflection coefficient phase angle is 180°.

3.3.4 Voltage Standing Wave Ratio, dB

By definition the decibel is fundamentally a power ratio:

$$\text{dB} = 10 \log_{10} \frac{P_1}{P_2} \tag{3-10}$$

An extension of this use came into being soon after introduction [4] of the unit in 1929, namely,

$$\text{dB} = 20 \log_{10} \frac{V_1}{V_2} \tag{3-11}$$

It was correctly stated that if two voltages are impressed across the *same* resistance the square of their ratio will be equivalent to the ratio of powers in the resistance.

Later, it became popular to measure standing waves of voltage along a waveguide with a voltmeter calibrated in dB in accordance with Eq.(3-11) and to define the VSWR in "dB" as

$$\text{dBS} = 20 \log_{10} \frac{V_{max}}{V_{min}} \tag{3-12}$$

This is obviously a misuse of the term dB as originally intended, since the maximum and minimum voltages in the standing wave do not exist across the *same* input resistance. To further perpetuate this misuse of the term, manufacturers quite generally calibrate standing wave indicating devices in dB.

Since standing wave ratios are, today, so frequently measured in dB it seems advisable to recognize this special use of the term. Accordingly, the lowest scale in Fig. 3.4 shows the VSWR scale in dB.

CHAPTER 4

Losses, and Voltage-Current Representations

4.1 RADIAL LOSS SCALES

In Chap. 3 it was shown that entry and exit to the coordinates of the SMITH CHART are conveniently accomplished through the use of appropriately graduated peripheral and radial scales. The peripheral scales (which were described in Chap. 3) relate all angular positions on the chart coordinates, as measured from its center, to corresponding physical positions along a waveguide. These scales include two linear length scales, one progressing clockwise and the other counterclockwise, from zero to one-half wavelength around the chart circumference. A third peripheral scale measures the phase angle of the voltage reflection coefficient in relation to chart coordinates. Each point along each of the three peripheral scales was shown to apply to all chart positions radially in line therewith.

Radial scales on the SMITH CHART (described in Chap. 3) were shown to be related to the magnitude of reflections from the load, or from discrete reflection points along the waveguide. These scales include voltage (and power) reflection coefficient magnitude and voltage (or current) standing wave ratio. A simple relationship between the magnitude of the voltage reflection coefficient and the voltage standing wave ratio was given.

It will now be shown that the effect of both dissipative and nondissipative losses encountered in a waveguide may also be represented on the SMITH CHART by appropriately graduated radial scales. Dissipative losses which will be considered include *transmission loss* (two-way attenuation) and *standing wave loss factor* (transmission loss coefficient). Nondissipative losses include *reflection loss* and *return loss.*

A universal voltage-current overlay for the SMITH CHART impedance (and/or admittance) coordinates is described in the latter part of this chapter.

4.1.1 Transmission Loss

Transmission loss is defined as the power loss in transmission between two points along a waveguide. It is measured as the difference between the net power passing the first point and the net power passing the second. Thus, by definition, transmission loss includes dissipative losses incurred in the reflected, as well as in the forward-traveling wave, and it also includes losses due to radiation of electromagnetic energy from the waveguide. Transmission loss excludes nondissipative losses due to impedance mismatches, which will be discussed later. Alternatively, transmission loss may be defined as the ratio (in dB) of the net power passing the first point to the net power passing the second.

Transmission loss in waveguides is of three basic types, namely, conductor losses, dielectric losses, and radiation losses. The first two types result in power dissipation or *heat loss* in the waveguide. Except in open-wire transmission lines, radiation losses are generally small enough to be neglected [5]. Conductor losses in waveguides result from the flow of currents in the conductor resistance, whereas dielectric losses are due to the flow of conduction current in the dielectric conductance.

At high frequencies conductor resistance losses in waveguides wherein TEM waves are propagated (which includes coaxial and open-wire transmission lines) increase as the square root of the frequency due to *skin effect* [10]. Dielectric conductance losses, on the other hand, are directly proportional to the number of alternations of the electric field in the dielectric medium adjacent to the conductors in a given interval of time since for a given voltage the dielectric absorbs a fixed amount of energy each cycle. Consequently, these losses increase linearly with frequency. These two types of loss are essentially independent and additive, so that in this type of waveguide, the total heat loss increases with frequency at a rate which is between the square root and the first power, depending upon the materials of which the waveguide is constructed and the frequency of operation.

In cylindrical uniconductor waveguides, in which the dielectric adjacent to the conductor is entirely air or gas and in which waves are propagated in the dominant mode (for example, TE_{10} waves), losses are related to frequency in a much more complicated manner. Below the cutoff frequency (lowest transmittable frequency), losses approach infinity. Immediately above the cutoff frequency, transmission losses decrease with increased frequency at a much higher rate than the concurrent increase in conductor resistance resulting from skin effect.

The radial scale labeled "transmission loss" in Fig. 4.1 actually refers to one-way transmission loss (attenuation). One-way transmission loss is the loss in a conjugate-match terminated waveguide, and is the minimum possible dissipative loss. It is this one-way transmission loss, only, which must be considered in the determination of the effect of all dissipative losses upon input impedance.

The one-way transmission loss scale is plotted, for use on the SMITH CHART coordinates, in dB units. Specific values are purposely omitted so that the zero-dB point on this scale may be taken to be at any position along the scale corresponding to any selected point of entry on the chart coordinates. One-way transmission loss between this initial point of entry and any other point along the waveguide may then be measured on this

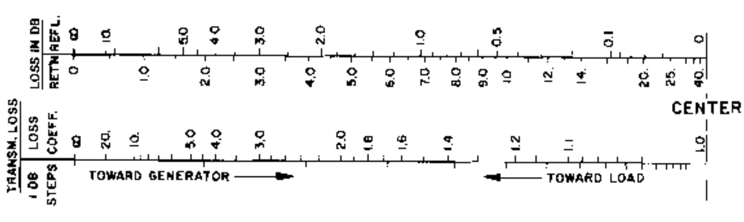


Fig. 4.1. Radial loss scales for SMITH CHART coordinates in Fig. 2.3 (see radial reflection scales in Fig. 3.4).

scale, with its consequent effect upon the input impedance (or admittance) coordinates. The proper direction to proceed along the one-way transmission loss scale depends upon whether one moves along the waveguide "toward generator" or "toward load."

For example, if one enters the SMITH CHART at a point corresponding to a known load impedance point and then moves "toward generator" for an observation of input impedance, the one-way transmission loss will *decrease* the radial distance from the center of the chart at which the input impedance is indicated on the basis of length moved. The scale of one-way transmission loss "Transmission Loss–1 dB steps" is labeled to show the direction in which it is effective in relation to the direction which the observer moves from the point of entry on the chart.

The one-way transmission loss scale for the SMITH CHART is most easily derived from a consideration of the magnitude of the power reflection coefficient. As discussed in Chap. 3, the power reflection coefficient is the ratio of reflected to incident wave power at any selected reference point along a waveguide. It is numerically equal to the square of the voltage reflection coefficient ρ at that point. The power reflection coefficient may be represented on SMITH CHART coordinates by an overlay of concentric circles (similar to those shown on the voltage reflection coefficient overlay in Fig. 3.5) whose radii vary from zero at the center to unity at the rim of the overlay. The radius of individual circles in this family is given in Fig. 3.4.

At any selected point of entry on this power reflection coefficient overlay, and consequently on the SMITH CHART coordinates, forward-traveling wave energy may be assumed to exist. This will be attenuated by the one-way transmission loss as the power flows toward the termination. The termination may conveniently be taken to be at any point on the outer rim of the power reflection coefficient overlay (or SMITH CHART) corresponding to 100 percent reflection. As explained in Chap. 1, the forward-traveling wave energy encounters only the characteristic impedance of the waveguide after leaving the initial point of entry. Upon arrival at the above termination the wave energy is completely reflected. The reflected wave energy continues to encounter the characteristic impedance of the waveguide. Since the propagation path is common to both the forward and the reflected wave energy, the latter, in its backward path to the initial point of entry, will be attenuated in the same ratio as was the incident wave energy. At the initial point of entry the power reflection coefficient is, thus, a measure of the two-way transmission loss, expressed as a power ratio. One-half of this, therefore, represents the one-way transmission loss, viz.,

$$\text{dB} = \frac{1}{2}(-10 \log_{10} \rho^2) \tag{4-1}$$

or

$$\text{dB} = -10 \log_{10} \rho \tag{4-2}$$

If the terminating impedance is other than one which produces complete reflection in a waveguide, such as an open- or short-circuit, or a pure reactance (or susceptance), the one-way transmission loss in the waveguide between any two sampling points where the voltage reflection coefficient is, respectively, ρ_1 and ρ_2, is

$$\text{dB} = -10 \log_{10}(\rho_2 - \rho_1) \tag{4-3}$$

Since only dB units are plotted on the one-way transmission loss scale, it will always be necessary to interpolate between scale divisions for points of entry and exit on the SMITH CHART coordinates which do not fall exactly at the scale division points.

4.1.2 Standing Wave Loss Factor

As previously discussed, additional dissipative losses occur in waveguides when there is reflection from the load, over the one-way transmission loss incurred when there are no reflections, even though the generator impedance may be conjugate-matched to the input impedance of the waveguide. This increased dissipative loss (heat loss) is incurred by the reflected power from the load. Reflected power results in reflected voltage and current waves, resulting in standing waves of voltage and current.

If a waveguide is one or more wavelengths long, the average increase in dissipative loss due to standing waves in a region extending plus or minus one-half wavelength from the point of observation may be expressed as a coefficient or factor of the one-way transmission loss per unit length.

The *standing wave loss factor* can be derived from a consideration of the amplitude and shape of the standing waves along a waveguide. The existence of a standing-wave loss factor and the fact that its amplitude will vary with the degree of mismatch of the load, and/or the standing wave ratio, is apparent from Fig. 3.6. This figure, as previously indicated, shows the relative amplitudes and shapes of several standing waves of current or voltage, all of which will transmit the same net power to the load along a lossless waveguide. For any given standing wave pattern the conductor losses vary along the waveguide in proportion to the square of the current at successive points. Since a uniform waveguide has uniform resistance there are always increased conductor losses in the region of current maxima points which more than counteract the reduction in conductor loss in the region of adjacent current minima points. (Note that the area between the current maxima loops and unity is greater than the area between the current minima loops and unity.)

Also, since dielectric losses are proportional to the square of the voltage and since the dielectric conductance is uniformly distributed, there are always increased dielectric conductance losses (if the waveguide has any dielectric conductance losses to start with) in the region of voltage maxima points. This more than compensates for the reduction in dielectric loss in the region of adjacent voltage minima points.

The dielectric conductance loss at any given point along a waveguide is, as previously stated, proportional to the square of the voltage at the point, while conductor loss is proportional to the square of the current. Since the standing voltage and current wave shapes are identical, it is evident that the percentage increased losses from these two causes are the same. For this reason a single transmission loss coefficient scale may be used to represent added percentage loss as a function only of the standing wave ratio. This will hold whether the losses are initially composed of conductor loss, dielectric loss, or any proportion of each. The standing wave loss factor is represented as a single radial scale on Fig. 4.1, under the caption "Transmission Loss–Loss Coef."

The added loss, which may be evaluated from the standing wave loss factor, does not enter into the calculation of the input impedance characteristic of the waveguide, and consequently when the unattenuated input impedance value on the SMITH CHART is being corrected to take losses into account, this factor should not be considered.

Expressed in terms of the standing wave ratio S, the standing wave loss factor (transmission loss coefficient) is

$$\text{Loss ratio, } \frac{\text{mismatched}}{\text{matched}} = \frac{1 + S^2}{2S} \tag{4-4}$$

This coefficient provides the means for determining the smoothed distribution of the transmission loss (total dissipation losses) in a waveguide when standing waves are present. Smoothing is accomplished by integrating the losses over plus or minus one-half wavelengths from the point of observation, then averaging the results over the same length and, finally, expressing the increase over the minimum attenuation on the basis of an equivalent increase in an elemental length at the point. Thus, spatially repetitive variations in transmission loss within each successive half standing wavelength are equated to an equivalent smoothed loss by the standing wave loss coefficient.

The standing wave loss coefficient scale can also be used to obtain, to a close approximation, the ratio of transmission loss to attenuation for an entire waveguide length in which the standing wave ratio changes along its length due to attenuation. For this the scale value is simply observed at a point midway between the values which represent conditions at the end points of the waveguide section. For example, in a waveguide in which the standing wave ratio is 3.0 at the load end and 1.5 at the generator end, the standing wave loss coefficient is, respectively (from Figs. 3.4 and 4.1, or from Fig. 14.9), 1.667 and 1.083. A point on this scale positioned midway between these two scale positions (not scale values) indicates an average loss increase for the entire length of waveguide to be 1.28 times the attenuation or 1.07 dB ($10 \log_{10} 1.28 = 1.07$ dB). This compares, within the limits of ability to read the scales, with the exact value indicated by the differences in the reflection losses at the same two positions, viz., 1.25 dB - 0.18 dB = 1.07 dB. The maximum variation of loss within a standing wavelength, from the attenuation per unit length, falls between the limits of S and $1/S$ where S is the standing wave ratio. It reaches these peak values only when the loss is due entirely to either conductor or dielectric loss and when radiation losses are absent.

The dissipative loss per unit length of waveguide due to conductor resistance is proportional to the square of the current at the point, whereas the dielectric loss per unit length of waveguide is proportional to the square of the voltage. Since both conductor resistance and dielectric conductance are evenly distributed in a uniform waveguide, when standing waves are present these two sources of loss reach peak values one-quarter wavelength apart. Their combined effect at any position along the standing waves can be evaluated by simply adding them at that position as illustrated in Prob. 4-1.

4.1.3 Reflection Loss

In a lossless waveguide transmission system, if a matched impedance condition is assumed between a generator and waveguide characteristic impedance, and between the waveguide characteristic impedance and the load, there will be no reflected power in the waveguide and consequently no reflection loss. If the load impedance only is then changed, so that a mismatch between the waveguide characteristic impedance and load occurs, there will be a reduction of power delivered to the load. The reduction in load power as the load impedance is changed is a measure of the reflection loss. This may be expressed as a ratio of the reflected to the absorbed power.

In terms of the voltage reflection coefficient, the reflection loss in dB is:

$$\text{Refl. Loss, dB} = -10 \log_{10}(1 - \rho^2) \qquad (4\text{-}5)$$

Under the mismatched load condition the generator will deliver less power to the waveguide. The reduction of power introduced to the waveguide at the generator end is the same as the reflection loss at the load. In other words, the reflection loss at the load can be referred back along the waveguide to the generator terminals and it will have a constant value at all positions which is independent of the waveguide length.

Reflection loss is a nondissipative type of loss representing only the unavailability of power to the load due to the mismatch of impedances originating, in this case, between the waveguide and load. If a lossless transformer is inserted at any point in the transmission system between the load and generator to provide a conjugate match of the impedances, as seen in either direction at the point of its insertion, then the reflection loss may be canceled by a negative reflection loss (sometimes called *reflection gain*) introduced by the transformer.

If the waveguide is not lossless, under the above mismatched conditions at the load, the reflection loss will decrease from its initial value at the load to something less than this value at the generator end of the waveguide. The difference between these two values of reflection loss, as may be read on the radial reflection loss scale for the SMITH CHART on Fig. 4.1, will correspond exactly to the increased dissipation in the waveguide due to the reflection of power from the load.

4.1.4 Return Loss

Return loss is a nondissipative loss term frequently used to describe the degree of mismatch of a load which is closely matched to the characteristic impedance (or characteristic admittance) of a waveguide and which, therefore, produces a small reflection coefficient or standing wave ratio.

Return loss is a measure of the ratio, in dB, of the power in the incident and reflected waves, i.e.,

$$\text{Return loss, dB} = 10 \log_{10} \rho^2 \qquad (4\text{-}6)$$

In other words, return loss is the power reflection coefficient expressed in dB. Thus, a waveguide which is nearly match-terminated will have a relatively large return loss, whereas one which is badly mismatched will have a small return loss.

Return loss is a term which is frequently used when working with waveguide directional couplers whose output ports contain sample portions of the incident and reflected power in the main waveguide.

Return loss is plotted as a radial scale for the SMITH CHART coordinates, ranging between zero dB at the outer rim and infinite dB at the center of the chart. This scale is shown in Fig. 4.1.

4.2 CURRENT AND VOLTAGE OVERLAYS

For a given transmitted power, the input impedance locus on the SMITH CHART resulting in constant current magnitude coincides with the locus of constant series resistance. Similarly, the complex admittance locus resulting in a given voltage magnitude coincides with the locus of constant conductance. The constant current and constant voltage loci are thus represented by two separate families of circles, all of which are centered on the resistance (and/or conductance) axis of an overlay chart for the SMITH CHART in Fig. 3.3. As shown in Fig. 4.2, the constant voltage circles are an exact image of the constant current circles as reflected in a vertical plane passing through the center of the overlay chart.

Corresponding current and voltage curves in Fig. 4.2 in each of the two families have the same magnitude since they are "normalized" to correspond to the current and voltage in a waveguide having a characteristic impedance of one ohm (or a characteristic admittance of one mho), *when transmitting one watt of power.* The actual voltage is

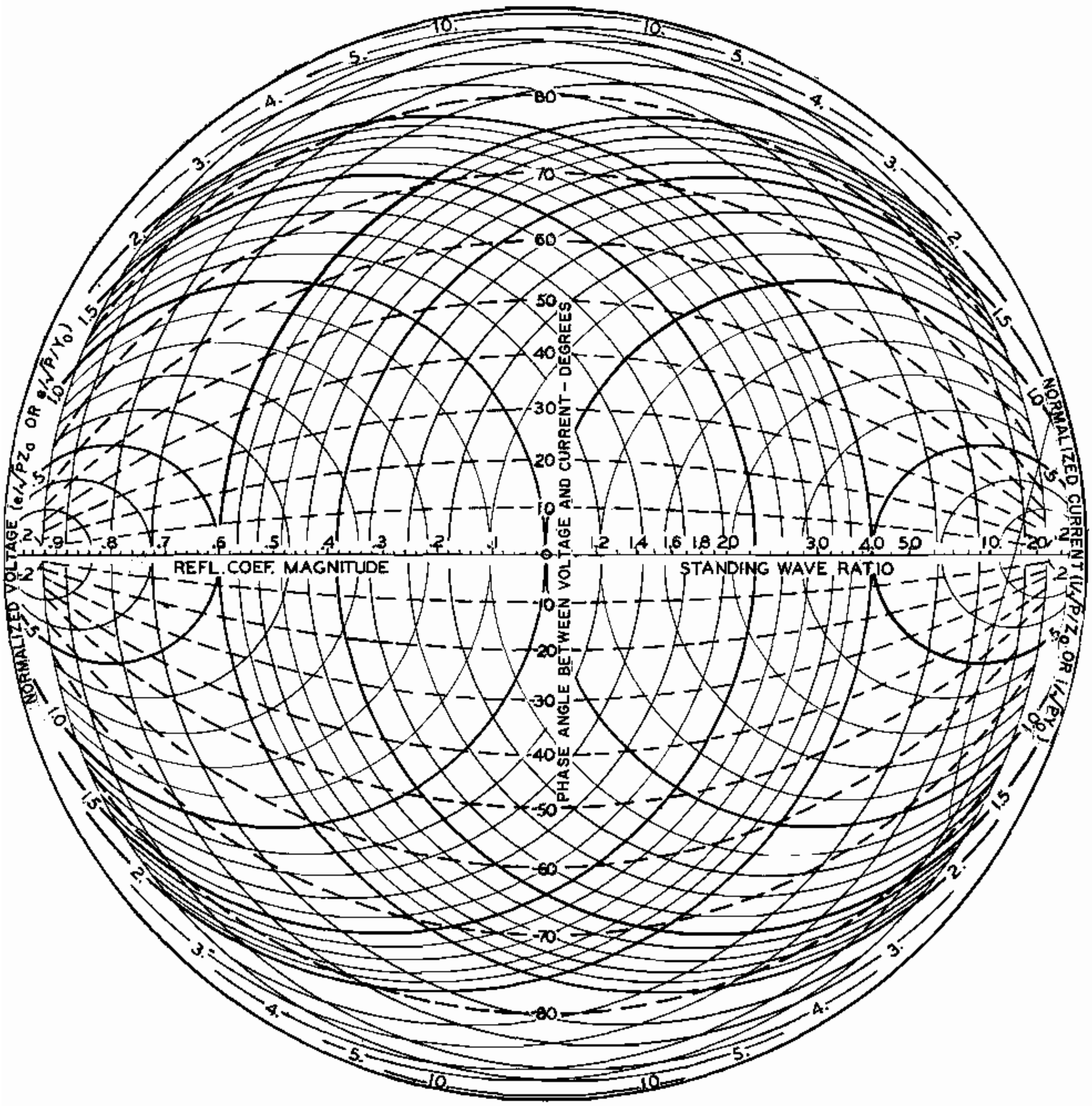


Fig. 4.2. Normalized voltage or current at any point along a waveguide (overlay for Charts A, B, or C in cover envelope).

obtained by multiplying the normalized voltage by the square root of the product of the power and the characteristic impedance (or by the square root of the ratio of the power and the characteristic admittance). The actual current is obtained by multiplying the normalized current by the square root of the ratio of the power and the characteristic impedance (or by the square root of the product of the power and the characteristic admittance). In waveguide transmission in the TEM mode, such as propagated in coaxial and open-wire transmission lines, the normalized complex impedance into which power flows is the ratio of the normalized steady-state voltage to the normalized steady-state longitudinal current times the cosine of the phase angle between the voltage and current.

The phase angle between the voltage across a waveguide and the longitudinal current observed at the same point will depend upon the waveguide input impedance, or admittance. This can vary between the limits of plus and minus 90°. This phase angle may be represented as a family of curves shown on the voltage-current overlay in Fig. 4.2, which apply to either impedance or admittance coordinates of the SMITH CHART. This phase angle is independent of the power, the characteristic impedance, or the characteristic admittance of the waveguide.

When the voltage-current overlay is applied to the impedance coordinates the zero voltage point on the overlay corresponds to zero input impedance. When applied to the admittance coordinates the zero voltage point corresponds to infinite admittance. In either case, in the inductive region of the chart the voltage always leads the current, and the phase angle between voltage and current is, by convention, positive. In this region the voltage across the inductive reactance (or inductive susceptance) component of the input impedance (or admittance) always leads the voltage across the resistance (or conductance) component by 90°.

Similarly in the capacitive region of the SMITH CHART the voltage always lags the current, and the phase angle between voltage and current is negative. In this region the voltage across the capacitive reactance (or capacitive susceptance) component of the input impedance (or admittance) always lags the voltage across the resistance (or conductance) component by 90°.

From Fig. 4.2 the shape of the current and voltage standing waves of any amplitude may be plotted (as shown on Fig. 3.6). This is accomplished by drawing the desired standing wave circle on the SMITH CHART coordinates and by then observing its intersecting points with the various normalized current or voltage loci on the overlay corresponding to positions along the waveguide (as measured along the peripheral length scale). It may be seen from this overlay that only when a waveguide is match-terminated is uniform current and voltage obtained throughout its length.

The radial transmission loss scales described in this chapter will, of course, apply to the current and voltage overlay curves on Fig. 4.2. However, since the transmission loss in one-half wavelength of waveguide is generally small at high frequencies, such losses will not generally have a significant effect upon the standing wave shape in a given region along a waveguide, except in the minimum region of a standing wave having a large amplitude ratio.

When using the universal voltage-current overlay (Fig. 4.2) on the SMITH CHART coordinates, one should remember that the position of all voltage minima points along a waveguide always coincides with the position of minimum input impedance along the resistance axis. Thus, the peripheral length scale on the SMITH CHART in Fig. 3.3, progressing clockwise and labeled "wavelengths toward generator," indicates wavelengths toward generator from a VSWR minimum position when its zero position is aligned with zero voltage at the left-hand end of the overlay axis.

If the voltage-current overlay is applied to the admittance coordinates of the SMITH CHART the position of zero voltage on the overlay coincides with the position of maximum admittance. In this case, the peripheral length scale indicates wavelength toward generator *from a VSWR maximum position.*

PROBLEMS

4-1. A lossy waveguide is conducting power to a mismatched load with produces a standing wave ratio S near the load end whose value is 3.0. One-third of the total attenuation is due to dielectric loss and the remaining two-thirds is due to conductor loss. Determine graphically the distribution of the combined losses along one-half wavelength of the standing wave near the load end of the guide.

Solution:

1. On SMITH CHART A in the cover envelope, draw a 3.0 standing wave ratio circle. The radius of this circle is obtained from the SWR scale across the bottom of the chart.

2. Superimpose Chart A on the normalized voltage and current overlay (Fig. 4.2), and at the intersection of the 3.0 standing wave ratio circle with the normalized voltage and current curves on the overlay, observe and tabulate their amplitudes, as in Table 4.1, at eight points spaced one-sixteenth wavelength apart.

3. From the above data plot, as in Fig. 4.3, the normalized voltage and current standing wave shapes (curves C and A, respectively).

4. Square the amplitude values at the data points on both voltage and current curves and tabulate the results, as in Table 4-1. Plot loss distribution curves through the squared amplitude values (curves D and B, respectively) as in Fig. 4.3. These latter curves represent the loss distribution due to dielectric and conductor losses, respectively, each of which would represent the overall distribution only in the complete absence of the other.

5. Multiply the loss distributions by the originally specified 1/3 and 2/3 factors, respectively, and tabulate the individual and the combined losses, as

Table 4.1. Data for Prob. 4-1.

DIST. IN WAVELENGTHS FROM E_{MIN} TOWARD LOAD	0/16	1/16	2/16	3/16	4/16	5/16	6/16	7/16	8/16
NORMALIZED CURRENT (I)	1.732	1.622	1.302	0.852	0.577	0.852	1.302	1.622	1.732
CONDUCTOR LOSS (PROPORTIONAL TO I^2)	3.000	2.630	1.696	0.726	0.333	0.726	1.696	2.630	3.000
NORMALIZED VOLTAGE (E)	0.577	0.852	1.302	1.622	1.732	1.622	1.302	0.852	0.577
DIELECTRIC LOSS (PROPORTIONAL TO E^2)	0.333	0.726	1.696	2.630	3.000	2.630	1.696	0.726	0.333
2/3 CONDUCTOR LOSS	2.000	1.754	1.131	0.484	0.222	0.484	1.131	1.754	2.000
1/3 DIELECTRIC LOSS	0.111	0.242	0.565	0.876	1.000	0.876	0.565	0.242	0.111
COMBINED CONDUCTOR AND DIELECTRIC LOSS	2.111	1.996	1.696	1.360	1.222	1.360	1.696	1.996	2.111

in Table 4.1. Plot the combined loss distribution as tabulated in Table 4.1 as the dot-dash curve (E) in Fig. 4.3. Note that this curve varies cyclically about a mean value whose amplitude closely approximates that of the "standing wave loss coefficient" (horizontal line F).

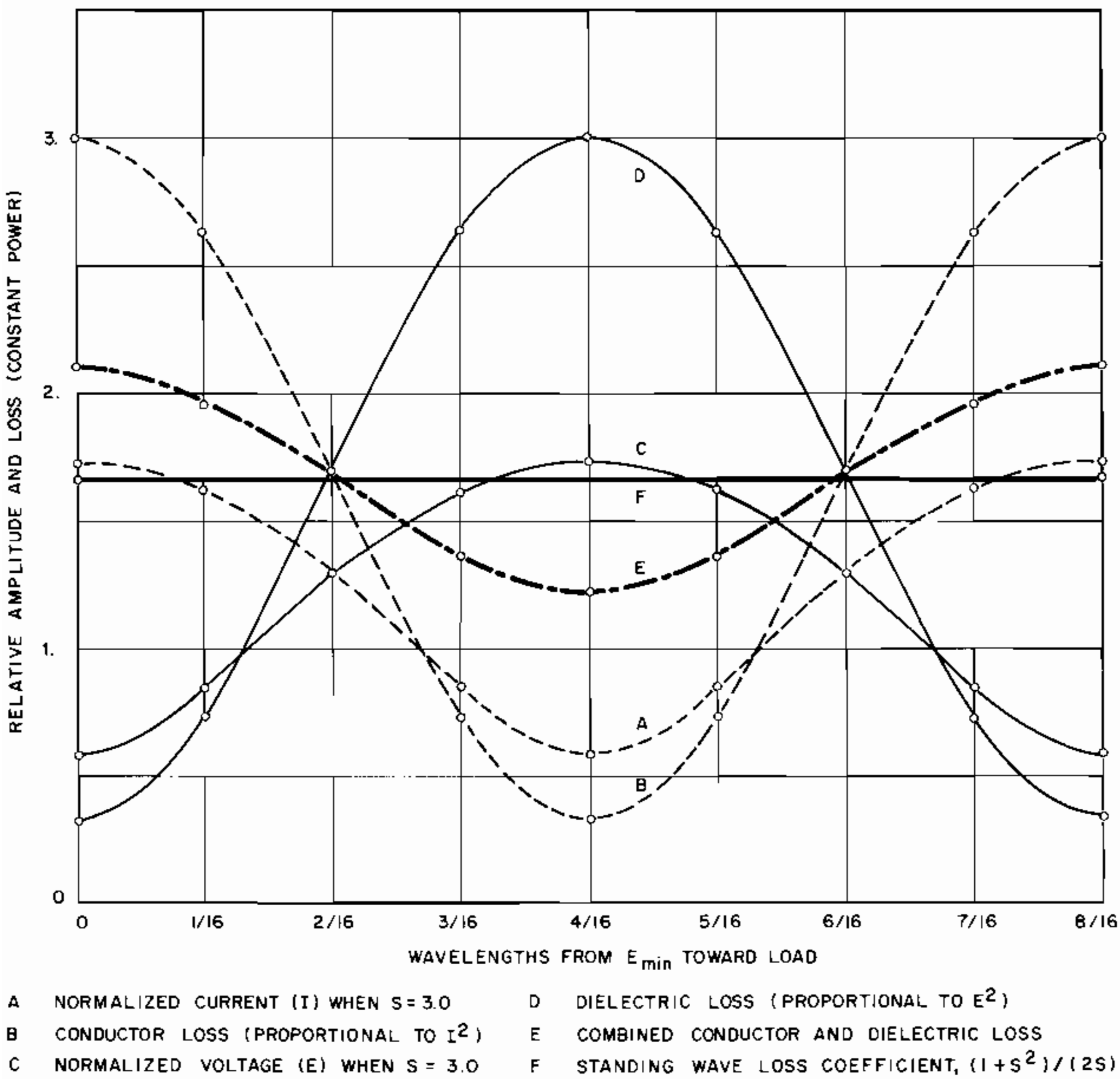


Fig. 4.3. Solution for Prob. 4-1, waveguide loss distribution when S = 3.0.

CHAPTER 5

Waveguide Phase Representations

5.1 PHASE RELATIONSHIPS

The phase relationship between the reflected and incident traveling voltage waves, and between the reflected and incident traveling current waves, at various positions along a waveguide has been discussed briefly in Chaps. 1 and 3. In Chap. 4 an additional phase relationship, namely, that between the steady-state (standing wave) voltage and the steady-state (standing wave) current at various coincident points along a waveguide was discussed. The two former relationships are, respectively, the *phase angle of the voltage reflection coefficient* and the *phase angle of the current reflection coefficient.* The latter is the phase angle of the *power factor*—the power factor itself being thus defined as the cosine of this angle [35]. All three of the foregoing phase relationships have been shown to be graphically representable as overlays on the impedance coordinates of the SMITH CHART.

It has also been shown that the voltage-current phase relationship (phase angle of power factor) overlay (Fig. 4.2) can be applied to a SMITH CHART whose coordinates are labeled to represent *either* impedances or admittances (such as the SMITH CHART of Fig. 3.3). Directions were indicated therein for properly orienting the overlay on the specific coordinates selected, and for interpreting the sign of the indicated phase angle.

In this chapter some fundamental waveguide phase conventions will first be reviewed. Following this, more generalized uses of the peripheral scale labeled "angle of reflection coefficient" (Fig. 3.3) will be presented. Next, a discussion of the voltage, current, and power transmission coefficient, with generalized SMITH CHART overlays therefor, will be given. Finally, some additional waveguide voltage and waveguide current phase relationships will be discussed and presented in the form of general-purpose overlays for the SMITH CHART. These latter phase relationships are of fundamental importance in the design of waveguide components and antennas employing phased radiating elements excited through waveguide feed systems.

5.2 PHASE CONVENTIONS

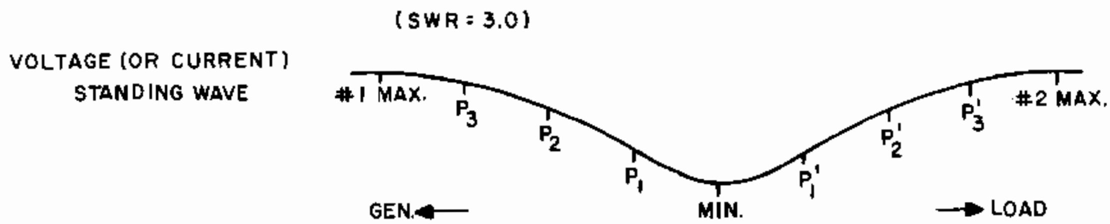
In waveguide terminology, *phase* describes the particular stage of progress of any periodically alternating quantity. It may also describe the relative progress of two such quantities. The two quantities may be at the same or at separate positions along a waveguide and they may or may not be of the same kind. Waveguide quantities which are customarily related by phase include currents, voltages, or current vs. voltage.

Absolute phase is expressed as the total number of cycles (including any fractional number) separating the two quantities, wherein one complete cycle is 2π radians or 360°. Relative phase is frequently of more interest. This is the absolute phase less the largest integral number of 2π radians (or 360°) which separates the quantities. The unit of phase is, therefore, the radian, or the electrical degree.

Relative phase is represented graphically as the angle between two periodically rotating vector quantities. If the periodicity of the two quantities is the same, as in all waveguide applications to be considered in this book, the relative phase is independent of real time variation. If the two quantities are not exactly in-phase (0°) or out-of-phase (±180°), one of them is considered to have a relative phase *lead* or *lag* over the other.

Increasing phase lag (or decreasing phase lead) is represented by a clockwise rotation of a voltage or a current vector. Conversely, decreasing phase lag (or increasing phase lead) is represented by a counterclockwise rotation of a vector.

Table 5.1. Voltage or Current Phase Relations Along a Lossless Waveguide When SWR = 3.0 (Refer to Fig. 5.1).



POINTS OF OBSERVATION (along standing wave, .0625 λ apart)	#1 MAX.	P_3	P_2	P_1	MIN.	P'_1	P'_2	P'_3	#2 MIN.
PHASE OF INCIDENT WAVE (rel. to incid. wave @ min.)	+90	+67.5	+45	+22.5	0	-22.5	-45	-67.5	-90
PHASE OF REFLECTED WAVE (rel. to refl. wave @ min.)	-90	-67.5	-45	-22.5	0	+22.5	+45	+67.5	+90
PHASE OF REFL. COEFF. (ratio refl. to incid. wave)	0	+45	+90	+135	±180	-135	-90	-45	0
PHASE OF TRANSM. COEFF. (ratio transm. to incid. wave)	0	+14	+25.5	+29.5	0	-29.5	-25.5	-14	0
PHASE OF STANDING WAVE AT A PT. (rel. to standing wave @ min.)	90	82	71.5	52	0	-52	-71.5	-82	-90

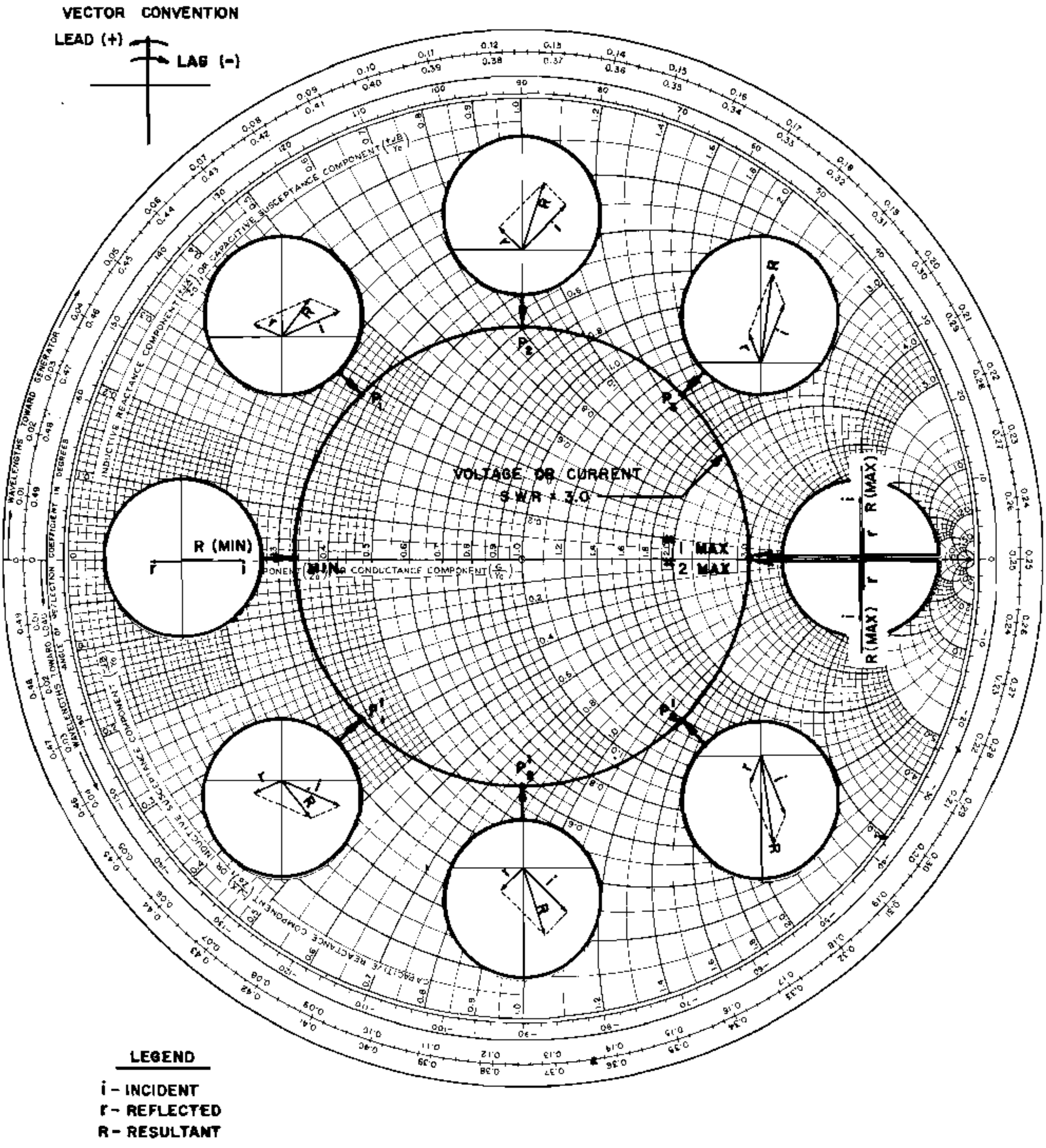


Fig. 5.1. Vector representation of phase relations for voltages on impedance coordinates, or currents on admittance coordinates, of SMITH CHART when SWR = 3.0 (refer to Table 5.1).

A relative phase lag of one vector over another is indicated by a negative sign (−) on the lagging vector, whereas a relative phase lead of one vector over another is indicated by a positive sign (+).

In accordance with the above convention, Fig. 5.1 shows a SMITH CHART upon which eight specific vector representations of the voltage or current on the impedance or admittance coordinates, respectively, are plotted.

These representations are for eight uniformly spaced positions within one-half wavelength of waveguide, accompanying a standing wave whose amplitude ratio is 3.0. They illustrate the amplitude and phase relationships of the incident, reflected, and resultant wave components.

The various relationships involving phase at any position along a waveguide which have been, or will be, discussed herein include:

1. Phase of the *incident* wave relative to the incident wave at the nearest standing wave minimum position.

2. Phase of the *reflected* wave relative to the reflected wave at the nearest standing wave minimum position.

3. Phase of the *reflection coefficients* (phase of reflected voltage or current wave at any position relative to the incident wave at the same position).

4. Phase of the *transmission coefficients* (phase of transmitted voltage or current wave at any position relative to the incident wave at the same position).

5. Phase of *standing wave* (phase of resultant of incident and reflected wave at a point along a standing wave relative to resultant wave at the standing wave minimum position).

All of the above relationships are representable, and may be evaluated by graphical means for any specified position along a waveguide, and for any specified standing wave ratio (or load impedance). The numerical results for the eight examples are given in Table 5.1.

5.3 ANGLE OF REFLECTION COEFFICIENT

It was previously indicated that the angle of the reflection coefficient scale (Fig. 3.3) applies to the voltage reflection coefficient phase angle on impedance coordinates. More generalized uses for this scale, as drawn, will be shown which are consistent with the previously discussed conventions.

Zero relative phase angle for the voltage reflection coefficient occurs at all voltage maxima positions along a waveguide, at which points the impedance is maximum (and the admittance is minimum). At these points the reflected and incident voltage waves are in phase. Likewise, zero relative phase angle for the current reflection coefficient occurs at the current maxima positions along a waveguide where the impedance is minimum (and the admittance is maximum). At these points the reflected and incident current waves are in phase.

Thus, as shown on Fig. 3.3, the angle of the reflection coefficient scale applies not only to the voltage reflection coefficient on the impedance coordinates, *but also to the current reflection coefficient of the admittance coordinates.*

A simple rule which applies to any combination of voltage or current reflection coefficient and impedance or admittance coordinates is that *zero on the reflection coefficient phase angle scale should always be aligned with a maximum of the corresponding standing wave.*

5.4 TRANSMISSION COEFFICIENT

A term which is used less frequently than reflection coefficient but which is nevertheless useful in many waveguide applications is *transmission coefficient.* This term, like reflection coefficient, may be applied to any two associated quantities at any given position along a waveguide. The chosen quantities must, of course, be specified. In waveguides, the term transmission coefficient is most frequently applied to voltage and current, although it may also be applied to power. The value of the transmission coefficient, like that of the reflection coefficient, will depend upon the associated quantities selected, the frequency, and the mode of transmission.

The voltage transmission coefficient is defined as the complex ratio of the resultant of the incident and reflected voltage to the incident voltage. Similarly, the current transmission coefficient is defined as the complex ratio of the resultant of the incident and reflected current to the incident current.

Since power has no "phase," the power transmission coefficient is simply the scalar ratio of the transmitted to incident power. This is constant at all positions along a lossless waveguide. The power transmission coefficient is numerically equal to one minus the power reflection coefficient, which was described in Chap. 3. Figure 5.2 shows a radial scale for the SMITH CHART coordinates in Fig. 2.3 representing this parameter. This scale applies equally to the impedance and admittance coordinates.

A graphical representation of the relationship between the complex voltage (or current) reflection coefficient ρ/α and the complex voltage (or current) transmission coefficient τ/β is shown in Fig. 5.3.

From the geometry of Fig. 5.3 the magnitude of τ in terms of ρ and α is seen to be

$$|\tau| = [\rho^2 + 1 - 2\rho \cos(180^\circ - \alpha)]^{1/2} \qquad (5\text{-}1)$$

Similarly, the magnitude of ρ in terms of τ and β is

$$|\rho| = [\tau^2 + 1 - 2\tau \cos\beta]^{1/2} \qquad (5\text{-}2)$$

Since ρ is always between zero and unity, it will be observed from Fig. 5.3, or from Eqs. (5-1) and (5-2), that the magnitude of τ must always be between zero and two. Scales representing the magnitude of τ as a voltage (or current) ratio, and in dB, are shown in Fig. 5.2. In the region along a waveguide where the voltage transmission coefficient is greater than unity, resulting in a gain, the current transmission coefficient will be less than unity, resulting in a compensating loss.

Also from the geometry of Fig. 5.3, the phase angle β in terms of ρ and α is seen to be

$$\beta = \tan^{-1} \frac{\rho \sin(180^\circ - \alpha)}{1 - \rho \cos(180^\circ - \alpha)} \qquad (5\text{-}3)$$

Similarly, the phase angle α in terms of τ and β is

$$\alpha = 180^\circ - \tan^{-1} \frac{\tau \sin\beta}{1 - \tau \cos\beta} \qquad (5\text{-}4)$$

At any given position along the waveguide the phase angle of τ and ρ will always have the same sign.

Both the voltage and the current transmission coefficient may be represented by a single overlay on the coordinates of the SMITH CHART shown in Fig. 3.3. Such a general-purpose transmission coefficient overlay is

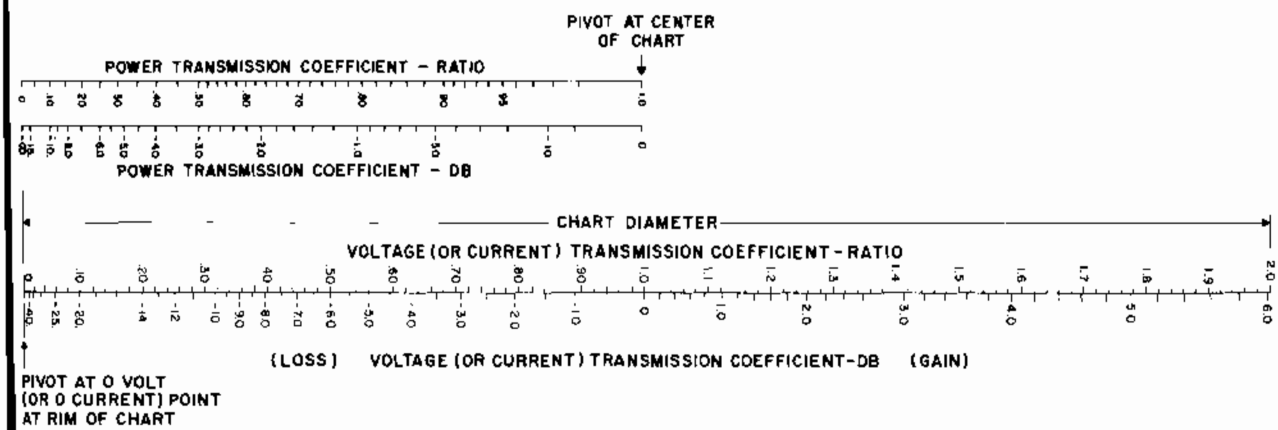


Fig. 5.2. Power, voltage, and current transmission coefficient magnitude scales for SMITH CHART coordinates in Fig. 2.3.

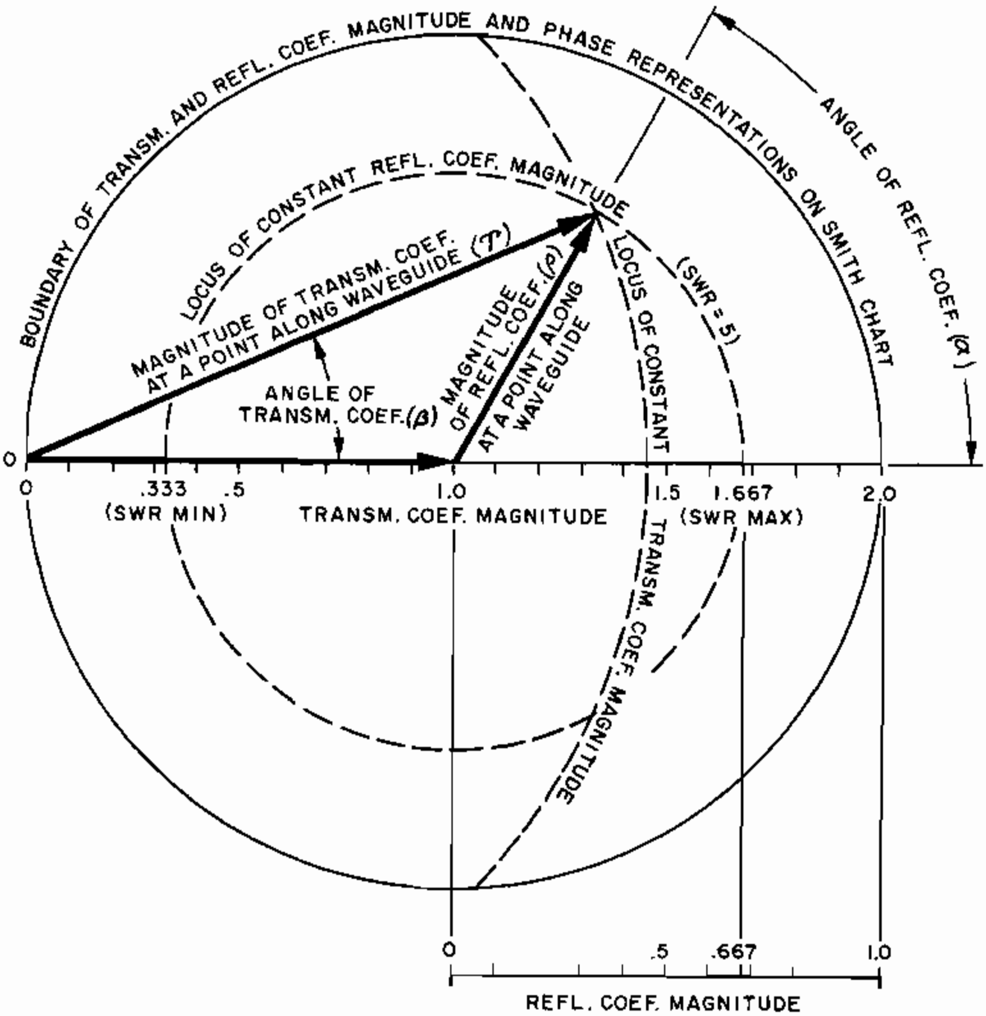


Fig. 5.3. Representation of voltage or current reflection coefficient magnitude and phase, and voltage or current transmission coefficient magnitude and phase on a SMITH CHART (diagram shows conditions at a point along a standing wave whose ratio is 5.0).

shown in Fig. 5.4. As oriented in Fig. 5.4, in relation to Fig. 3.3, it represents the voltage transmission coefficient on the impedance coordinates, or the current transmission coefficient on the admittance coordinates. When rotated 180° from this orientation, the overlay of Fig. 5.4 represents the voltage transmission coefficient on the admittance coordinates, or the current transmission coefficient on the impedance coordinates of this same chart. *The convergence point of all transmission coefficient phase angles on this overlay should always be aligned radially with the corresponding standing wave minima points.*

From the transmission coefficient overlay the shape vs. amplitude of all standing waves of voltage or current along a waveguide may be plotted *for a constant incident wave amplitude.*

Standing wave shapes for three specific standing wave amplitude ratios have been plotted in Fig. 1.3. Note that the maximum voltage (or current) never exceeds twice the incident voltage (or current) at any point along a waveguide.

The requirement for a constant incident wave on a waveguide is commonly satisfied by inserting a large, for example, 20-dB or more, dissipative attenuator, whose characteristic

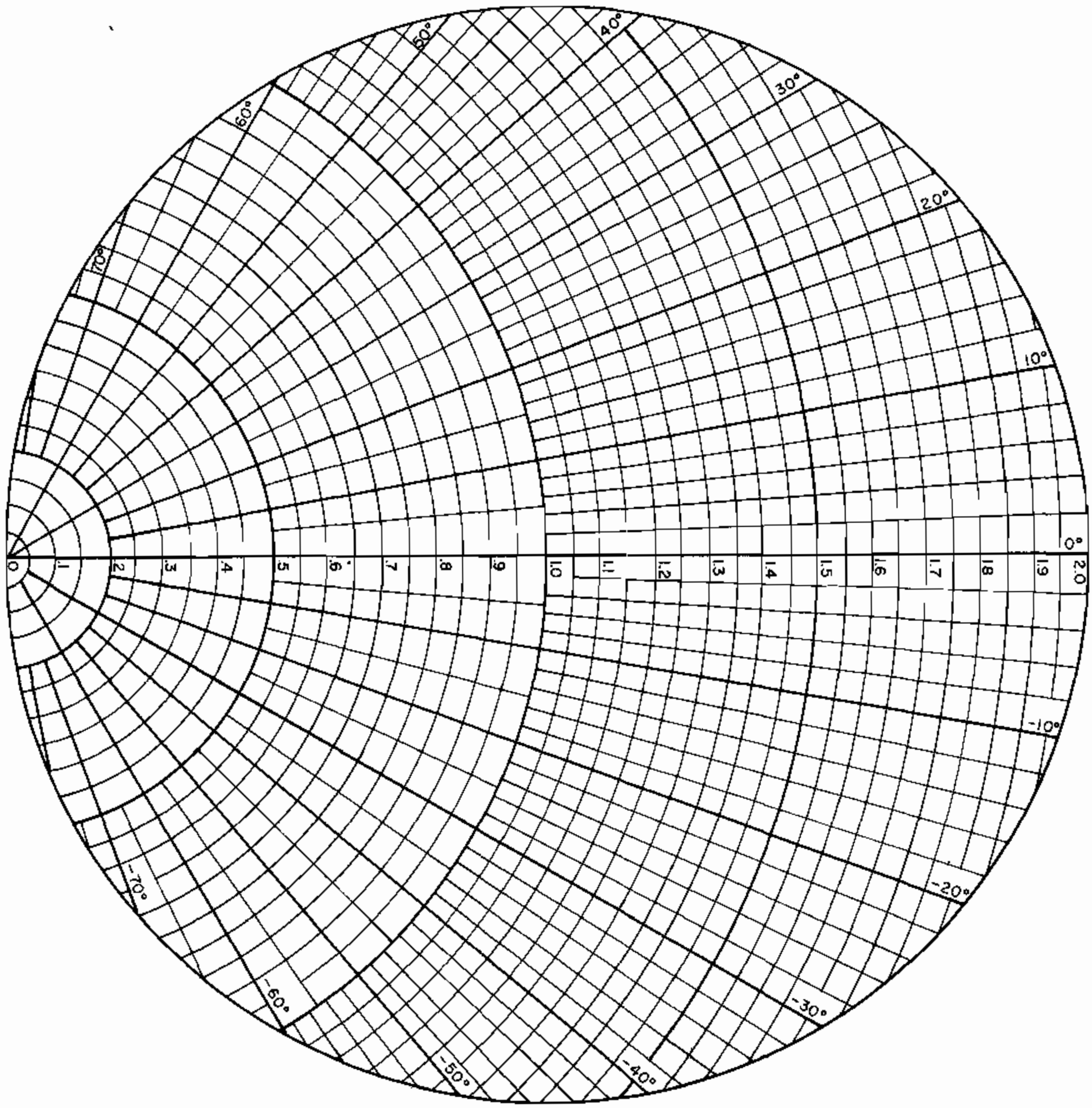


Fig. 5.4. Complex voltage or current transmission coefficient along a waveguide (overlay for Charts A, B, or C in cover envelope).

impedance matches the waveguide, between the generator and the waveguide. In this way reflected waves from the load do not return to the generator with sufficient power to significantly affect its output power. Consequently, the incident wave on the waveguide is held constant and independent of the load impedance or admittance characteristics.

The phase angle of the voltage (or current) transmission coefficient is a measure of the extent that the resultant wave departs in its phase relationship with the incident wave. The maximum departure from an in-phase relationship depends upon the standing wave ratio, being smallest for small standing waves and reaching a maximum of 90° for an infinite standing wave ratio.

5.5 RELATIVE PHASE ALONG A STANDING WAVE

Another useful waveguide phase relationship is the relative phase angle between the standing wave voltages (or currents) at any two positions along a waveguide. This phase relationship is particularly important in the design of phased-array antennas. It should not be confused with the relative phase at two points along a traveling wave, nor with the relative phase angle of the voltage (or current) reflection (or transmission) coefficients at the two points.

The standing voltage (or current) wave at each of two separated positions along a waveguide is the vector sum of the incident and the reflected voltage (or current) waves at the respective positions (Fig. 5.1). The relative phase between these two resultant voltages (or currents) is not linearly related to their physical separation. It is a function of the degree of mismatch of the waveguide and load (standing wave ratio) and the positions along the waveguide relative to the position of some reference point on the standing wave.

While any reference position is possible, the most generally useful position is the minimum position of the standing wave nearest to the point of measurement, either toward the load or toward the generator. The selection of a minimum reference position along a standing wave on a waveguide makes it possible to plot, and to unambiguously identify, the phase angle on a family of relative-phase curves for all standing voltage or current waves along a waveguide. Such a plot, as shown in Fig. 5.5, may conveniently be used as an overlay for the SMITH CHART impedance or admittance coordinates. From the reference phase shift at each of the two points the relative phase of the standing wave voltage or current at the two points is readily obtainable.

Mathematically, the relationship for the phase of the standing wave voltage or current at a point relative to that at the nearest minimum point is

$$\phi = \tan^{-1}\left(S \tan \frac{2\pi l}{\lambda}\right) \tag{5-5}$$

where l/λ is + in the direction of the generator from the minimum point. Thus, if l/λ (the distance from the standing wave minimum point to the point in question) is in the direction of the generator (positive direction) the phase angle at the point in question is positive, i.e., it leads the phase at the minimum point. In the opposite direction, i.e., toward the load from the minimum, l/λ is negative and the phase at all points in the region between the minimum and the following maximum lags that at the minimum.

The overlay of Fig. 5.5 was machine-plotted. Equation (5-5) was first rewritten to express ϕ in terms of the voltage reflection coefficient magnitude ρ and its phase angle α. (See Eq. (3-8).) The resulting equation is

$$\rho = \frac{\sin[\phi - (\alpha/2)]}{\sin[\phi + (\alpha/2)]} \tag{5-6}$$

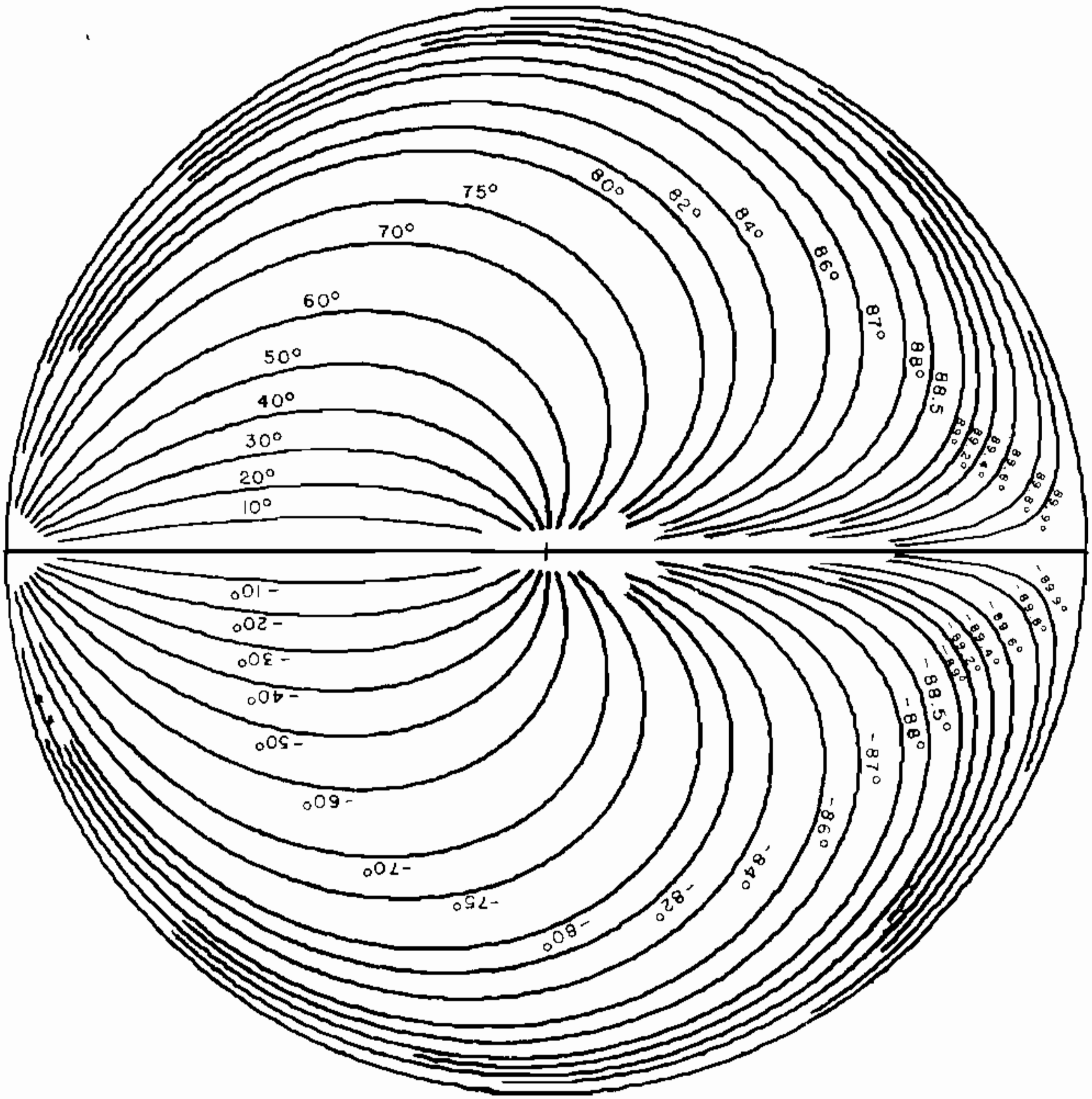


Fig. 5.5. Phase of voltage or current at any point along a waveguide relative to voltage or current, respectively, at nearest minimum point (overlay for Charts A, B, or C, in cover envelope).

The x and y components of the phase angle ϕ are

$$x = \rho \cos\alpha \tag{5-7}$$

$$y = \rho \sin\alpha \tag{5-8}$$

The relative phase of the current at any two positions along a standing current wave of a given amplitude ratio on a waveguide is identical to the relative phase of the voltage at any two corresponding positions along a standing voltage wave of the same amplitude ratio. Accordingly, the phase overlay of Fig. 5.5 may be used to obtain either relative voltage or relative current phase relations. This overlay may also be used on either the impedance or the admittance coordinates.

A simple rule to follow in orienting the overlay of Fig. 5.5, for either voltage or current phase relationships on the SMITH CHART impedance or admittance coordinates, is to *observe that the overlay is always oriented so that the phase change vs. length of waveguide is greatest in the region of the respective standing wave minima.*

The eight vector phase relationships plotted in Fig. 5.1 may conveniently be used as check points on the proper orientation of any of the phase overlays which have so far been discussed. As shown, they represent voltages on impedance coordinates or currents on admittance coordinates. When rotated, as a group, through 180° with respect to their present positions on the coordinates, they represent voltages on the admittance coordinates or currents on the impedance coordinates.

5.6 RELATIVE AMPLITUDE ALONG A STANDING WAVE

The relative amplitude of the voltage (or current) at any two points along a standing wave are uniquely related to the relative phase at the two points. Hence it is possible to plot the family of amplitude ratio curves in Fig. 5.6 which are uniquely related to the family of phase curves in Fig. 5.5.

All of the rules which have been given for application of the phase curves of Fig. 5.5 to voltage or current on impedance or admittance coordinates also apply to the amplitude curves of Fig. 5.6.

PROBLEMS

5-1. One radiating element of an array antenna is connected via a coaxial cable to a junction point of a "corporate" feed system. The known conditions are: (1) electrical length of the cable is 6.279 wavelengths, (2) attenuation is 1.0 dB, (3) normalized load impedance of cable is 1.6 - j 1.35 ohms. (See insert in Fig. 5.7.)

Using a SMITH CHART, determine (1) the standing wave ratio in the cable at its load end, (2) the normalized sending end impedance Z_s/Z_0, (3) the standing wave ratio in the cable at the sending end, (4a) the voltage transmission coefficient τ_E at each end of the cable, (4b) the voltage insertion phase φ_E, (5a) the current transmission coefficient τ_I at each end of the cable, and (5b) the current insertion phase φ_I.

Solution:

1. As shown in Fig. 5.7, locate on SMITH CHART A in the cover envelope the normalized load impedance Z_l/Z_0 = 1.60 - j 1.35, and construct a standing wave circle centered on the chart coordinates, passing through this impedance point. Observe that the radius of this circle, as laid out on the SWR scale across the bottom, shows that at the load end of the cable the standing wave ratio S is 3.0.

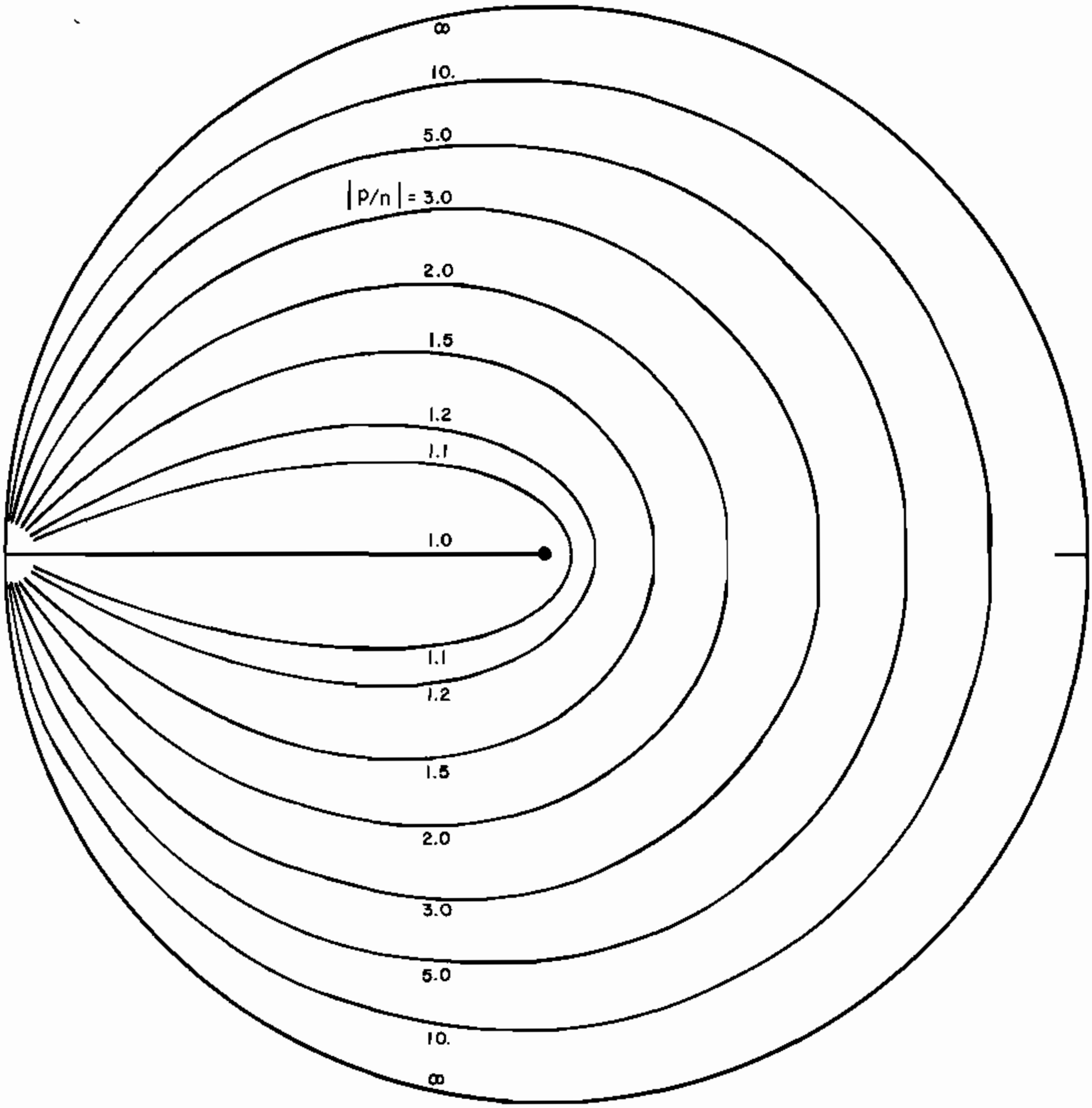


Fig. 5.6. Amplitude of voltage or current at any point along a waveguide relative to voltage or current, respectively, at nearest minimum point (overlay for Charts A, B, or C in cover envelope).

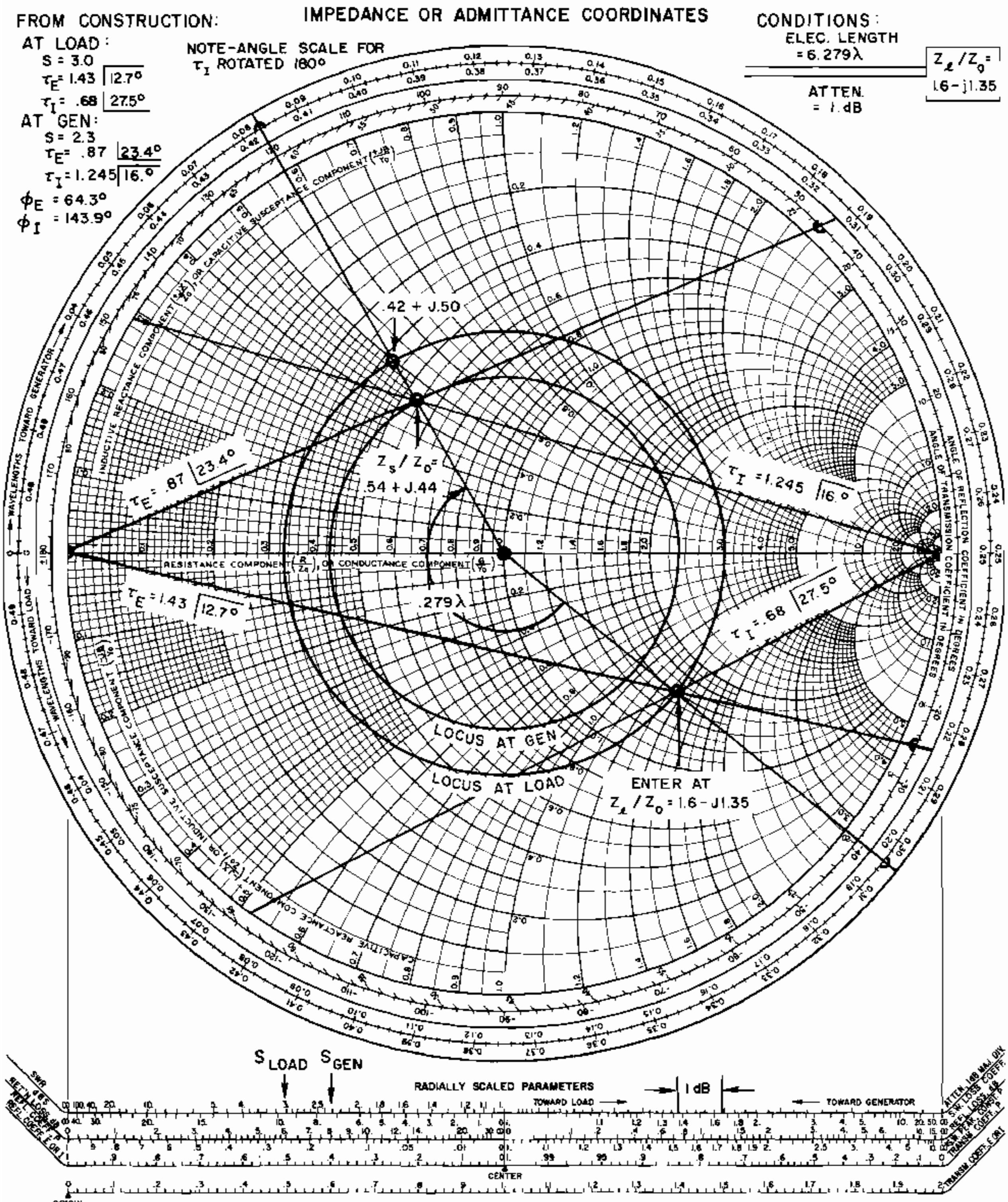


Fig. 5.7. Construction for Prob. 5-1, transmission coefficients and insertion phase.

2. Move clockwise around this standing wave circle (toward generator) a distance equal to 0.279 wavelength, as measured on the outermost peripheral scale, to the sending end position, where in the absence of attenuation the sending end impedance would be $0.42 + j\ 0.50$. (Note: the largest integral number of half wavelengths is subtracted from the total electrical length since relative—not absolute—phase is required.) Correct for 1.0-dB attenuation by moving radially toward the center of the chart coordinates a distance corresponding to 1.0-dB major divisions as obtained from the attenuation scale across the bottom, to the position where the normalized sending-end impedance Z_s/Z_0 is $0.54 + j\ 0.49$.

3. The standing wave ratio at $Z_s/Z_0 = 0.54 + j\ 0.49$ is seen to be 2.3 when measured as the radial distance on the chart coordinates, employing the SWR scale across the bottom.

4a. Construct straight lines from the origin of the chart coordinates through the load and sending-end impedance points, respectively, intersecting the innermost peripheral voltage transmission coefficient angle scale. Determine the magnitudes of the voltage transmission coefficient at each end of the cable using the magnitude scale across the bottom, and thus observe that at the load end of the cable $\tau_E = 1.43\angle 12.7°$, and at the sending end $\tau_E = 0.87\angle 23.4°$.

4b. The voltage insertion phase φ_E is the phase change undergone by the voltage traveling wave (total angle of a match-terminated cable reduced to an equivalent electrical length less than one-half wavelengths) plus the net difference in the angles of the voltage transmission coefficients at the load and sending ends, respectively. Thus, in this example,

$$\varphi_E$$
$$= \{0.279 \times 360° + [(-12.7°) - (+23.4°)]\}$$
$$= 64.3°$$

5a. Use the complex transmission coefficient overlay (Fig. 5.4) to represent the current transmission coefficients by inverting it (rotating it 180°) with respect to the impedance coordinates of Fig. 5.7. Then observe from this overlay that at the normalized load impedance point, where $Z_l/Z_0 = 1.6 - j\ 1.35$, the current transmission coefficient τ_I is $0.68\angle 27.5°$, and at the sending-end impedance point, where $Z_s/Z_0 = 0.54 + j\ 0.49$, the current transmission coefficient is $1.245\angle 16.0°$.

5b. The current insertion phase φ_I is obtained analogously to that of the voltage insertion phase as described in Prob. 4b, and is found to be

$$\varphi_I$$
$$= \{0.279 \times 360° + [(+27.5) - (-16.0°)]\}$$
$$= 143.9°$$

CHAPTER 6

Equivalent Circuit Representations of Impedance and Admittance

6.1 IMPEDANCE CONCEPTS

In this chapter the concept of waveguide input impedance (and admittance) is presented first. This is followed by a consideration of the input impedance (or admittance) relationships of a waveguide to those of simple series or parallel circuits which present equivalent impedance (or admittance) at a given frequency.

Two overlays for conventional SMITH CHART coordinates which provide alternative coordinate forms, useful in specific waveguide applications, will be described. One of these coordinate overlays displays normalized input impedance components presented by the equivalent parallel-circuit (rather than the conventional series-circuit) elements, and/or normalized input admittance components presented by the equivalent series-circuit (rather than the conventional parallel circuit) elements. The other overlay displays normalized polar coordinate components of impedance, i.e., magnitude and phase angle.

A graphical method for combining two normalized polar impedance vectors in parallel, which utilizes special polar coordinates, is included.

6.2 IMPEDANCE-ADMITTANCE VECTORS

The input impedance at any specified position along a waveguide is, fundamentally, the complex ratio of voltage to current at that position; the input admittance is the reciprocal of this ratio. Any sinusoidally varying voltage or current at any point in a waveguide may be represented by the projection of a uniformly rotating vector on a fixed axis. The ratio between voltage and current is a stationary vector whose magnitude is the ratio of voltage magnitude to current magnitude, and whose angle is the difference between the phase angles of the voltage and current vectors. Thus, waveguide input impedance and its inverse (waveguide input admittance) may be regarded as stationary vectors. At a given position along a waveguide these two vectors have reciprocal magnitudes and equal phase angles of opposite sign.

The terms *waveguide input impedance* and *waveguide input admittance* were defined and discussed briefly in Chap. 2. The "normalization" of these terms to the waveguide characteristic impedance or characteristic admittance, respectively, was also discussed therein. Also in Chap. 2, the conversion of a normalized waveguide input impedance to an equivalent normalized waveguide input admittance, and vice versa, on conventional SMITH CHART coordinates was described.

6.3 SERIES-CIRCUIT REPRESENTATIONS OF IMPEDANCE AND EQUIVALENT PARALLEL-CIRCUIT REPRESENTATIONS OF ADMITTANCE ON CONVENTIONAL SMITH CHART COORDINATES

Waveguide input impedance and input admittance vectors may be expressed in either complex or polar notation. On conventional SMITH CHART coordinates, such as the coordinates in Fig. 6.1, the complex notation is used. On this chart the normalized impedance presented by series-circuit combinations of resistive and inductive (or capacitive) circuit elements is expressed in complex notation by

$$\frac{\mathbf{Z}_s}{Z_0} = \frac{R_s}{Z_0} \pm \frac{jX_s}{Z_0} \tag{6-1}$$

Similarly, the normalized input admittance presented by parallel-circuit combinations of these respective circuit elements is expressed by

$$\frac{\mathbf{Y}_p}{\mathbf{Y}_0} = \frac{G_p}{Y_0} \mp \frac{jB_p}{Y_0} \tag{6-2}$$

The vector diagrams superimposed on the conventional SMITH CHART coordinates of Fig. 6.1 show normalized input impedance vectors $\mathbf{Z}_s/\mathbf{Z}_0$ and (diametrically opposite) normalized input admittance vectors $\mathbf{Y}_p/\mathbf{Y}_0$ at eight positions equally spaced along one-half wavelength of waveguide. In this example, a waveguide termination is arbitarily chosen which produces a standing wave ratio of 3.0. The normalized impedance vector $\mathbf{Z}_1/\mathbf{Z}_0$, for example, represents the impedance of a series circuit which is equivalent, at a given frequency, to the parallel circuit represented by the diametrically opposite admittance vector $\mathbf{Y}_1/\mathbf{Y}_0$. Similarly, the normalized impedance vector $\mathbf{Z}_2/\mathbf{Z}_0$ represents the impedance of a series circuit which is equivalent, at a given frequency, to the parallel circuit represented by the diametrically opposite admittance vector $\mathbf{Y}_2/\mathbf{Y}_0$.

In the upper half of the SMITH CHART of Fig. 6.1 normalized impedances are represented at a single frequency by series circuit combinations of resistive and inductive elements, that is, $R/Z_0 + jX/Z_0$. The voltage across any inductive circuit leads the current flowing into the circuit by a phase angle between 0° and 90°. As discussed in Chap. 5, a lead is indicated graphically by a counterclockwise rotation of a vector, and the sign of the angle is positive. Impedance vectors in the upper half of the SMITH CHART of Fig. 6.1 are therefore assigned positive phase angles. Conversely, impedances in the lower half of Fig. 6.1 are assigned negative phase angles.

The above convention for the representation of impedance vectors also applies to the representation of admittance vectors. Thus, admittances in the upper half of Fig. 6.1 are represented by parallel-circuit combinations of conductance and capacitance, that is, $G/Y_0 + jB/Y_0$. This follows from the fact that in all capacitive circuits the current flowing into the circuit leads the voltage across the circuit by an angle between 0° and + 90°. Since admittance is the reciprocal of impedance, i.e., the ratio of current to voltage, capacitive admittance vectors have a positive angle and lie in the upper half of the SMITH CHART.

The complex impedance and complex admittance represented, respectively, by series

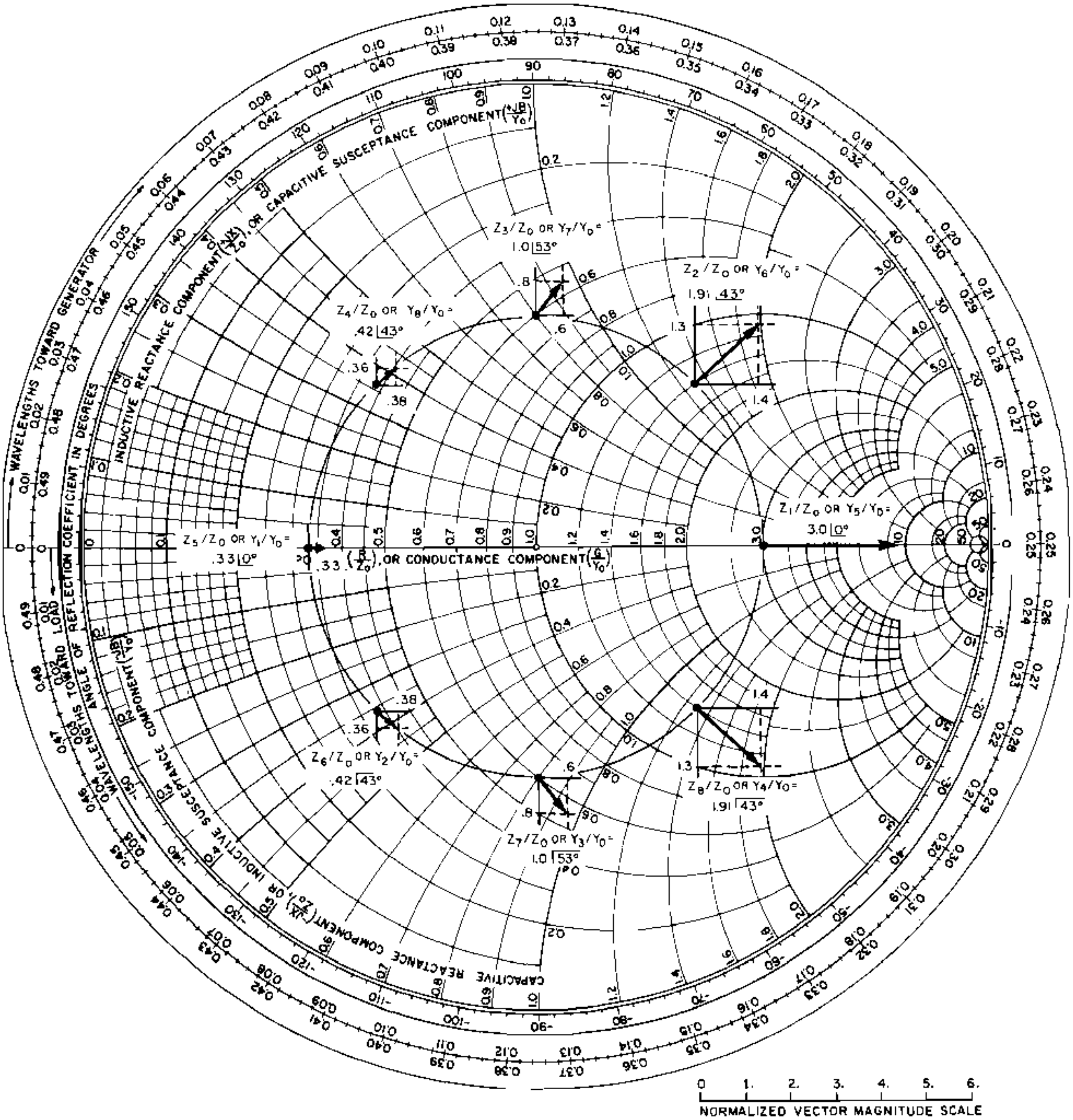


Fig. 6.1. Normalized impedance and admittance vector relationships along a waveguide when SWR = 3.0.

combinations of resistive and reactive circuit elements, and parallel combinations of conductive and susceptive circuit elements, are shown in Fig. 6.2(a) and 6.2(b). A specific example is illustrated in Fig. 6.3. The two equivalent circuit positions (A and B) shown in this latter figure are diametrically opposite one another at equal chart radius.

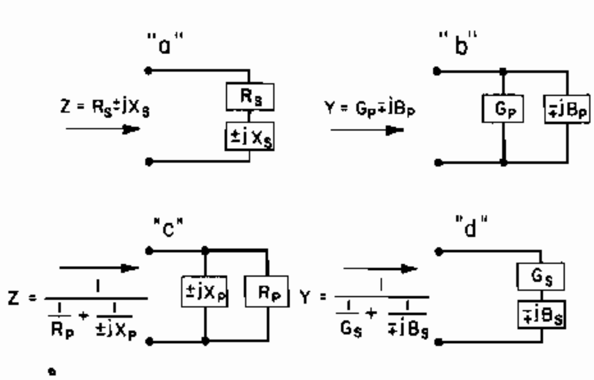


Fig. 6.2. **Equivalent series and parallel circuits.**

As may be seen from Fig. 6.2(a) and (b), the usual representation of waveguide input impedance (in terms of its series circuit component values) facilitates the calculation of the resultant input impedance when additional series elements are to be added. Similarly, the usual representation of waveguide input admittance in terms of its parallel-circuit component values facilitates the calculation of the resultant input admittance when additional parallel elements are to be added.

6.4 PARALLEL-CIRCUIT REPRESENTATIONS OF IMPEDANCE AND SERIES-CIRCUIT REPRESENTATIONS OF ADMITTANCE ON AN ALTERNATE FORM OF THE SMITH CHART

In some applications it is useful to represent waveguide input impedance by a parallel combination of resistive and reactive primary circuit elements [Fig. 6.2(c)]. Similarly, it is sometimes useful to represent waveguide input admittance by series combination of conductive and susceptive primary circuit elements [Fig. 6.2(d)].

Figure 6.2(c) shows a parallel combination of a resistance R_p and an inductive or capacitive reactance $\pm jX_p$, resulting in an input impedance Z. The input impedance of this, as of any parallel circuit, is equal to the reciprocal of the sum of the reciprocals of the resistive and reactive components, viz.,

$$Z = \frac{1}{1/R_p + 1/\pm jX_p} \tag{6-3}$$

Equation (6-3) can be rationalized to obtain the component parts of Z which represent resistance and reactance of the equivalent series circuit shown in Fig. 6.2(a) ($R_s \pm jX_s$), viz.,

$$R_s \pm jX_s = \frac{R_p X_p^2}{R_p^2 + X_p^2} \pm j\frac{R_p^2 X_p}{R_p^2 + X_p^2} \tag{6-4}$$

Similarly, as shown in Fig. 6.2(d), a particular combination of a conductance G_s in series with an inductive or capacitive susceptance $\mp jB_s$ results in an input admittance Y, which is equal to the reciprocal of the sum of the reciprocals of the conductive and susceptive components of the series circuit, viz.,

$$Y = \frac{1}{1/G_s + 1/\mp jB_s} \tag{6-5}$$

Equation (6-5) may similarly be rationalized to obtain the component parts of Y which represent the conductance and susceptance of the equivalent parallel circuit shown in Fig. 6.2(b) (G_p and B_p), viz.,

$$G_p \mp jB_p = \frac{G_s B_s^2}{G_s^2 + B_s^2} \mp j\frac{G_s^2 B_s}{G_s^2 + B_s^2} \tag{6-6}$$

The conventional SMITH CHART coordinates shown in Fig. 6.1 are not suitable for portraying, directly, the parallel-circuit component values of impedance or the series circuit component values of admittance. However, these latter component values may be plotted directly on an alternate form of coordinates shown in Fig. 6.4. The modification of the conventional SMITH CHART to obtain this alternate form involves the redesignation of all normalized coordinate values with their reciprocal values, and the rotation of coordinates through 180° with respect to the peripheral scales. Rotation of the coordinates through an angle of 180° is necessary so that fractional wavelength designations on the

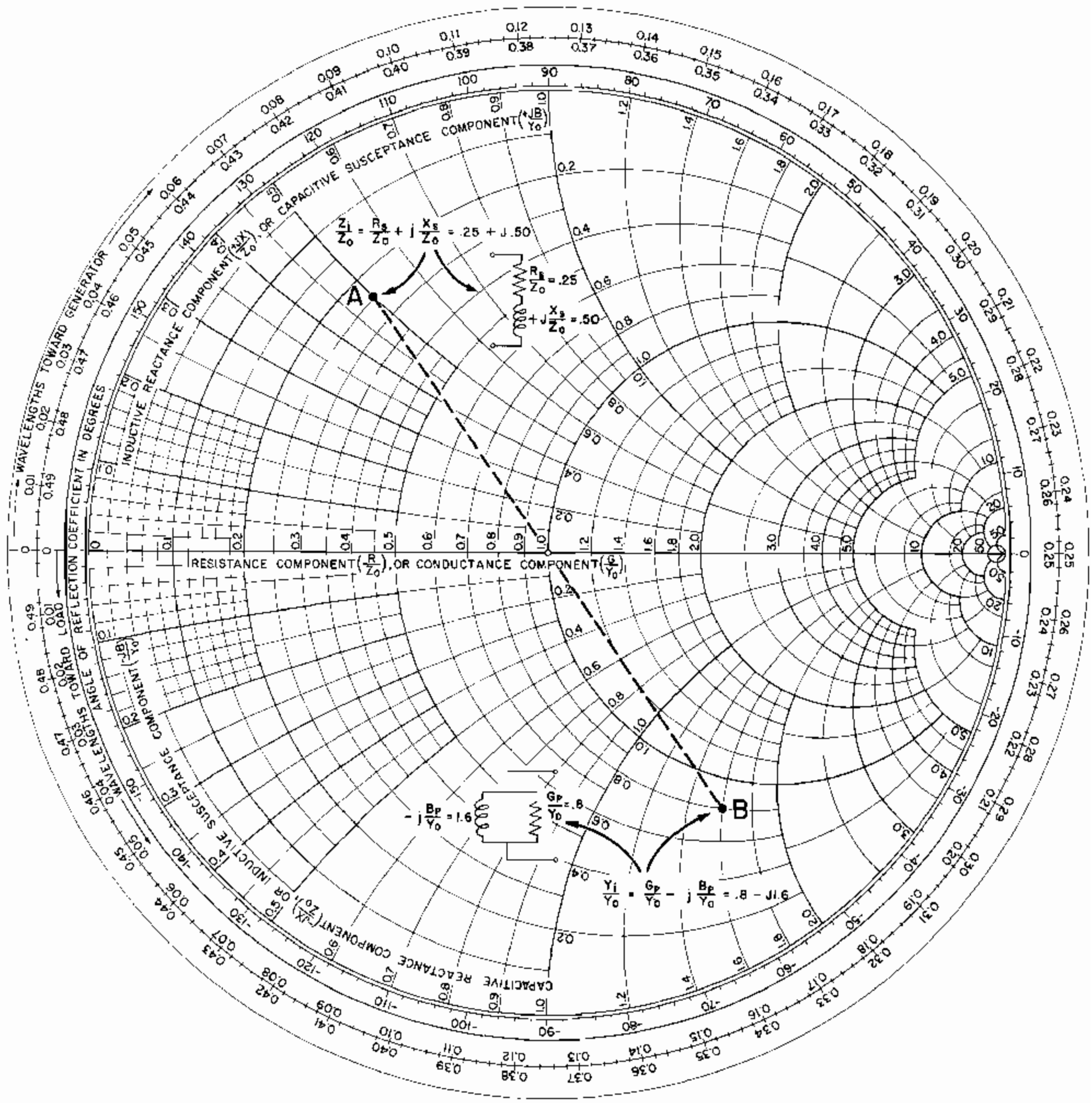


Fig. 6.3. Series-circuit representation for an impedance (A) and equivalent parallel-circuit representation for an equivalent admittance (B) at the same position along a waveguide, on SMITH CHART in Fig. 3.3.

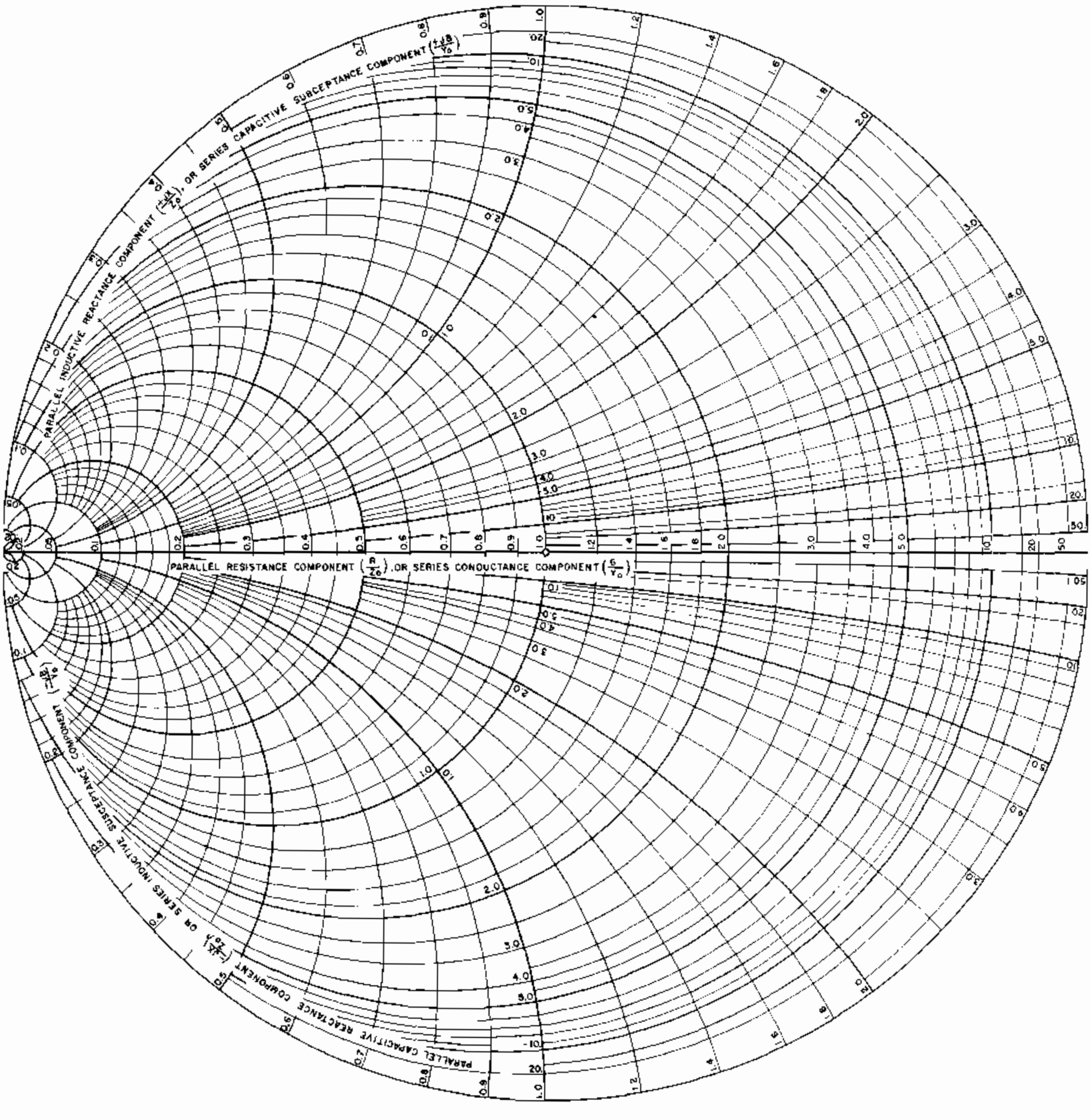


Fig. 6.4. Alternate form of SMITH CHART coordinates displaying rectangular components of equivalent parallel-circuit impedance (or of series-circuit admittance) (overlays for Charts A and C in cover envelope).

peripheral scales of this alternate form SMITH CHART refer to voltage nodal points on its impedance coordinates (and to current nodal points on its admittance coordinates), as on the conventional SMITH CHART. Also, as will be shown later, this permits the use of this alternate form of SMITH CHART coordinates to be used as an overlay for the conventional coordinates for converting series to equivalent parallel-circuit components.

The two positions A and B indicated on the alternate form of SMITH CHART coordinates (Fig. 6.6) give, respectively, the parallel-circuit impedance components, and the series-circuit admittance components of equivalent circuits. Equivalent circuit positions on the coordinates of either Fig. 6.3 or Fig. 6.6 must always be located diametrically opposite each other at equal chart radius.

6.5 SMITH CHART OVERLAY FOR CONVERTING A SERIES-CIRCUIT IMPEDANCE TO AN EQUIVALENT PARALLEL-CIRCUIT IMPEDANCE, AND A PARALLEL-CIRCUIT ADMITTANCE TO AN EQUIVALENT SERIES-CIRCUIT ADMITTANCE

On the conventional SMITH CHART of Fig. 6.3, point A represents the impedance of a circuit whose normalized series component values may be read directly from the chart coordinates. On the alternate form of SMITH CHART of Fig. 6.6, point A, at the same location, represents the impedance of a circuit whose equivalent normalized parallel component values may also be read directly from the chart coordinates. Thus, any two coincident points on these respective charts correspond to equivalent series and parallel-circuit combinations whose normalized resistive and reactive components are readable from the respective charts at a single frequency.

Thus, the alternate form of SMITH CHART coordinates shown in Fig. 6.6 (drawn without circuit diagrams or peripheral scales in Fig. 6.4) provides an overlay for conventional SMITH CHART coordinates (Fig. 3.3) which permits graphically converting component values of series-circuit normalized impedance to equivalent component values of parallel-circuit normalized impedance. Similarly, this overlay provides a convenient means for graphically converting component values of parallel-circuit normalized admittance to equivalent component values of series-circuit normalized admittance.

When the overlay of Fig. 6.4 is provided with the peripheral scales on Fig. 3.3 the resulting chart (Fig. 6.5) becomes an alternate and basic form of SMITH CHART to which all of the overlays applicable to the series component chart (Fig. 3.3 or Fig. 8.6) are equally applicable.

The general-purpose, parallel-impedance (or series-admittance) chart shown in Fig. 6.5 is reproduced as one of four translucent plastic charts whose function is described in the Preface and which is contained with the other three in an envelope in the back cover of this book (Chart B).

6.6 IMPEDANCE OR ADMITTANCE MAGNITUDE AND ANGLE OVERLAY FOR THE SMITH CHART

Complex impedance or admittance vectors $R \pm jX$ or $G \mp jB$, respectively, may also be expressed in polar form, that is, $Z\underline{/\pm\theta}$ or $Y\underline{/\mp\theta}$. The magnitude of the normalized impedance vector Z/Z_0 in terms of its rectangular components is

$$\left|\frac{Z}{Z_0}\right| = \left[\left(\frac{R}{Z_0}\right)^2 + \left(\frac{X}{Z_0}\right)^2\right]^{1/2} \tag{6-7}$$

The phase angle θ is

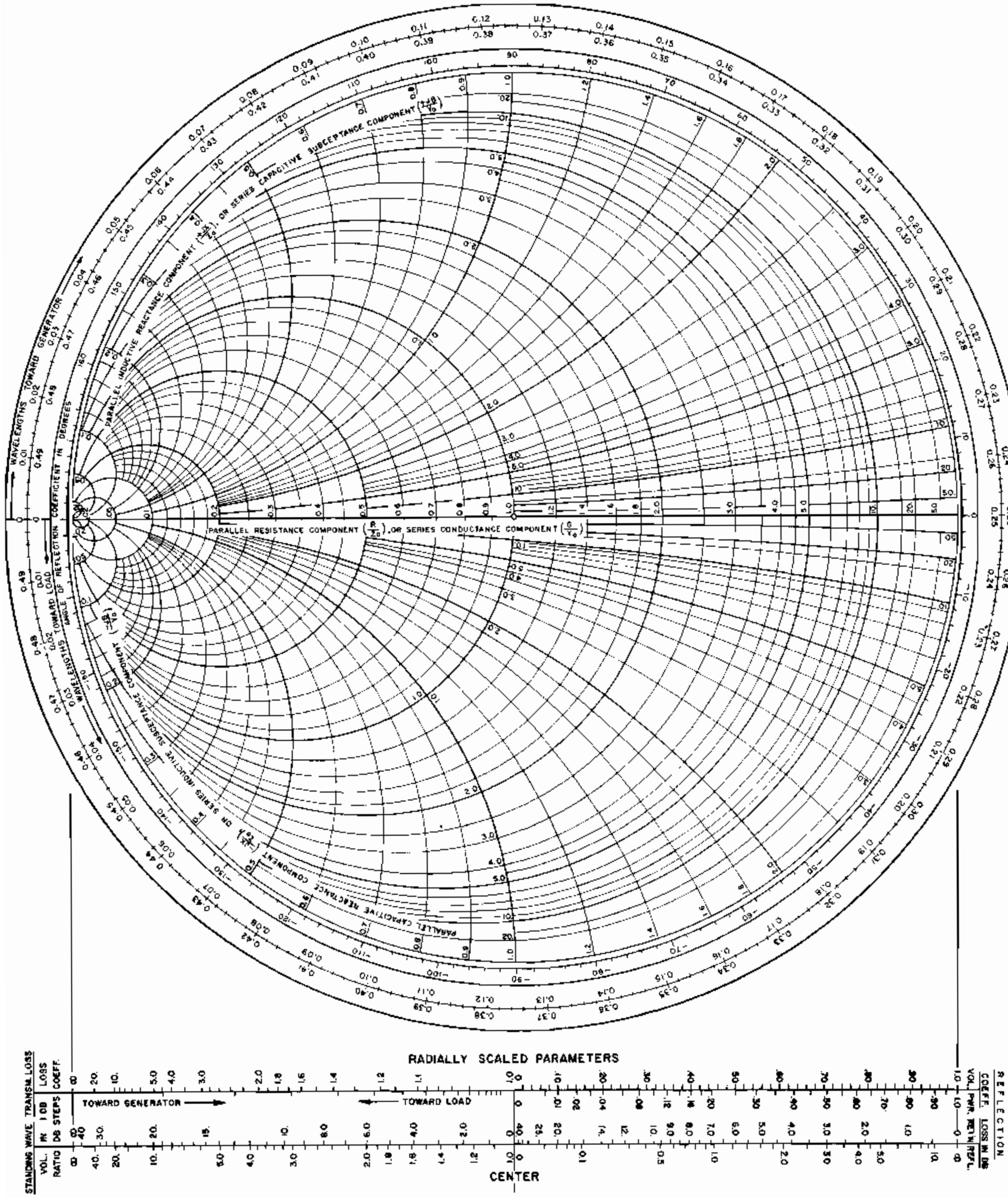


Fig. 6.5. Alternate form of SMITH CHART with coordinates of Fig. 6.4 (see Chart B in cover envelope).

$$\theta = \pm \arctan\left(\frac{X}{R}\right) \tag{6-8}$$

Similarly, the magnitude of the normalized admittance vector $\mathbf{Y}/\mathbf{Y}_0$ in terms of its rectangular components is

$$\left|\frac{\mathbf{Y}}{\mathbf{Y}_0}\right| = \left[\left(\frac{G}{Y_0}\right)^2 + \left(\frac{B}{Y_0}\right)^2\right]^{1/2} \tag{6-9}$$

The phase angle θ is

$$\theta = \mp \arctan\left(\frac{B}{G}\right) \tag{6-10}$$

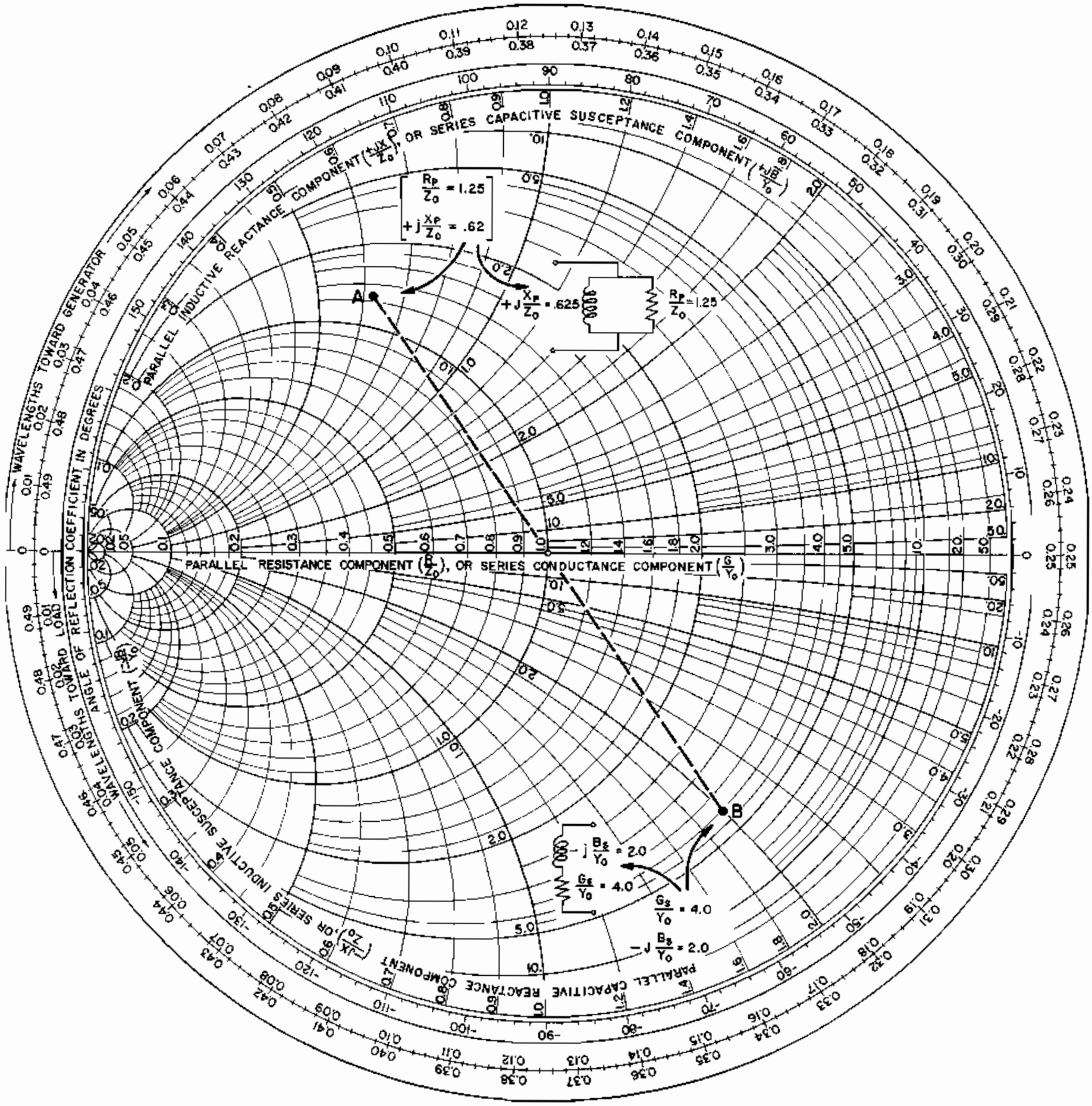


Fig. 6.6. Parallel-circuit representation for an impedance (A) and equivalent series-circuit representation for an admittance (B) at the same position along a waveguide, on SMITH CHART in Fig. 6.5.

The normalized resistance component R/Z_0, and reactance component X/Z_0, respectively, of an impedance expressed in polar notation, that is, $Z/\underline{\pm\theta}$, is

$$\frac{R}{Z_0} = \left|\frac{Z}{Z_0}\right| \cos(\pm\theta) \tag{6-11}$$

and

$$\frac{X}{Z_0} = \left|\frac{Z}{Z_0}\right| \sin(\pm\theta) \tag{6-12}$$

The phase angle of the normalized impedance vector (angle between the rotating voltage and current vectors) is also the *angle of the power factor.* This is represented graphically as an overlay for the SMITH CHART coordinates by the family of dotted curves in Fig. 4.2. This overlay also displays loci of the normalized voltage and normalized current vector extremities.

Both the magnitude and the phase angle of the normalized impedance and admittance vectors are plotted in Fig. 6.7, which is useful as an overlay for SMITH CHART A or B in the cover envelope. As oriented in Fig. 6.7, with relation to the orientation of the SMITH CHART coordinates this overlay permits the conversion of the normalized rectangular components of impedance to equivalent normalized magnitude and phase angle. When rotated through 180° from the orientation shown in Fig. 6.7, with respect to these same SMITH CHART coordinates, it permits the conversion of the normalized rectangular components of admittance to equivalent normalized magnitude and phase angle. Thus, Fig. 6.7 provides a useful means for graphically converting components of any complex impedance or admittance from the rectangular to the equivalent polar form, and vice versa. With the addition of the peripheral scales, Fig. 6.7 becomes a complete alternate form of transmission line chart commonly known as the Carter chart [9].

This general-purpose polar impedance (or admittance) chart shown in Fig. 6.8 is reproduced as the third of four translucent plastic charts whose function is described in the Preface and which is contained with the other three in an envelope in the back cover of this book (Chart C).

6.7 GRAPHICAL COMBINATION OF NORMALIZED POLAR IMPEDANCE VECTORS

The resultant of two normalized impedance vectors, expressed in polar notation, e.g.,

$$\frac{Z_1}{Z_0}\underline{/\pm\theta_1} \quad \text{and} \quad \frac{Z_2}{Z_0}\underline{/\pm\theta_2}$$

whose representative circuit elements are connected in series, may readily be obtained graphically by the familiar parallelogram construction. In this case, all vector magnitudes are directly proportional to their lengths, and the plot is most conveniently made on ordinary polar coordinate paper with a linear radial scale. The left half of a complete polar plot is used only for impedances having negative real parts (see Chap. 12).

Figure 6.9 shows a polar coordinate system on which it is possible to plot directly the resultant of two normalized impedance vectors whose representative circuit elements are connected in parallel. The resultant is determined by the same parallelogram construction used for series elements. On such a plot all vector magnitudes are inversely related to their lengths by a constant. The value of this constant is selected on the plot of Fig. 6.9 to provide a convenient general-purpose scale range. This may be varied as required to extend the range of the plot.

The significance of polar impedance vectors whose angles lie between + 90 and + 180°, or between − 90 and − 180°, is that the resistive

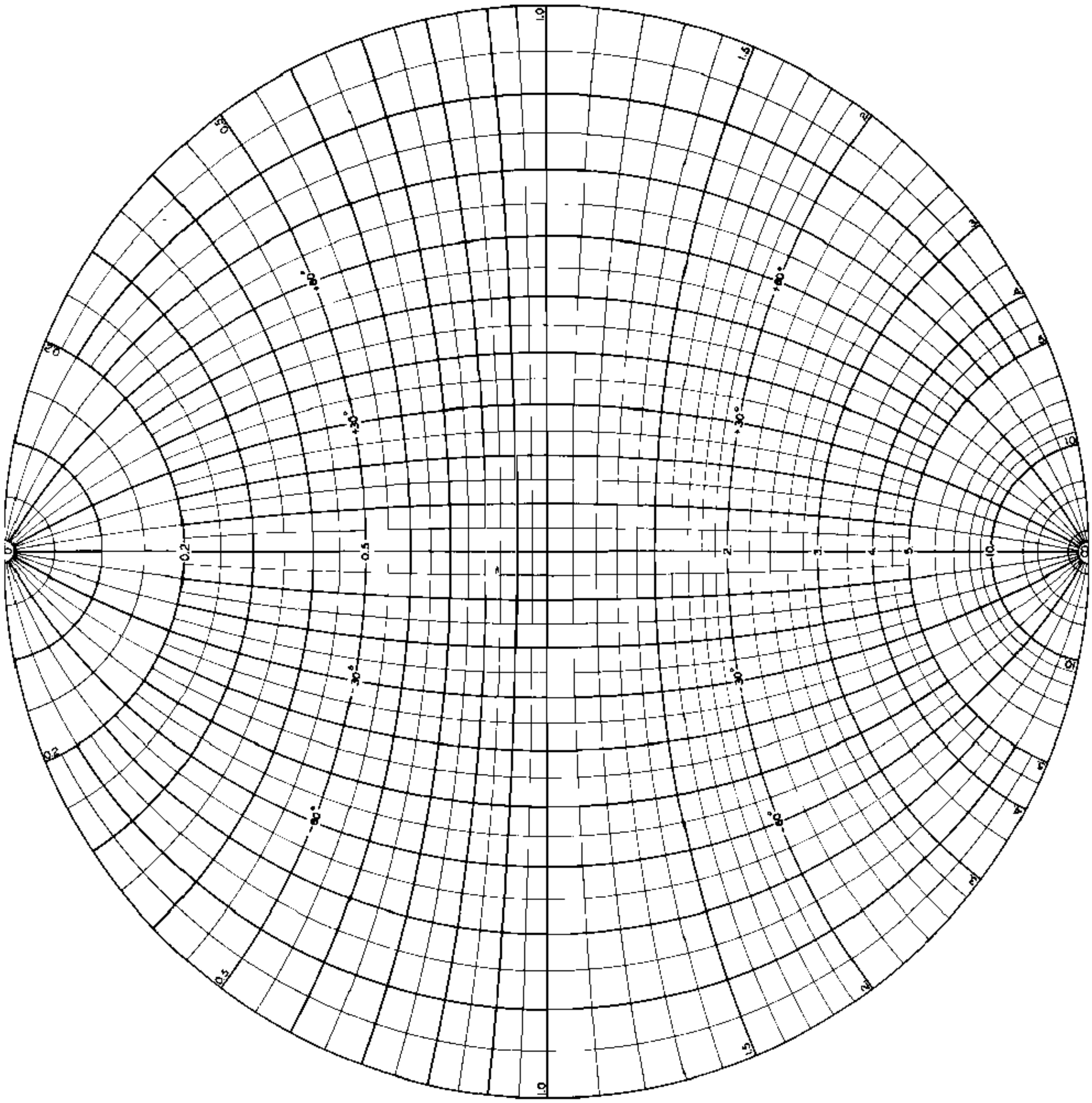


Fig. 6.7. Carter Chart coordinates displaying polar components of equivalent series-circuit impedance (or of parallel-circuit admittance) (overlay for Smith Charts A and B in cover envelope).

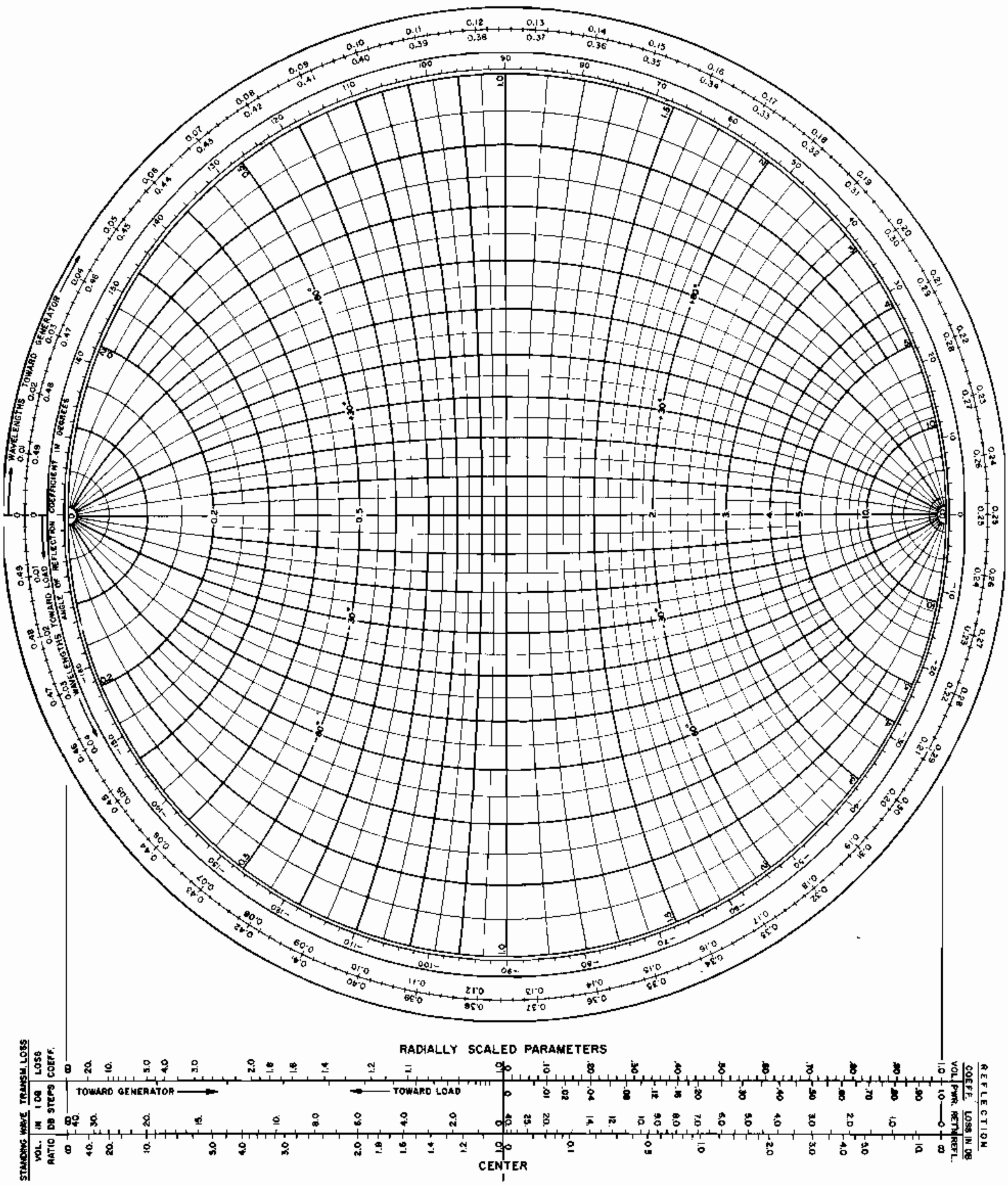
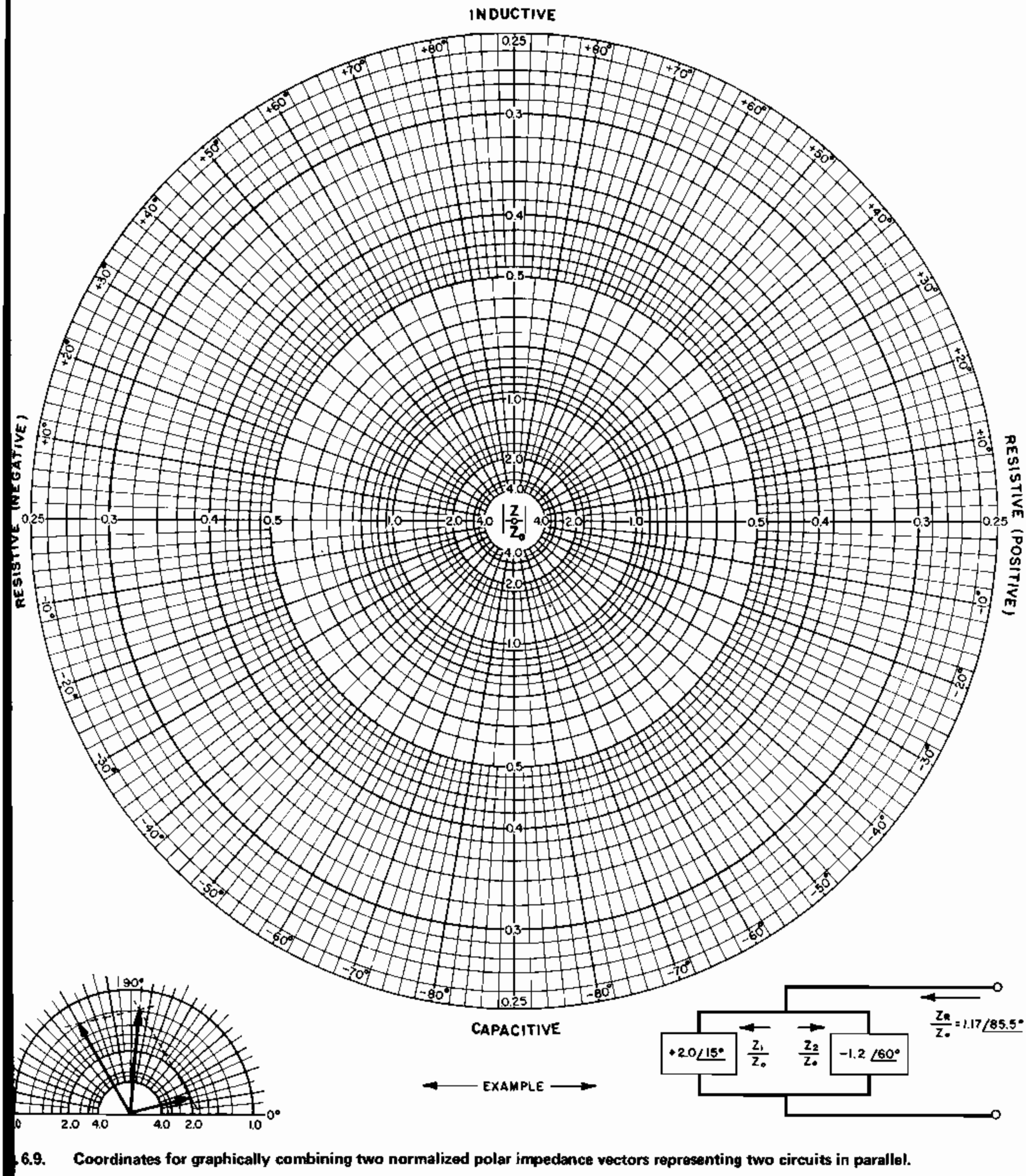


Fig. 6.8. Carter Chart with coordinates of Fig. 6.7 (see Chart C in cover envelope).



6.9. **Coordinates for graphically combining two normalized polar impedance vectors representing two circuits in parallel.**

component of the impedance represented thereby is negative. Thus, in the example shown in Fig. 6.9, the normalized polar impedance vector $\mathbf{Z}/\mathbf{Z}_0 = 1.2 \angle 120°$ represents an impedance having a negative resistance component $-R/Z_0 = 0.6$, and an inductive reactance component $+jX/Z_0 = 1.04$, that is, $1.2 \angle 120° = -0.6 + j\,1.04$, as obtained from Eqs. (6-11) and (6-12), respectively.

Figure 6.9 is useful in problems involving polar impedance vectors, such as those represented in Fig. 6.7.

CHAPTER 7

Expanded Smith Charts

7.1 COMMONLY EXPANDED REGIONS

The accuracy which can be obtained in plotting and reading out data on the coordinates of the SMITH CHART, as with any other chart, can be improved by simply increasing its size to provide space for a finer and more expanded coordinate grid. Where it is impractical to increase the overall chart size to the desired extent, small regions of special interest may be enlarged as much as is desired.

In this chapter, enlargements of the more frequently used regions (see Fig. 7.1) of the SMITH CHART will be presented and discussed, and special applications for these will be given. The graphical representation of the properties of stub sections of waveguide which are operated near their resonant or antiresonant frequency, as may be readily portrayed on two of these expanded charts, will be discussed in some detail.

The specific region on a SMITH CHART which is most commonly expanded is perhaps a circular region at its center concentric with its perimeter. This region, typically as shown in Fig. 7.2 or 7.3, is of special interest because it encompasses all possible input impedances, and equivalent admittances, along a waveguide when the load reflection coefficient or standing wave ratio is less than the value at the perimeter of the expanded central area. The expanded central portion of a SMITH CHART utilizes the same peripheral scales as the complete chart; all radial scales have the same value at the center and are linearly expanded to correspond to the linear radial expansion of the coordinates.

Other regions of the SMITH CHART which are sometimes expanded to provide greater plotting accuracy are the small (approximately rectangular) regions shown at the top of Fig. 7.1 which encompass the two diametrically opposite poles of the chart and which are bisected by the real axis. These regions (Figs. 7.4 and 7.5) are particularly useful for the representation of the electrical properties of waveguide stubs operated near their resonant or antiresonant frequency. These electrical properties include input impedance (or input admittance), frequency, bandwidth, attenuation, and Q, as will be more fully described herein.

In addition to the above conventional SMITH CHART expansions, two nonconformal transformations of the SMITH CHART coordinates will be described which, in effect,

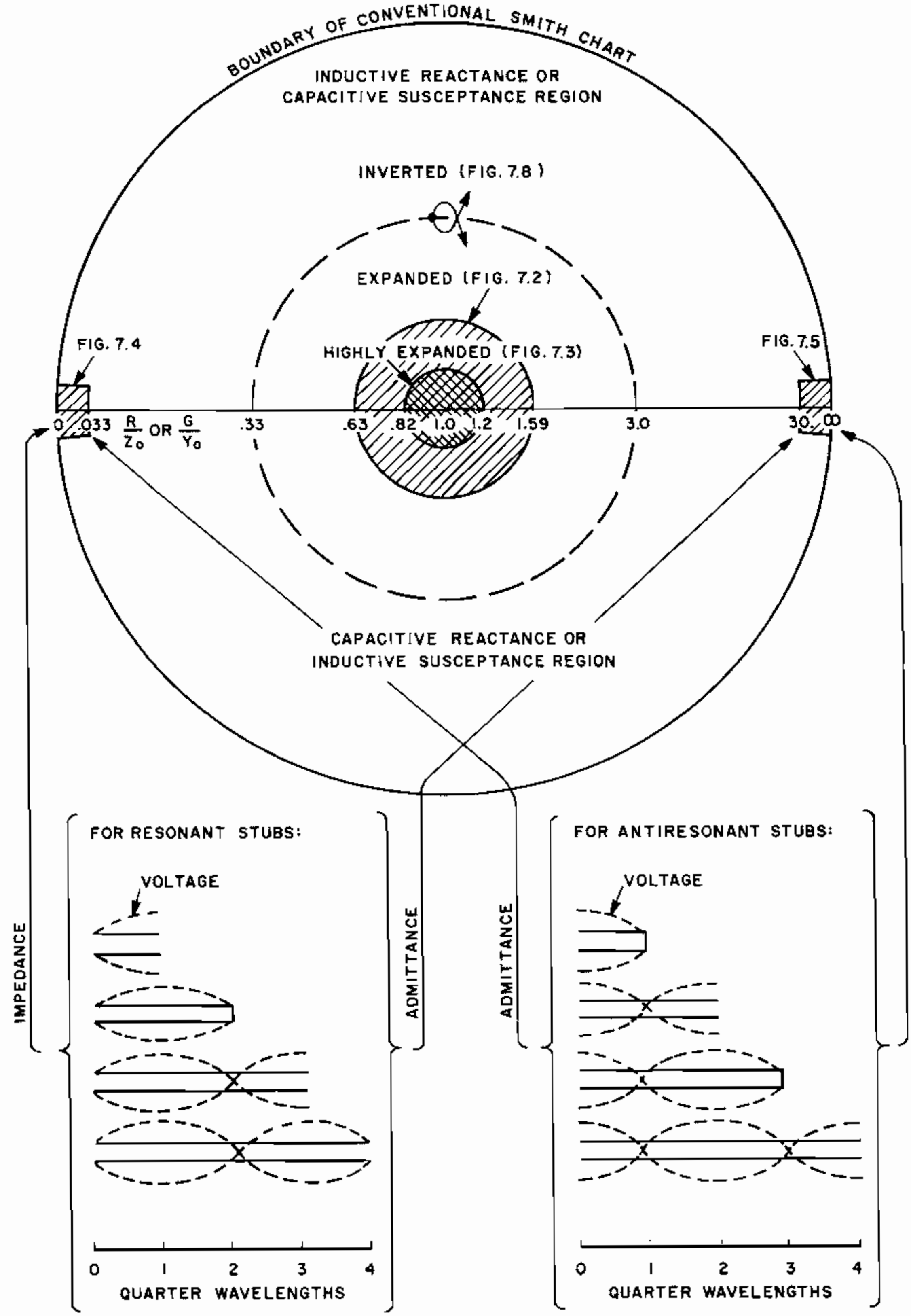


Fig. 7.1. Commonly expanded regions of the SMITH CHART.

result in expansion of certain regions of the chart coordinates at the expense of a compression of adjoining regions. One of these (Fig. 7.7) results from a transformation of the usually linear radial reflection coefficient magnitude scale into a linear radial standing wave ratio scale. On these coordinates the standing wave ratio is expressed as a number ranging from 0 to 1.0 rather than 1.0 to ∞ as is more customary in this country,* and the circular coordinates are distorted to permit this standing wave ratio scale, rather than the reflection coefficient scale, to vary linearly from unity at the chart center to zero at its rim. This, in effect, radially expands the region near the center of the conventional chart and radially compresses the region near its rim.

The other transformation of the usual SMITH CHART coordinates (Fig. 7.8) results from their being radially inverted about the linear reflection coefficient magnitude circle whose value is 0.5. This transforms the circular central region to a band adjacent to the perimeter, and vice versa; no radial expansion is provided, however.

7.2 EXPANSION OF CENTRAL REGIONS

While any degree of linear expansion of the central area of the coordinates of the SMITH CHART is possible, two expansions of this region have been selected which satisfy the large majority of applications. The first of these is shown in Fig. 7.2. This incorporates a 4.42/1 radial expansion of the central area of the SMITH CHART of Fig. 3.3, and displays reflection coefficient magnitudes between 0 and 1/4.42, that is, 0.226. This chart is suitable for displaying waveguide input impedance or admittance characteristics accompanying mismatches which produce standing wave ratios less than 1.59 (4 dB).

The chart of Fig. 7.3 is a more highly expanded version of Fig. 7.2. It incorporates a 17.4/1 radial expansion of the central area of the SMITH CHART of Fig. 3.3 and displays reflection coefficient magnitudes between 0 and 1/17.4, that is, 0.0573. It is suitable for displaying waveguide input impedance and admittance characteristics accompanying very small mismatches (standing wave ratios of less than 1.12, or 1 dB).

Like the complete SMITH CHART of Fig. 3.3, both of the above expanded charts display series components of impedance and/or parallel components of admittance. The peripheral scales are unchanged from those on the complete chart, and indicate distances from voltage nodal points when used to display impedances, and distances from current nodal points when used to display admittances. The radial scales have the same center values as those for the complete chart but are linearly expanded by the radial expansion factors indicated above.

7.3 EXPANSION OF POLE REGIONS

Figure 7.4 is a linear expansion of the SMITH CHART coordinates in the pole region where the normalized impedance, or normalized admittance, has a real component less than 0.0333 and/or a reactive component less than $\pm j\ 0.02355$. Similarly, Fig. 7.5 is a linear expansion in the opposite pole region where the normalized impedance, or normalized admittance, has a real component greater than 30 and/or a reactive component greater than $\pm j\ 42.46$.

At the scale size plotted in Figs. 7.4 and 7.5 the radius of the coordinates of a complete SMITH CHART would be 84.6 inches. Figures 7.4 and 7.5 can, therefore, display as much detail per square inch within its area as can

*In the United Kingdom, for example, the standing wave ratio is usually expressed as a number less than unity.

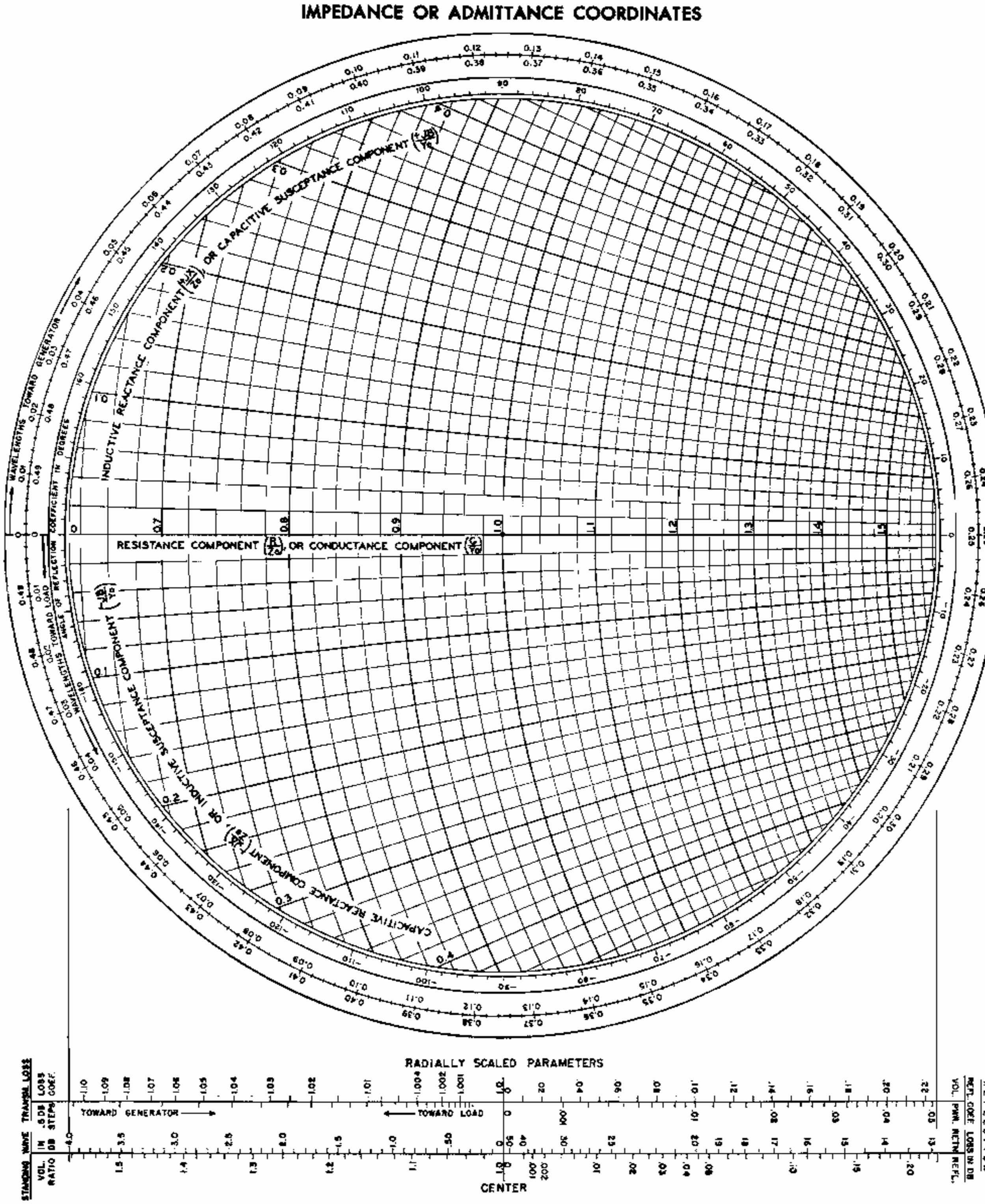


Fig. 7.2. Expanded central region of SMITH CHART.

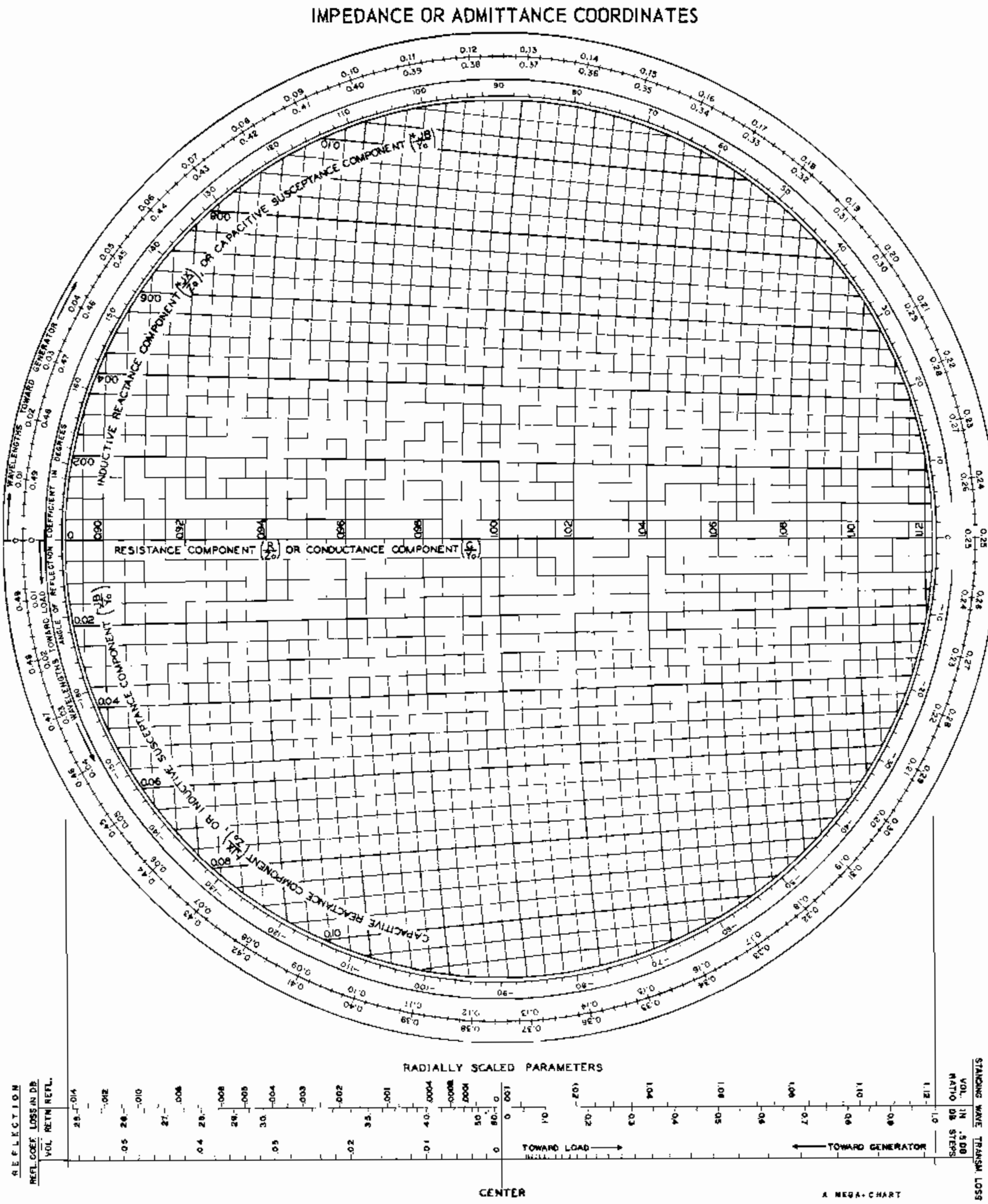


Fig. 7.3. Highly expanded central region of SMITH CHART.

be displayed on the corresponding area of a complete SMITH CHART whose coordinates are over 150 square feet in area.

The regions of the SMITH CHART shown in Figs. 7.4 and 7.5 are particularly useful for determining the input impedance (or admittance) characteristics vs. frequency of waveguide stubs operating near resonance or antiresonance.

In Fig. 7.5 short portions only of a large spiral, which could be drawn in its entirety only on a complete SMITH CHART (nearly vertical dashed curves), have been drawn. These curves trace the input impedance (or input admittance) locus on the enlarged chart coordinates as the frequency is varied within ± 1.5 percent of the resonant frequency. Due to the high degree of enlargement of coordinates the dotted spiral curves closely approximate arcs of circles centered on the 84.6-inch-radius SMITH CHART. Their departure is so slight that it may, for all practical purposes, be ignored and each curve may thus be used to represent a particular value of waveguide attenuation (one-way transmission loss) determined by its intersection with this scale at the bottom, which value holds essentially constant over the length of each arc. On Fig. 7.4 the curves of constant series resistance approximate very closely the input impedance (or input admittance) locus so that the dotted portions of the large spirals representing the locus of constant attenuation are not required to be drawn as they are on Fig. 7.5.

The use of Figs. 7.4 and 7.5 is described in more detail following a brief discussion of the relationship of waveguide attenuation to the series and parallel resonant impedance and/or admittance of short- and open-circuited waveguides.

7.4 SERIES-RESONANT AND PARALLEL-RESONANT STUBS

Resonance (specifically, electrical amplitude resonance) may be defined as a condition that exists in any passive electrical circuit containing inductance and capacitance when its combined reactance is zero. This reactance may be lumped as in conventional circuit elements or it may be distributed as along a waveguide.

Any uniform stub section of waveguide whose characteristic impedance (or characteristic admittance) is (1) essentially real, (2) short- or open-circuited at its far end, and (3) an integer number of quarter wavelengths long electrically has electrical properties which closely parallel those of simple series or parallel resonant circuits. At the resonant (mid-band) frequency the input impedance, or admittance, to such a stub is purely resistive, or purely conductive, respectively. If this input resistance is appreciably lower than the characteristic impedance of the waveguide it is called a *resonant* stub; if it is appreciably higher than the characteristic impedance of the waveguide it is called an *antiresonant* stub. Similarly, if the input conductance is appreciably higher than the characteristic admittance of the waveguide the stub is called a *resonant* stub, or if it is appreciably lower the stub is called an *antiresonant* stub.

The combination of the electrical length of a waveguide stub and its termination (open- or short-circuited) determines whether it will be resonant or antiresonant. In either case, the electrical length of the stub must be an integer number of quarter wavelengths. (See Fig. 7.1.)

Near its resonant frequency the equivalent circuit for a uniform waveguide stub is a series-resonant circuit; near its antiresonant frequency the equivalent circuit is a parallel-resonant circuit.

The attenuation (one-way transmission loss) of any uniform waveguide uniquely determines the normalized resonant impedance or admittance (Z_{min}/Z_0 or Y_{max}/Y_0) and/or the normalized antiresonant impedance or admittance (Z_{max}/Z_0 or Y_{min}/Y_0) of stubs constructed thereof. Thus

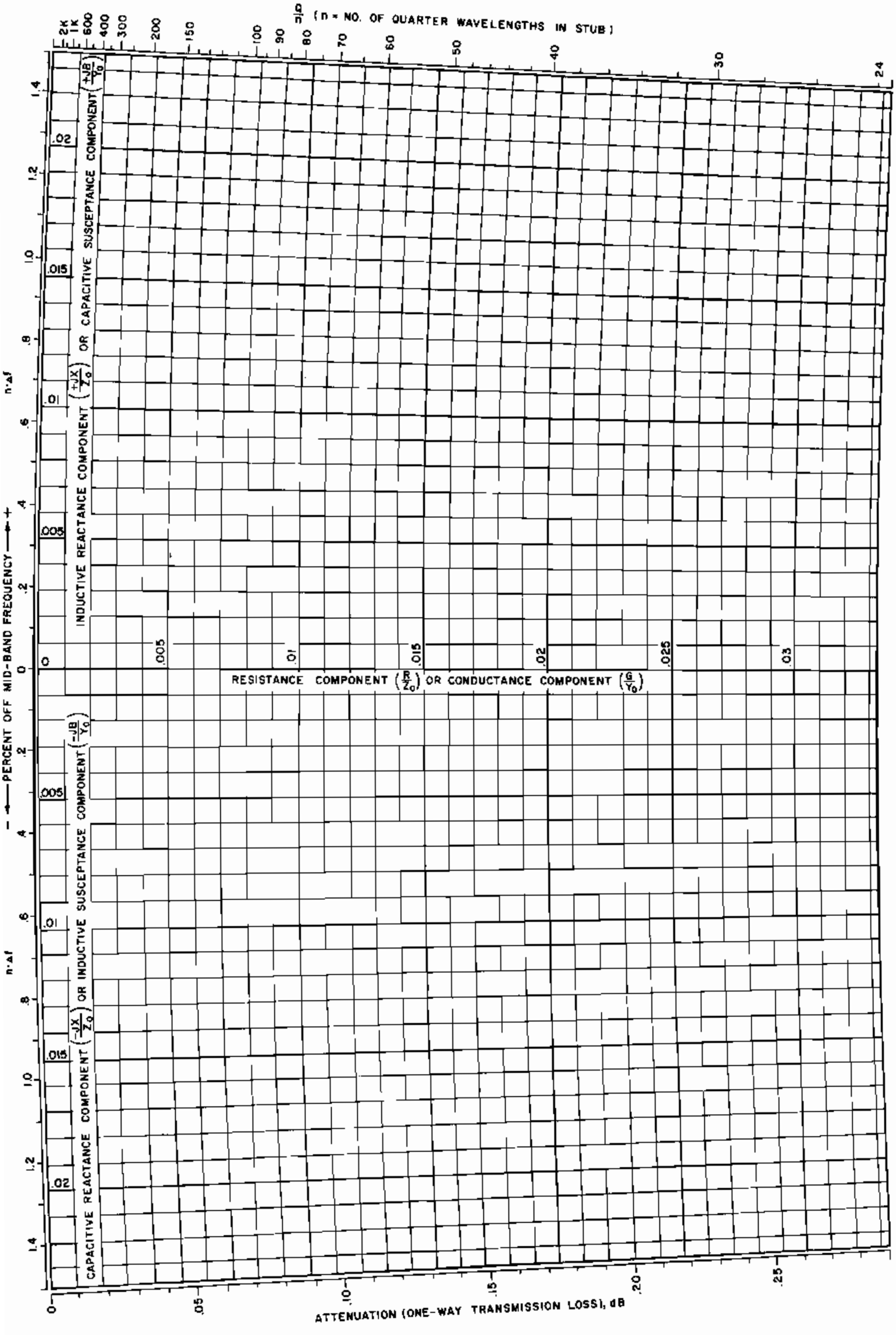


Fig. 7.4. Expanded zero-pole region of SMITH CHART.

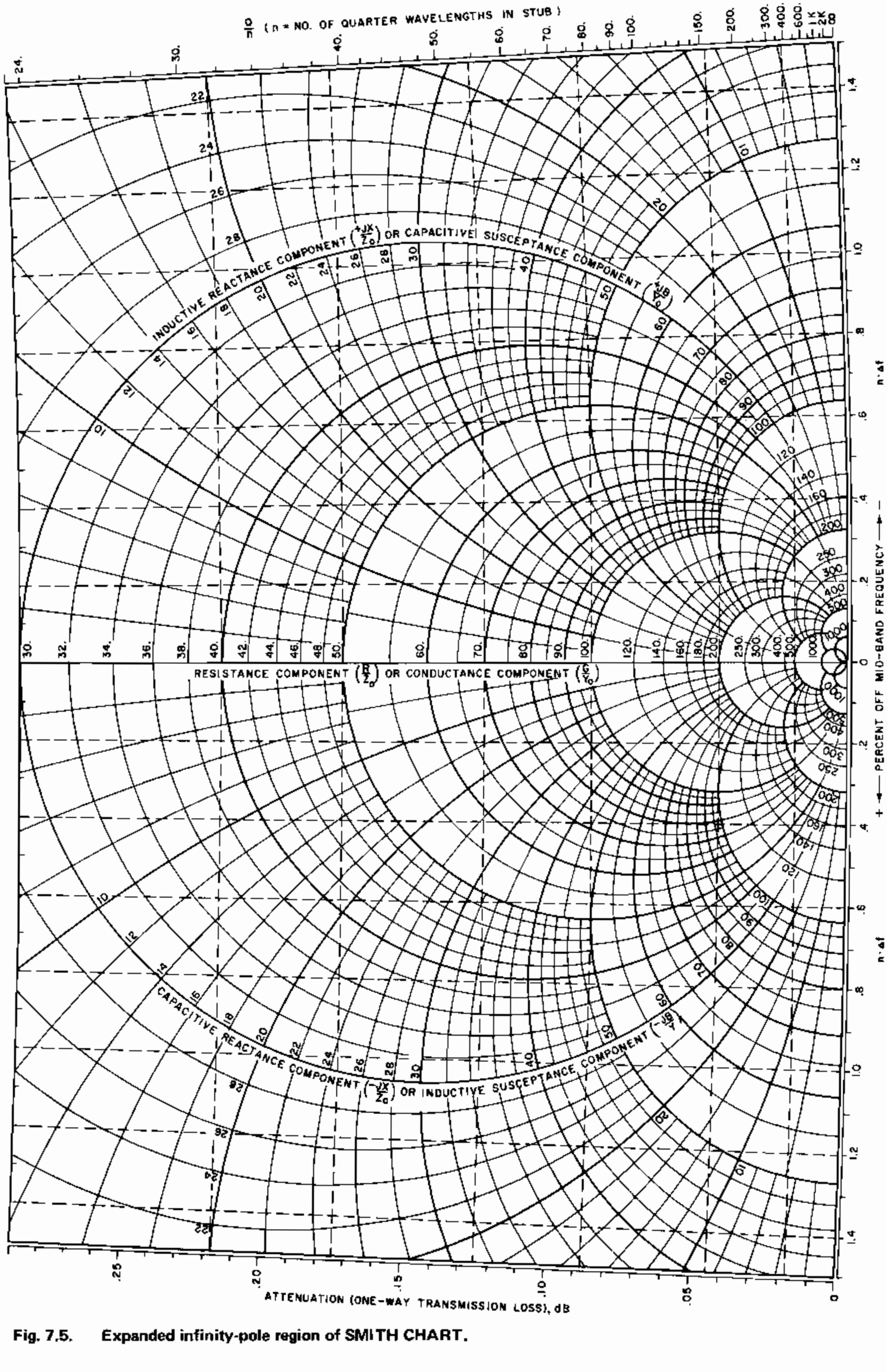


Fig. 7.5. Expanded infinity-pole region of SMITH CHART.

$$\frac{Z_{min}}{Z_0} = \frac{Y_{min}}{Y_0} = \frac{\log^{-1}(\text{dB}/10) - 1}{\log^{-1}(\text{dB}/10) + 1} \qquad (7\text{-}1)$$

and

$$\frac{Z_{max}}{Z_0} = \frac{Y_{max}}{Y_0} = \frac{\log^{-1}(\text{dB}/10) + 1}{\log^{-1}(\text{dB}/10) - 1} \qquad (7\text{-}2)$$

If $Z_{min} \ll Z_0$ and $Y_{min} \ll Y_0$, a good approximation for Eq. (7-1) is

$$\frac{Z_{min}}{Z_0} = \frac{Y_{min}}{Y_0} \cong \frac{\text{dB}}{8.686} \qquad (7\text{-}3)$$

Similarly, if $Z_{max} \gg Z_0$ and $Y_{max} \gg Y_0$, a good approximation for Eq. (7-2) is

$$\frac{Z_{max}}{Z_0} = \frac{Y_{max}}{Y_0} \cong \frac{8.686}{\text{dB}} \qquad (7\text{-}4)$$

Equation (7-3) is accurate to within 0.3 percent if the normalized resonant impedance (Z_{min}/Z_0) or antiresonant admittance (Y_{min}/Y_0) is less than 1/30, and Eq. (7-4) is accurate to within 0.3 percent if the normalized antiresonant impedance Z_{max}/Z_0 or resonant admittance Y_{max}/Y_0 is greater than 30.

The shortest possible resonant or antiresonant waveguide stub is one-quarter wavelength long. For a given size and type of waveguide this length stub has the lowest attenuation. As seen from Eq. (7-3) it will therefore have the lowest resonant impedance or admittance; or, as seen from Eq. (7-4), it will have the highest antiresonant impedance or admittance.

For example, a resonant waveguide stub which is one-half wavelength long (short-circuited at its far end) has twice the attenuation of a resonant waveguide stub which is a quarter wavelength long (open-circuited at its far end). For the same size and type of waveguide the half-wavelength stub will therefore have a resonant impedance which is twice as high as that of the quarter-wavelength stub.

7.5 USES OF POLE REGION CHARTS

As previously stated, the charts of Figs. 7.4 and 7.5 are enlargements of the pole regions of the SMITH CHART.

The chart of Fig. 7.4 is of particular use for tracing the normalized input impedance locus of waveguide stubs as a function of frequency deviations from the resonant (midband) frequency. Alternatively, it is useful for tracing the normalized input admittance locus of stubs as a function of frequency deviations from the antiresonant (midband) frequency.

Similarly, the chart of Fig. 7.5 is of particular use for tracing the normalized input impedance locus of waveguide stubs as a function of frequency deviations from the antiresonant (midband) frequency. It is also useful for tracing the normalized input admittance locus of stubs as a function of frequency deviations from the resonant (midband) frequency.

The nearly horizontal dashed lines of Fig. 7.5 (omitted in Fig. 7.4 because they parallel the lines of constant resistance and are therefore not necessary) are drawn as a convenience in translating chart values of normalized impedance or admittance to the nearly vertical scale on the chart coordinates which indicates deviation from the midband frequency.

On both Figs. 7.4 and 7.5 the stub attenuation is indicated directly on scales across the bottom. The scale values refer to the total attenuation of a stub regardless of the number of quarter wavelengths therein. To obtain a value of Q from the Q/n scales across the top of either of these charts it is only necessary to multiply the Q/n scale values by the number n of quarter wavelengths in the stub. Similarly, to obtain a specific value for Δf from the "percent off midband frequency" scale ($\Delta f \times n$) at the side of these charts it is only necessary to divide the scale values by the number n of quarter wavelengths in the stub.

The use of these charts is further illustrated by an example: Assume that we have a waveguide stub which is one-quarter wavelength long and short-circuited at its far end. Assume also that the antiresonant input impedance of the stub is known to be 200 times Z_0, the characteristic impedance of the waveguide, and we wish to know its impedance at a frequency 0.5 percent off resonance. Since the stub is only one-quarter wavelength long ($n = 1$), enter the chart of Fig. 7.5 directly at 0.5 percent on the $n\Delta f$ scale. (Had the stub been three-quarter wavelengths long ($n = 3$), for example, the chart would be entered at 3×0.5 percent or 1.5 percent on this scale.) Erect a perpendicular to the chart perimeter through the 0.5 percent point on the $n\Delta f$ scale and extend it to intersect the nearly vertical dashed curve which passes through $R/Z_0 = 200$. At this latter point on the chart coordinates the impedance in complex notation is found to be $(58 - j\,90)\ Z_0$, and the absolute magnitude is the square root of the sum of the squares or 107 Z_0. This procedure is repeated to obtain as many data points on the impedance-frequency relationship as desired. The result, in the example given, is a plot as shown on Fig. 7.6.

If it is found that the range of the chart is too small it may be extended, within limits, by multiplying the scales properly. For instance, suppose that the antiresonant impedance of the waveguide in question is 2,000 Z_0. The "200" curve in Fig. 7.5 may be used if the resistance and reactance circle

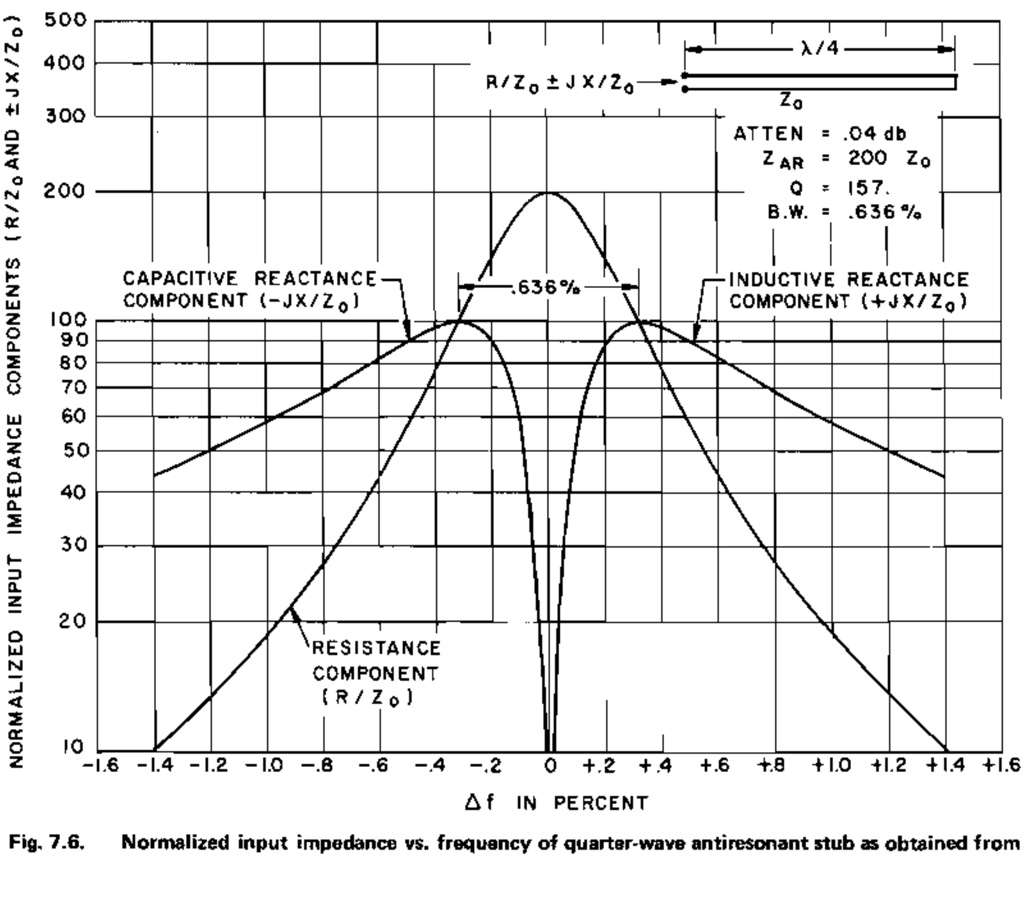


Fig. 7.6. Normalized input impedance vs. frequency of quarter-wave antiresonant stub as obtained from Fig. 7.5.

values (and the Q/n scale values) are multiplied by 10 and the "percent off midband frequency" ($n\Delta f$) scale is divided, i.e., expanded, by 10. This can be checked by noting that the impedance point $(50 - j\,50)\ Z_0$ occurs at 0.64 percent off resonance while the point $(500 - j\,500)\ Z_0$ occurs at 0.064 percent off resonance. The range of the "percent off midband frequency" scale may be increased a limited amount by division of the resistance and reactance circle values. Caution must be used, however, as the errors may be large if the extension is carried too far. Figure 7.5 is similarly used for obtaining the input admittance characteristics of stub waveguides at frequencies near resonance.

7.5.1 Q of a Uniform Waveguide Stub

The Q of a uniform waveguide stub may be defined as the ratio of its resonant or antiresonant (midband) frequency f_0 to its bandwidth. The bandwidth is defined as the total width of the frequency band within which the real part of the input impedance equals or exceeds the imaginary part. This corresponds to the half-power (3 dB) criterion for bandwidth of conventional tuned circuits; thus

$$Q = \frac{f_0}{\text{bandwidth}} \tag{7-5}$$

If the real part of the normalized resonant input impedance or normalized antiresonant input admittance (R_{min}/Z_0 or G_{min}/Y_0) of any uniform waveguide stub an integer number n of quarter wavelengths long (including any lumped resistive loading at either end) is less than approximately 0.03, for which condition Fig. 7.4 is applicable,

$$\frac{R_{min}}{Z_0} = \frac{G_{min}}{Y_0} \cong \frac{n\pi}{4Q} \tag{7-6}$$

Similarly, if the real part of the normalized antiresonant input impedance or normalized resonant admittance (R_{max}/Z_0 or G_{max}/Y_0) of any uniform waveguide stub is more than approximately 30.0, for which condition Fig. 7.5 is applicable,

$$\frac{R_{max}}{Z_0} = \frac{G_{max}}{Y_0} \cong \frac{4Q}{n\pi} \tag{7-7}$$

Along any lossless uniform waveguide R_{min}/Z_0 or G_{min}/Y_0 are the reciprocal values for R_{max}/Z_0 or G_{max}/Y_0, respectively. Furthermore, all pairs of reciprocal relationships when plotted on a SMITH CHART are at equal radius along the real axis. Thus a single scale for Q/n is applicable to both Figs. 7.4 and 7.5.

In the example given, the antiresonant impedance of a stub one-quarter wavelength long ($n = 1$) was assumed to be 200 Z_0. Its Q is determined by projecting this antiresonant impedance value vertically along the dashed line to the Q/n scale where it is found to be 157.

7.5.2 Percent Off Midband Frequency Scales on Pole Region Charts

Peripheral scales in Figs. 7.4 and 7.5 indicating "percent off midband frequency" show specific values of the function $100\ n\Delta f/f_0$, where Δf is the frequency deviation from the resonant or antiresonant (midband) frequency.

It is possible to plot such scales on these charts since this function is linearly related to the electrical length of the waveguide stub which is a linear radial parameter on the SMITH CHART. If, for example, the frequency of operation of a stub one-quarter wavelength long is changed 1 percent, from f_0 to $f_0 \pm \Delta f$, the electrical length of the stub is changed 1 percent of one-quarter wavelength, or ± 0.0025 wavelength. If the stub were three-quarter wavelengths long, this 1 percent frequency change would result in a stub length change of 3 × ± 0.0025 wavelength, and so on. The percent off midband frequency scales in Figs. 7.4 and 7.5 read directly for one-quarter wavelength stubs. For use at other stub lengths the scale values must first be divided by n.

7.5.3 Bandwidth of a Uniform Waveguide Stub

From Eq. (7-5) it will be seen that the Q of a uniform waveguide stub is simply its midband frequency divided by its bandwidth. Thus its bandwidth is its midband frequency divided by Q. In the foregoing example, the Q of the one-quarter wavelength stub was 157; its bandwidth is, therefore, 1/157 = 0.636 percent.

The band edges are equally removed on a percentage basis from the midband frequency; thus they occur in the above example at ± 0.318 percent. At the band edge frequencies, the stub input resistance and reactance is equal by definition. This may be checked by projecting these peripheral scale values (± 0.318 percent) nearly horizontal (parallel to the dashed lines) to the points where they intersect the nearly vertical dashed line passing through the antiresonant impedance value of 200 Z_0, at which points the impedance values (100 ± j 100) Z_0 will be found.

7.6 MODIFIED SMITH CHART FOR LINEAR SWR RADIAL SCALE

The chart shown in Fig. 7.7 is one of the many possible nonconformal transformations of SMITH CHART coordinates. Historically this chart, developed in 1936, was the first SMITH CHART. Although difficult to draw, the chart offers at least one significant advantage, in plotting data involving small standing wave ratios, over the conventional chart, viz., the region near its center is expanded in relation to the overall size of the chart. For the same overall chart diameter standing wave ratios up to 1.5 occupy an area 2.86 times as large as that occupied for this same standing wave ratio on the more easily drawn conventional SMITH CHART coordinates.

7.7 INVERTED COORDINATES

Another unorthodox method for achieving an expansion of the central area of the SMITH CHART is to invert the coordinates about the dotted line in Fig. 7.1, where the reflection coefficient magnitude equals 0.5. An inverted coordinate SMITH CHART is shown in skeleton form in Fig. 7.8.

All radial scales of the original chart (such as Fig. 3.3) remain unchanged but read in the opposite direction. The peripheral scales, their designations, and their sense of direction also remain unchanged. The large outer area of the inverted SMITH CHART coordinates represents an area of small reflection coefficients (standing wave ratios near unity). Points on the perimeter of the inverted coordinates represent a matched impedance termination.

Figure 7.9 shows a complete inverted SMITH CHART [14] with a finer coordinate grid structure than that shown in Fig. 7.8. The radial scales at the top of this figure are identical to those shown in Figs. 3.4 and 4.1 except that they are inverted and their lengths are reduced to correspond to the inverted chart radius.

Although inverted SMITH CHART coordinates may have advantages in special applications, they have several disadvantages. One of the disadvantages is that the families of

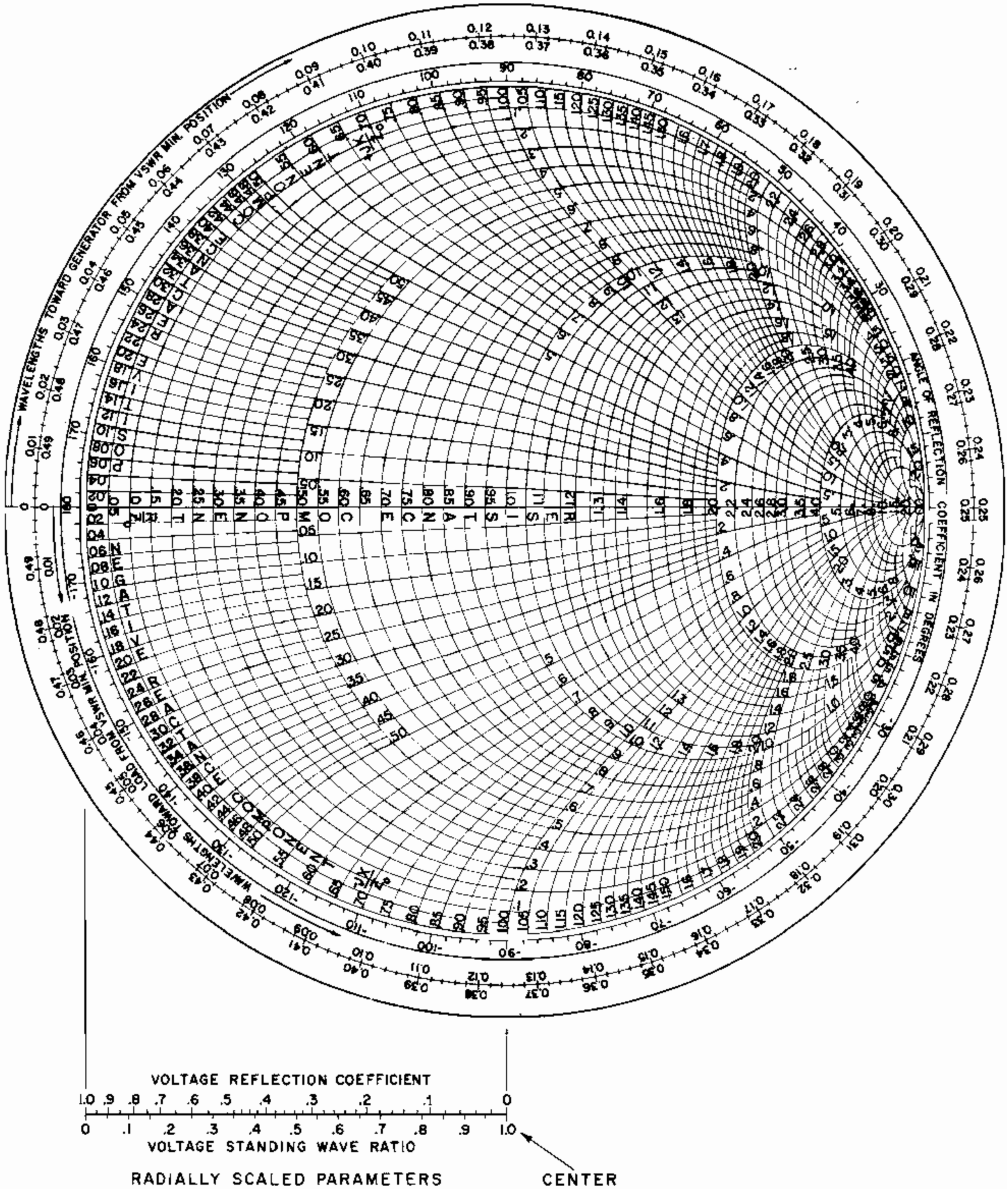


Fig. 7.7. **SMITH CHART (1936) with linear standing wave ratio scale.**

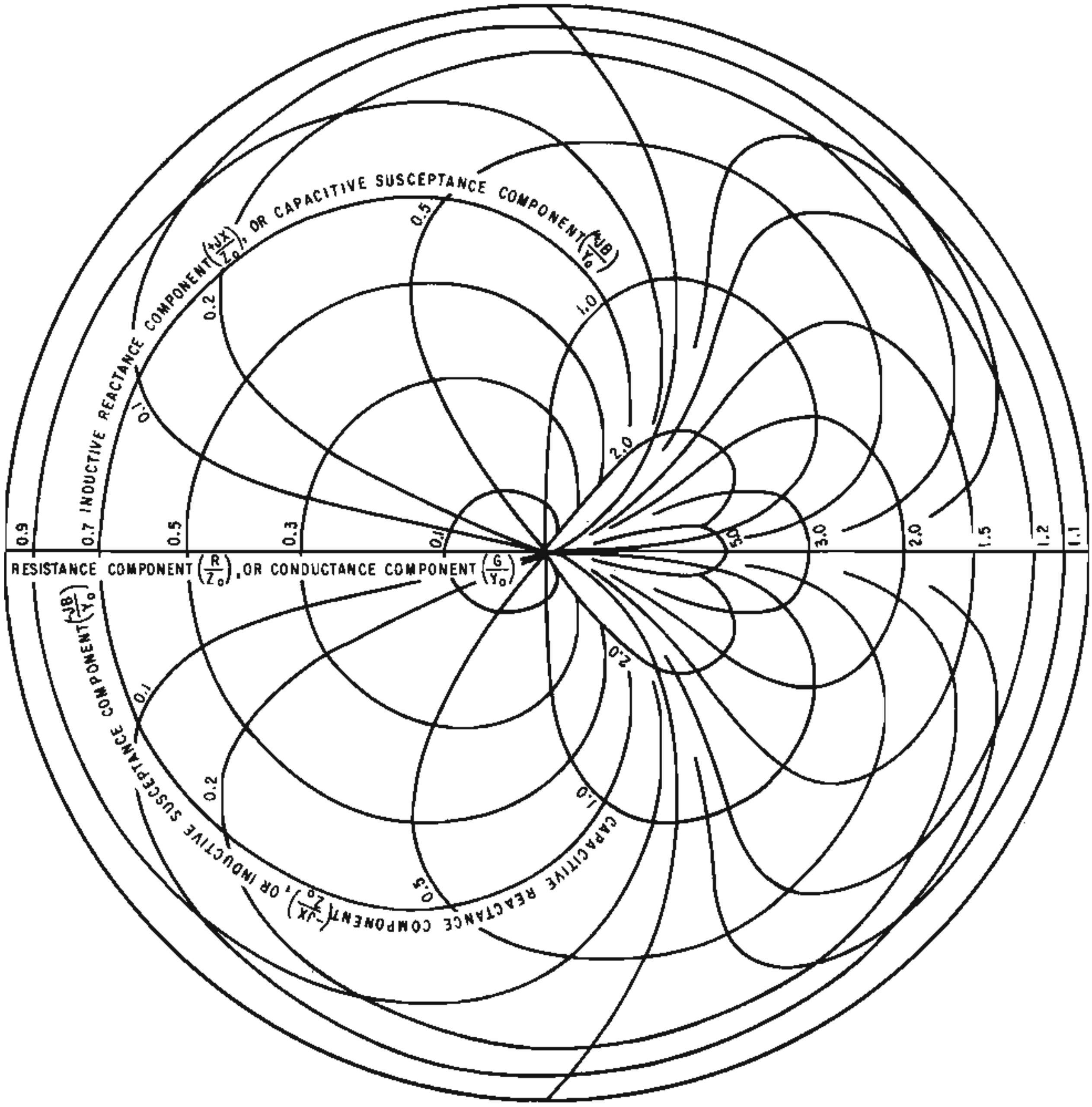


Fig. 7.8. Series impedance and parallel admittance components on inverted SMITH CHART (coordinates inverted about 0.5 reflection coefficient).

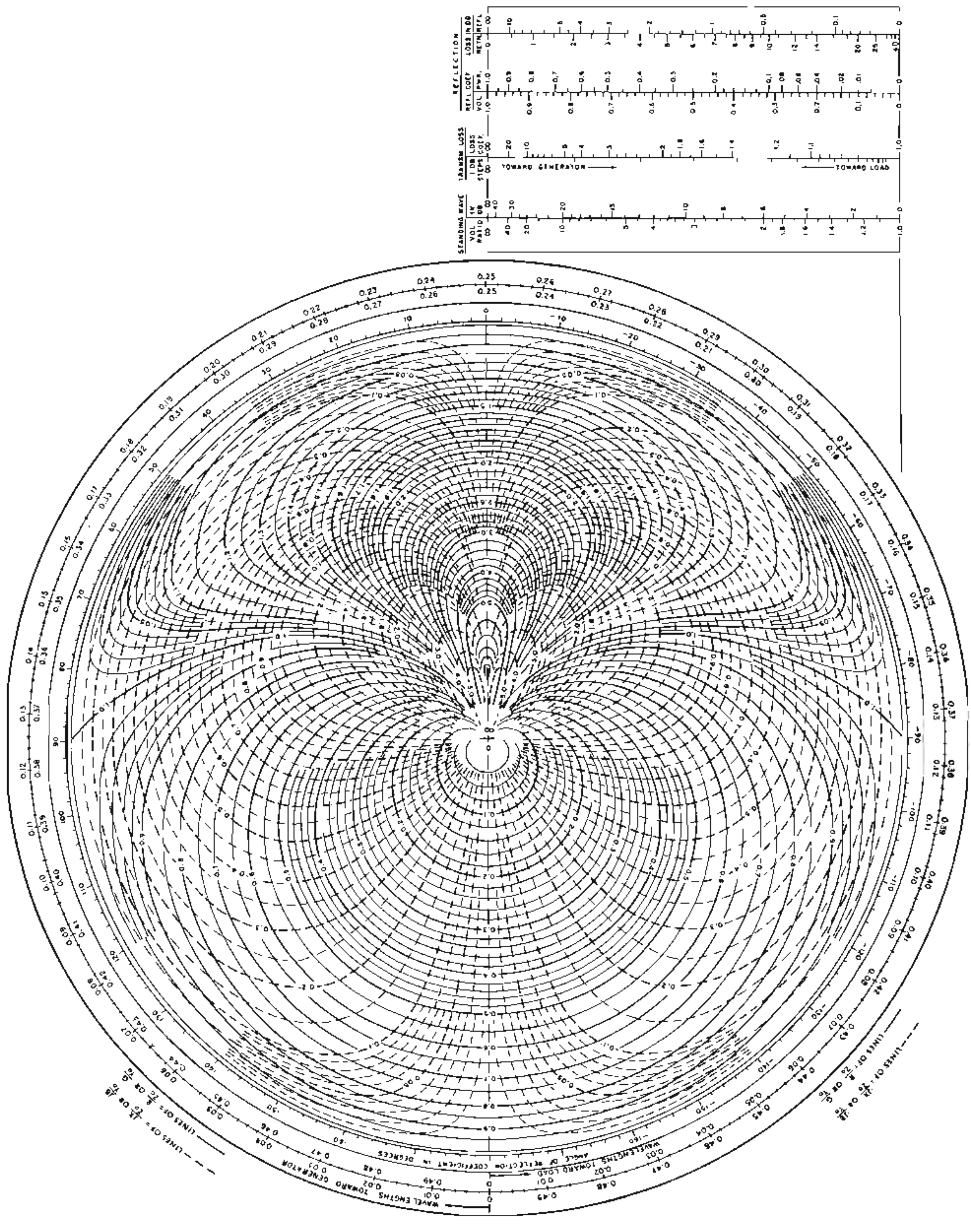


Fig. 7.9. **Inverted SMITH CHART with peripheral and radial scales.** (*Reproduced by permission of W. C. Blanchard of the Bendix Corporation and the Editors of Microwaves.*)

impedance or admittance component curves of the inverted SMITH CHART are not circles as on the conventional chart of Fig. 3.3, and they are not orthogonal families of curves; consequently, like the chart of Fig. 7.7, it is not easy to draw. Also, the magnitude of the reflection coefficient (or of the standing wave ratio), which is measured from the perimeter of the chart toward the center, is not expanded, whereas the phase angle of the reflection coefficient is highly expanded. For these reasons inverted coordinates for the SMITH CHART are not generally useful.

The later coordinate transformations illustrate only two of many possible nonconformal transformations which may have special applications. In each of these, the circular orthogonal SMITH CHART coordinates are transformed into noncircular nonorthogonal coordinates which are difficult to construct and therefore not likely to achieve widespread use.

CHAPTER 8

Waveguide Transmission Coefficient τ

8.1 GRAPHICAL REPRESENTATION OF REFLECTION AND TRANSMISSION COEFFICIENTS

Numerous attempts have been made to find a simpler grid [125] than that of the SMITH CHART on which waveguide transmission and reflection functions could be displayed. This chapter presents one solution to the problem which does not alter the basic SMITH CHART coordinates and which because of its inherent simplicity has certain practical advantages.

8.1.1 Polar vs. Rectangular Coordinate Representation

Polar coordinates are generally chosen to graphically portray the voltage and current reflection and transmission coefficients along a waveguide as shown in Figs. 8.1 and 8.2, and as described more fully in Chaps. 3 and 5, respectively. Their component values (magnitude and phase) are directly related to the voltage and current at any position along a standing wave which can readily be observed and measured. Accordingly, they have a well-defined physical significance. It is not the purpose of this chapter to depreciate the polar coordinate representation of these coefficients since this is extremely useful, but rather to show how an alternative rectangular coordinate representation can offer simplifications and advantages in some applications. This chapter will also show how the transmission coefficient components (both polar and rectangular) can be represented on the SMITH CHART in a manner which will not obscure the basic impedance or admittance coordinates with additional superimposed curves.

Any vector can, of course, be graphically represented on either polar or rectangular coordinates. For the specific representation of the reflection and transmission coefficients the use of rectangular coordinates makes it possible to employ a single rather than a dual set of coordinates. This single set serves, alternatively, to represent either of these coefficients as they coexist at all positions along a waveguide. Furthermore, the rectangular

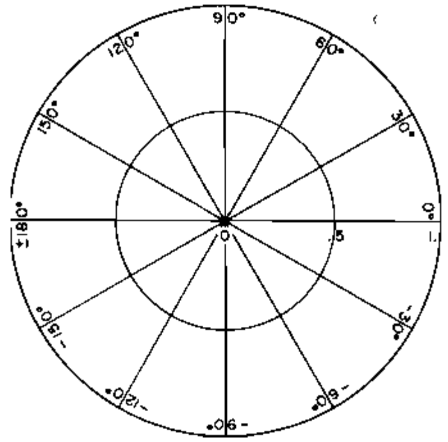


Fig. 8.1. Vol. refl. coeff.—polar coord.

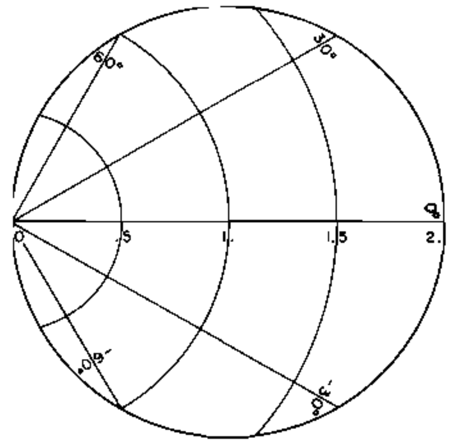


Fig. 8.2. Vol. transm. coeff.—polar coord.

component values of each of these coefficients are mathematically more simply related to each other, and to the impedance or admittance coordinate components of the SMITH CHART, than are the polar component values; also, as with polar coordinates, they are fully compatible with the SMITH CHART coordinates and therefore can be used as an overlay thereon. Evaluation of these coefficients in terms of their rectangular components provides the data necessary to generate a spot or a trace on conventional cathode ray tubes, and on mechanical (x, y) curve plotters. Finally, such a waveguide chart can be constructed with ordinary cross-section paper available to any engineer.

8.1.2 Rectangular Coordinate Representation of Reflection Coefficient ρ

If the voltage reflection coefficient whose polar magnitude ranges between 0 and 1.0, and whose phase angle ranges between 0 and $\pm$ 180° (see Fig. 8.1), is expressed in equivalent complex notation, it can then be plotted on rectangular coordinates as shown in Fig. 8.3. In the latter plot the X component ranges uniformly from − 1.0 to + 1.0, and the Y component ranges uniformly from + j 1.0 to − j 1.0. As in the polar representation, the zero value for this coefficient (no-reflection point) lies at the center of the plot.

The same radial scale as used on polar coordinates, representing the magnitude of the voltage reflection coefficient, and ranging from 0 at the center of the chart to 1.0 at its periphery, is applicable to the equivalent rectangular coordinate representation. Also, the same peripheral scales, representing the phase angle of the voltage reflection coefficient and the relative positions along the waveguide, are applicable thereto. Thus, the rectangular coordinate representation of Fig. 8.3 is directly applicable as an overlay for the more usual polar coordinate representation of Fig. 8.1.

8.1.3 Rectangular Coordinate Representation of Transmission Coefficient τ

In a manner somewhat analogous to that described above for the voltage reflection coefficient the voltage transmission coefficient, whose polar magnitude ranges between 0 and 2.0 and whose phase angle ranges from + 90 to − 90° (see Fig. 8.2), may be expressed in equivalent complex notation and then plotted on rectangular coordinates, as shown in Fig. 8.4. In this latter plot, however, the X component ranges uniformly from 0 to + 2.0, and the Y component ranges uniformly from + j1.0 to − j 1.0. As in the polar representation of the transmission coefficient, and in both polar and rectangular representations of the reflection coefficient, the no reflection point lies at the center of the respective circular plots.

The magnitude scale for the voltage transmission coefficient, whose zero value lies on the rim of the chart at the origin of the polar coordinates (see Fig. 8.2), is similarly applicable to the equivalent rectangular representation, and its zero value also lies at the origin of these latter coordinates. As in the case for the reflection coefficient, the peripheral angle scale representing the phase angle of the voltage transmission coefficient (referenced to the origin of the polar coordinates at the periphery of the chart) is applicable to this rectangular representation. Thus, the rectangular coordinate representation for the transmission coefficient in Fig. 8.4 is directly applicable as an overlay for the polar coordinate representation of Fig. 8.2.

8.1.4 Composite Rectangular Coordinate Representation of ρ and τ

It will be noted from Figs. 8.1 and 8.2 that, in polar form, the coordinate grids for the voltage reflection and transmission coefficients are not coincident with each other (one being displaced with respect to the other by the chart radius), while in the respective rectangular forms (Figs. 8.3 and 8.4) the coordinates, *per se*, are one and the same—the difference in this latter case being that the labeling thereon is different for the reflection and for the transmission coefficient real parts. Thus, a composite representation of those two coefficients on a common rectangular grid is possible in this case. Figure 8.5 shows a mutually compatible coordinate grid for both reflection and transmission coefficients ρ and τ, respectively. Radial and peripheral scales of the SMITH CHART relate points on this common grid to the standing wave ratio and wave position, and to other uniquely related guided-wave parameters, each of which has been discussed individually in previous chapters.

Through a comparison of the geometry of any common point on the polar coordinates

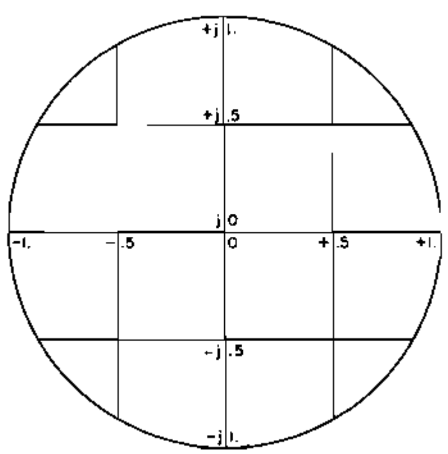


Fig. 8.3. Vol. refl. coeff.—rect. coord.

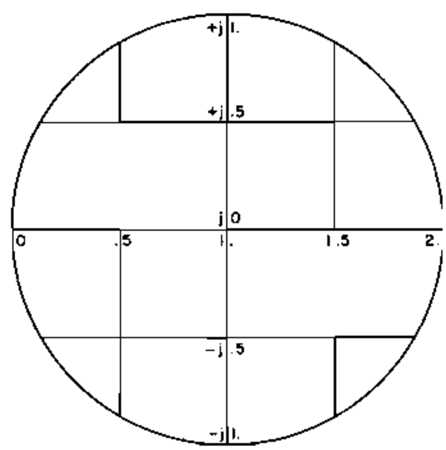


Fig. 8.4. Vol. transm. coeff.—rect. coord.

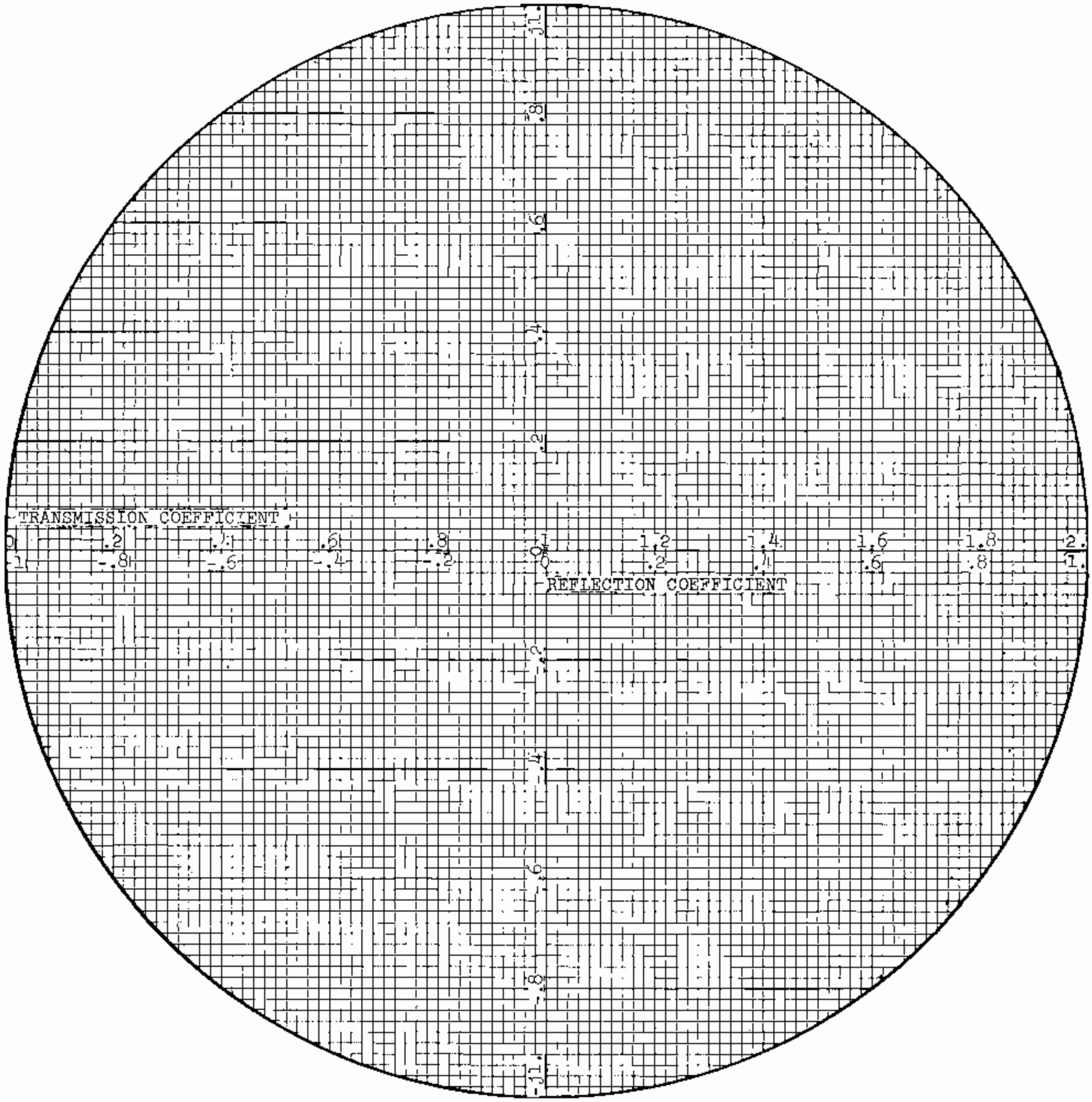


Fig. 8.5. Rectangular coordinate representation of transmission and reflection coefficients (overlay for Charts A, B, or C in cover envelope).

of Figs. 8.1 and 8.2 it will be seen that in polar form the magnitudes and phase angles of ρ relative to τ are given by the following quadratic trigonometric equations, as was further described in Chap. 5, viz.,

$$\tau = \sqrt{\rho^2 + 2\rho \cos\alpha + 1} \tag{8-1}$$

$$\rho = \sqrt{\tau^2 - 2\tau \cos\beta + 1} \tag{8-2}$$

$$\beta = \tan^{-1} \frac{\rho \sin\alpha}{\rho \cos\alpha + 1} \tag{8-3}$$

$$\alpha = \tan^{-1} \frac{\tau \sin\beta}{\tau \cos\beta - 1} \tag{8-4}$$

where τ and ρ are the magnitudes, and β and α are the respective phase angles, of the voltage transmission and voltage reflection coefficients, respectively, and in which $0 \leq \tau \geq 2$, and $0 \leq \rho \geq 1$.

On the rectangular coordinates of Figs. 8.3 and 8.4, and on the composite rectangular coordinates of Fig. 8.5, the same relationships are given by two simple linear equations, viz.,

$$\tau_X = \rho_X + 1 \tag{8-5}$$

and

$$\tau_Y = \rho_Y \tag{8-6}$$

where the subscripts X and Y indicate real and imaginary components, respectively, of the respective vector coefficients τ and ρ, and in which $-1 \leq (\tau_Y \text{ or } \rho_Y) \geq +1$.

From Eqs. (8-5) and (8-6) one observes the interesting fact that the real components of τ and ρ invariably differ by unity, and the imaginary components are invariably equal to each other at any given position along a waveguide.

The conversion of the component values of ρ to equivalent normalized waveguide complex input impedances is possible by virtue of the well-known vector relationship which plots the SMITH CHART

$$Z = \frac{1 + \rho}{1 - \rho} \tag{8-7}$$

where Z is the normalized complex input impedance and ρ is the complex voltage reflection coefficient.

The relationships expressed by Eqs. (8-1) through (8-6) are consistent with accepted definitions [11] for the voltage transmission coefficient and the voltage reflection coefficient (in a transmission medium), as applied to a transmission line or waveguide.

8.2 RELATION OF ρ AND τ TO SMITH CHART COORDINATES

The SMITH CHART of Fig. 8.6 includes the basic impedance or admittance coordinates, shown in Fig. 2.3, with the addition of four peripheral scales. The two outermost of these scales indicate the relative position along the waveguide in either direction from a voltage null position, and the two innermost scales indicate the angle of the reflection coefficient and the angle of the transmission coefficient, as plotted in polar form. As discussed in Chap. 5, when the SMITH CHART is used to represent impedances the peripheral reflection coefficient angle scale thereon refers to the voltage reflection coefficient and when this chart is used to represent admittances this scale refers to the current reflection coefficient. The same rule applies to the peripheral transmission coefficient angle scale in Fig. 8.6. A rotation of either of these scales about the center of the chart reverses its application, i.e., the scale will then apply to current as related to the impedance coordinates or voltage as related to admittance coordinates.

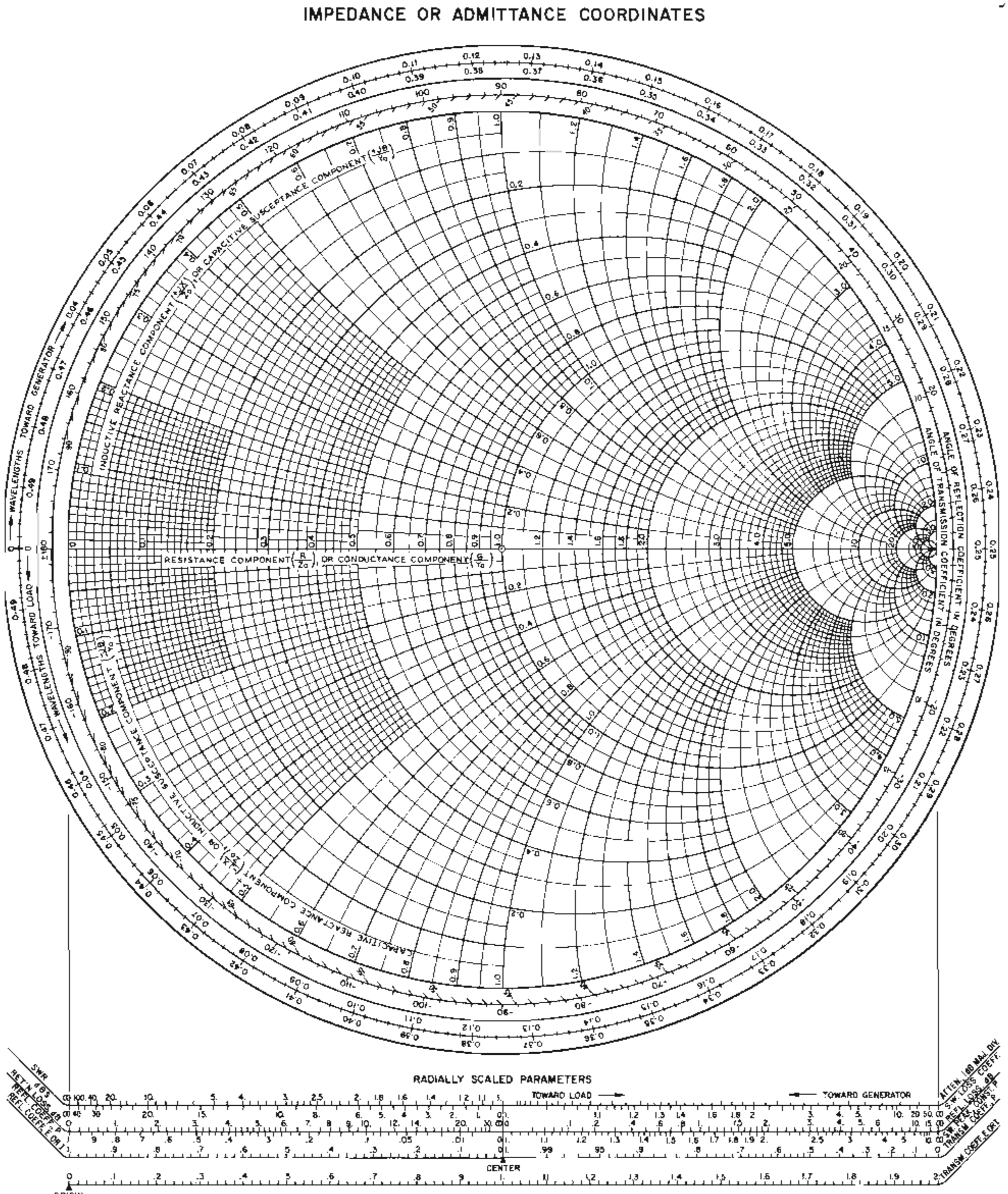


Fig. 8.6. SMITH CHART (1966) with transmission coefficient scales (see Chart A in cover envelope).

IMPEDANCE OR ADMITTANCE COORDINATES

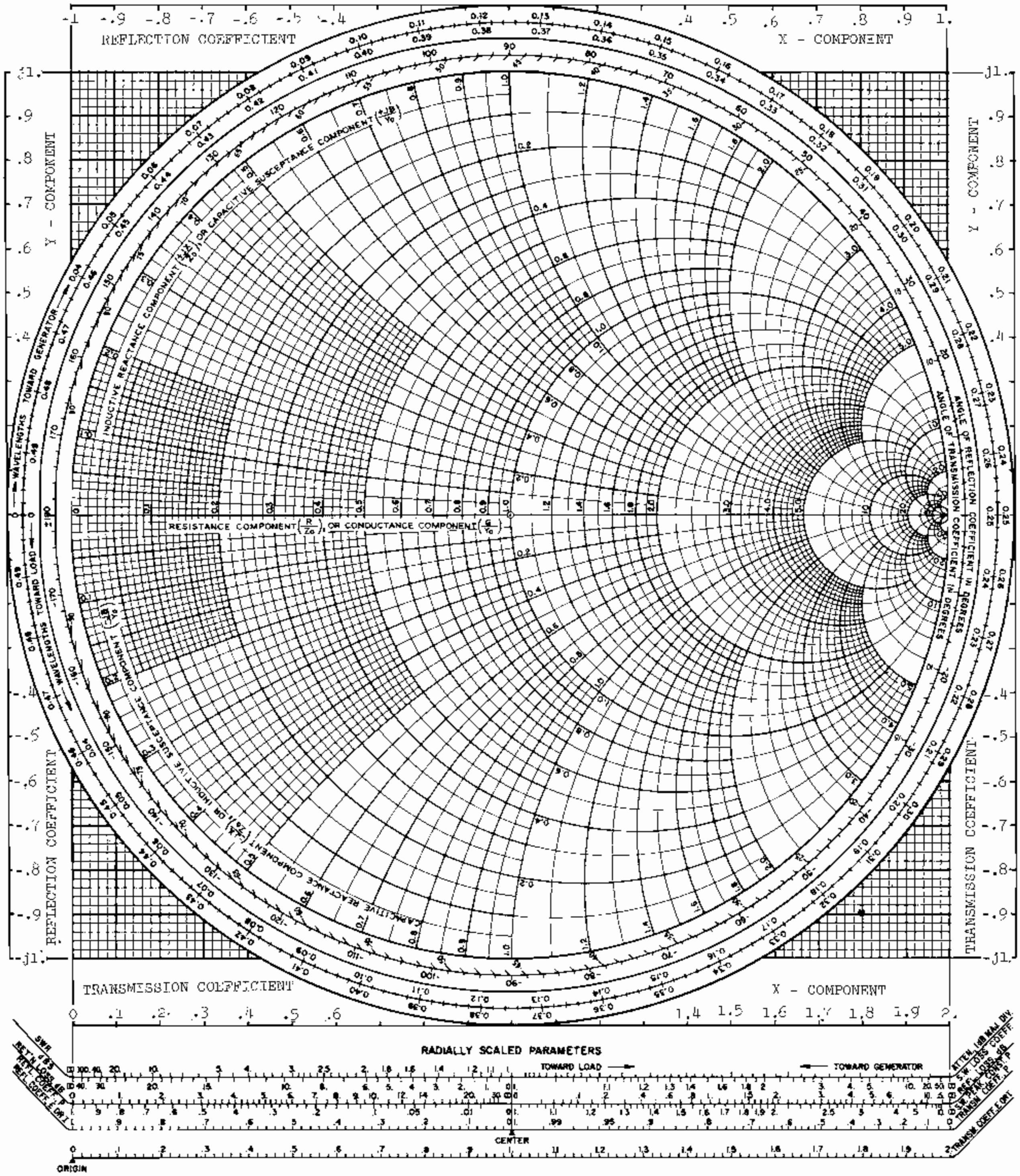


Fig. 8.7. SMITH CHART (1966) with polar and rectangular coordinate transmission and reflection coefficient scales.

Across the bottom of Fig. 8.6 all of the radial scales described in previous chapters are displayed. Also displayed is a polar coordinate transmission coefficient magnitude scale measuring in all cases from the origin of the coordinates.

The rectangular coordinates for ρ and τ described in this chapter can be added to Fig. 8.6 to produce the composite chart of Fig. 8.7. This addition involves only rectangular coordinate component scales for ρ and τ, to which points on the chart coordinates can readily be projected. The superposition of Figs. 8.5 and 8.6 to produce Fig. 8.7 is thus in effect accomplished without confusing the basic impedance or admittance coordinates.

As was the case with the polar coordinate scales, the rectangular component scales for ρ and τ apply to voltage reflection and transmission coefficients in relation to impedance coordinates and to current reflection and transmission coefficients in relation to admittance coordinates; and a rotation of either of these latter scales through 180° about the center of the chart reverses its application.

8.3 APPLICATION OF TRANSMISSION COEFFICIENT SCALES ON SMITH CHART IN FIG. 8.6

An example of the application of the polar coordinate transmission coefficient scales in Fig. 8.6 is shown in Fig. 8.8(a), which is a plot of the amplitude and phase of the voltage or current transmission coefficient attending a standing wave whose ratio is 3. The plot of this coefficient is identical to that of the standing wave. Figure 8.8(b) shows an equivalent plot for the rectangular components X and Y. If the rectangular component values X and Y of the voltage or current transmission coefficient τ are known, the polar magnitude is readily obtained from

$$|\tau| = (X^2 + Y^2)^{1/2} \tag{8-8}$$

and the polar angle from

$$\angle\beta = \tan^{-1} \frac{X}{Y} \tag{8-9}$$

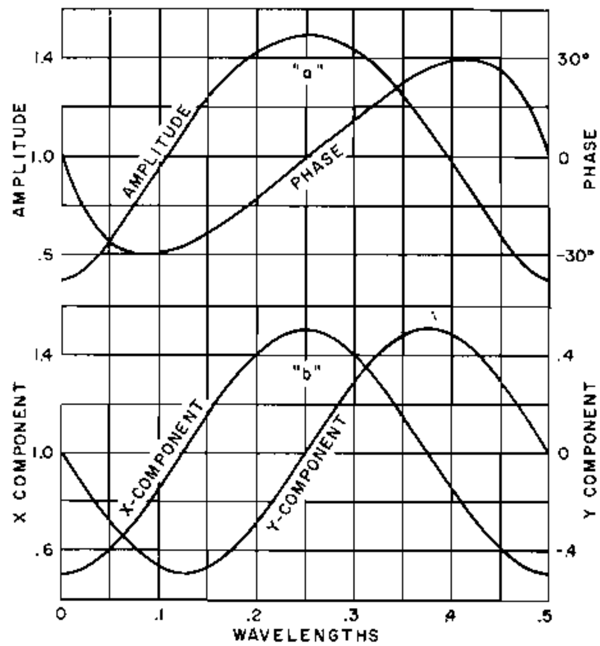


Fig. 8.8. Polar and equivalent rectangular components of waveguide transmission coefficient for SWR = 3.0.

The X- and Y-component curves (Fig. 8.8(b)) are seen to have identical shapes which are sinusoidal regardless of their maximum and minimum amplitude value, whereas the shapes of the amplitude and phase component curves (Fig. 8.8(a)) are a function of the magnitude of the SWR to which they apply, and approach sinusoidal shapes only when the SWR approaches infinity. Furthermore, X- and Y-component curves are always displaced, one from the other, by 45° and consequently may be drawn by inspection, whereas the displacement of the amplitude and phase curves of Fig. 8.8(a) varies with the SWR between the limits of 0 and 90°. Thus when the transmission coefficient is expressed in rectangular components a simple sine function plot of the standing wave ratio serves to display the shape of the X- and Y-component curves for a current or voltage standing wave of any ratio of maximum to minimum (SWR).

8.4 SCALES AT BOTTOM OF SMITH CHARTS IN FIGS. 8.6 AND 8.7

The linear scale representing the voltage or current transmission coefficient magnitude, which stems from the origin of the coordinates of the SMITH CHART, was seen to be related to both the magnitude and the angle of the voltage reflection coefficient by Eq. (8-1).

All of the remaining radially scaled parameters are related by simple algebraic formulas to the magnitude only of the voltage or current (1) reflection coefficient, (2) standing wave, and (3) traveling waves. While these relationships have been discussed individually in previous chapters for convenience, the formulas are grouped in Table 8.1.

Table 8.1. Waveguide Transmission—Reflection Formulas

TRANSMISSION – REFLECTION FORMULAS

SCALE	FUNCTION	TRAVELING WAVES	REFLECTION COEFFICIENT	STANDING WAVES
(1)	VOLTAGE REFL. COEF.	$\frac{r}{i}$	ρ	$\frac{S-1}{S+1}$
(2)	POWER REFL. COEF.	$(\frac{r}{i})^2$	ρ^2	$(\frac{S-1}{S+1})^2$
(3)	RETURN LOSS, dB	$10 \log_{10} (\frac{i}{r})^2$	$-10 \log_{10} \rho^2$	$-10 \log_{10} (\frac{S-1}{S+1})^2$
(4)	REFLECTION LOSS, dB	$10 \log_{10} \frac{i^2}{i^2 - r^2}$	$-10 \log_{10} (1-\rho^2)$	$-10 \log_{10} \left[1-(\frac{S-1}{S+1})^2\right]$
(5)	STDG. WAVE LOSS COEF.	$1+\frac{[(i+r)/(i-r)]^2}{2[(i+r)/(i-r)]}$	$\frac{1-\rho+\rho^2-\rho^3}{1-\rho-\rho^2+\rho^3}$	$\frac{1+S^2}{2S}$
(6)	STDG. WAVE RATIO, dB	$20 \log_{10} \frac{i+r}{i-r}$	$20 \log_{10} \frac{1+\rho}{1-\rho}$	$20 \log_{10} S$
(7)	MAX. OF STDG. WAVE	$(\frac{i+r}{i-r})^{1/2}$	$(\frac{1+\rho}{1-\rho})^{1/2}$	$\sqrt{S}$
(8)	MIN. OF STDG WAVE	$(\frac{i-r}{i+r})^{1/2}$	$(\frac{1-\rho}{1+\rho})^{1/2}$	$\frac{1}{\sqrt{S}}$
(9)	STANDING WAVE RATIO	$\frac{i+r}{i-r}$	$\frac{1+\rho}{1-\rho}$	S
(10)	ATTENUATION, dB	$-10 \log_{10} \frac{r}{i}$	$-10 \log_{10} \rho$	$-10 \log_{10} \frac{S-1}{S+1}$

i = INCIDENT WAVE AMPLITUDE
r = REFLECTED WAVE AMPLITUDE
ρ = REFLECTION COEFFICIENT
S = STANDING WAVE RATIO
} VOLTAGE OR CURRENT

CHAPTER 9

Waveguide Impedance and Admittance Matching

9.1 STUB AND SLUG TRANSFORMERS

As discussed in previous chapters, a mismatch between the load impedance and the characteristic impedance (or between the load admittance and the characteristic admittance) of a waveguide or transmission line causes a reflection loss at the load and increased dissipative losses along the entire length of the line. Also, a mismatch termination causes the transmission phase to vary nonlinearly with changes in frequency or line length, and increases the tendency of the waveguide or line to overheat and arc over at the current and voltage maximum points, respectively, when operating under high power.

Several commonly used devices for obtaining a *match* at the load end of a transmission line will be described in this chapter. These include the single and the dual matching *stub* [65], and the single and the dual matching *slug* transformers. Matching stubs or *building-out sections,* as they are sometimes called, are sections of transmission line, frequently of the same characteristic impedance (or characteristic admittance) as that of the main line, and either open- or short-circuited at their far end, connected in shunt with the main line at any one of several permissible positions in the general location where it is desired to provide the match. Slug transformers, on the other hand, are sections of line of appropriate characteristic impedance (or characteristic admittance) and length connected in series with, and forming a continuation of, the main line.

These devices are described in some detail herein since it is through its terminations alone that the transmission characteristics of waves along any given waveguide can be controlled. Furthermore, the SMITH CHART provides the ideal medium for visualization of the principle of operation of such transformers and for quantitatively determining their specific design constants.

9.2 ADMITTANCE MATCHING WITH A SINGLE SHUNT STUB

The single open- or short-circuited shunt matching stub whose length is continuously adjustable over a range of one-quarter wavelength, and whose position along the main line is adjustable over a range of one-half wavelength, is capable of correcting any mismatched condition whatsoever along the main line [8].

For its principle of operation refer to the SMITH CHART of Fig. 9.1. Therein it will be observed that the circle of unit conductance (C, A, P, B) passes uninterruptedly through both the center and the periphery of the admittance coordinates, at points C and P, respectively, and is centered on the zero-conductance axis. Thus two admittances, such as at points A and B, of equal magnitudes and of opposite sign, whose normalized conductance component is unity, always coexist within a quarter wavelength of each other along any transmission line regardless of the extent of the mismatch. At either of those positions a susceptance, whose magnitude is equal to that of the input susceptance but of opposite sign, can be shunted across the main line to cancel the main line susceptance, and thereby provide a match (unit conductance) for that section of the main line between the point of attachment of the stub and the generator end of the line.

Although the susceptance provided by a reactive circuit element will of course serve the purpose, a short-circuited or an open-circuited stub transmission line of suitable length in relation to its chosen characteristic admittance provides a practical, convenient, and controllable shunt susceptance. This matching principle is made use of in the so-called *slide-screw tuner* often used in microwave *plumbing.*

9.2.1 Relationships between Impedance Mismatch, Matching Stub Length, and Location

For a short-circuited matching stub of characteristic impedance Z_{0s} in shunt with a main line of characteristic impedance Z_0, the relationship between its required length L', the distance D toward the generator from a voltage maximum position along the main line (at which position the stub must be attached), and the standing wave ratio S are given by [8]

$$\frac{j(Z_{0s}/Z_0)\tan(2\pi L'/\lambda)}{(Z_{0s}/Z_0) + j\tan(2\pi L'/\lambda)} = \frac{S + j\tan(2\pi D/\lambda)}{1 + jS\tan(2\pi D/\lambda)} \tag{9-1}$$

in which λ is the wavelength.

Similarly, for an open-circuited stub line, the relationship is

$$\frac{-j(Z_{0s}/Z_0)\cot(2\pi L'/\lambda)}{(Z_{0s}/Z_0) - j\cot(2\pi L'/\lambda)} = \frac{S + j\tan(2\pi D/\lambda)}{1 + jS\tan(2\pi D/\lambda)} \tag{9-2}$$

The two possible positions within each half wavelength of main line for matching stubs, which are always located at equal distances D but in opposite directions from a voltage maximum (and therefore minimum) position, are a function only of the standing wave ratio S, as can be seen from Fig. 9.1. The "forbidden" stub locations in Fig. 9.1 within one-eighth wavelength of either side of a voltage maximum point correspond to the positions along the line where no shunt susceptance can provide a matched condition.

The mathematical relationship between D and S is simply

$$\tan\frac{2\pi D}{\lambda} = \pm(S)^{1/2} \tag{9-3}$$

The required length for a matching stub for a given value of S will depend upon whether a short-circuited or an open-circuited stub is to be used. The lengths of these two possible stub types will also depend upon the selected ratio of Z_{0s} to Z_0. Stub lengths less than

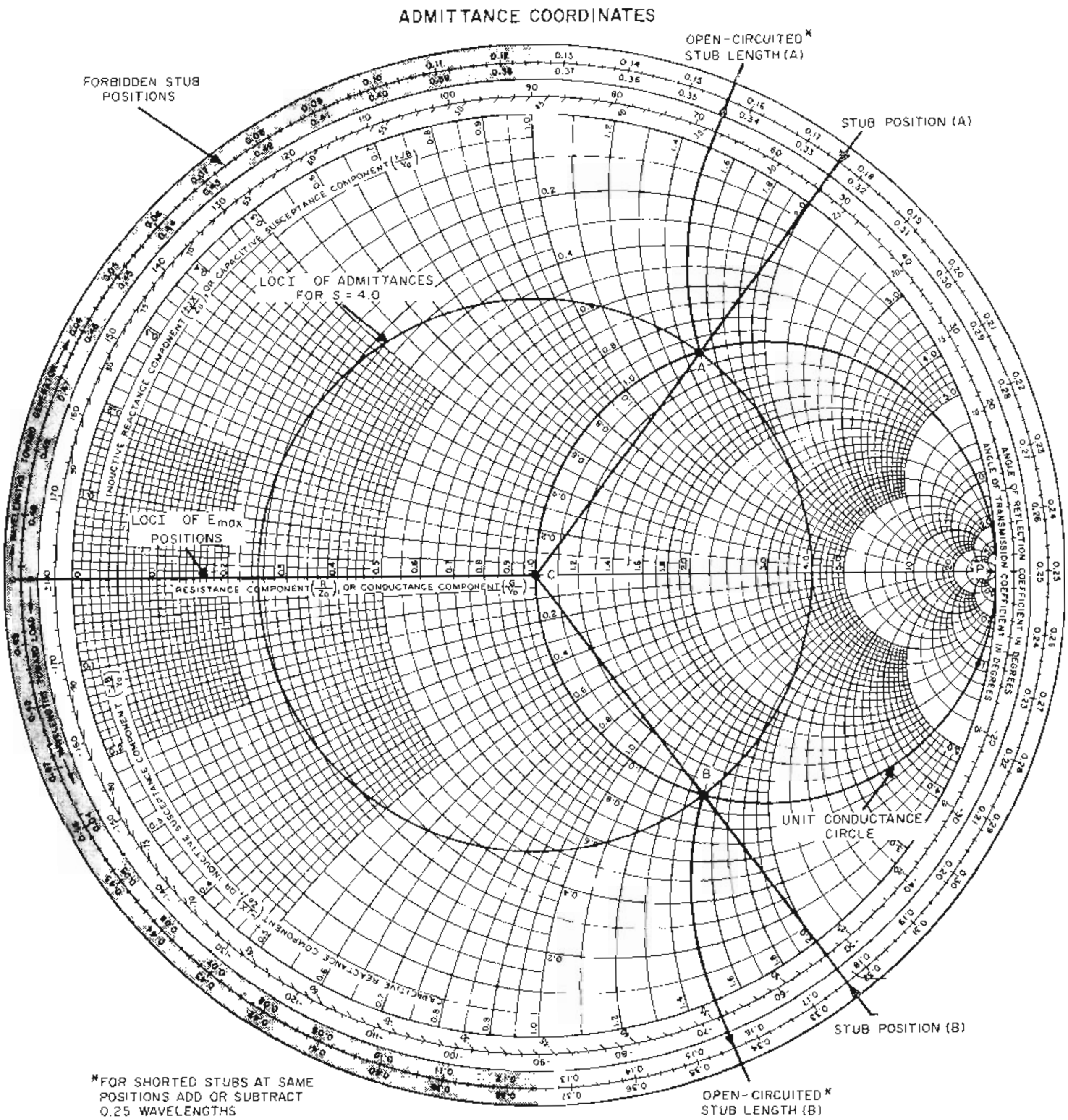


Fig. 9.1. Determination of required length and position of a single shunt matching stub.

one-quarter wavelength are generally preferred to equivalent stub lengths an integral number of half wavelengths longer since the shorter stub provides the wider operating bandwidth. Also the characteristic impedance of the stub is frequently made the same as that of the main line, so that the relationship between L and S as derived from Eqs. (9-1) and (9-2) is independent of the characteristic impedances of either the main line or the stub, in which case Eq. (9-1), for the shorted stub, reduced to

$$\frac{j\tan(2\pi L/\lambda)}{1 + j\tan(2\pi L/\lambda)} = \frac{S + j(S)^{1/2}}{1 + jS(S)^{1/2}} \tag{9-4}$$

in which L is the required length of stub of characteristic impedance Z_0. Similarly, Eq. (9-2), for the open stub, reduces to

$$\frac{-j\cot(2\pi L/\lambda)}{1 - j\cot(2\pi L/\lambda)} = \frac{S + j(S)^{1/2}}{1 + jS(S)^{1/2}} \tag{9-5}$$

Because of frequency dispersive effects a single stub is capable of providing a perfect impedance match only at a single frequency. However, for the applications in which radio frequency lines are generally employed, the operable bandwidth of a single stub matching transformer is generally adequate. Broader operating bands can be obtained through the use of two or more stages of impedance transformation which can be provided by the use of multiple stubs. In this latter case each stub may be adjusted to provide an incremental share of the overall match. A sufficient number of stubs may thus be used to achieve the desired bandwidth.

9.2.2 Determination of Matching Stub Length and Location with a SMITH CHART

The SMITH CHART in Fig. 9.1 will serve both to describe generally and to illustrate specifically the method for determination of stub length and position directly from a SMITH CHART.

1. Draw a circle, centered on the chart coordinates, whose radius corresponds to the standing wave ratio S on the main line. (Example: $S = 4.0$.)

2. Draw two radial lines each of which intersects the circle $G/Y_0 = 1.0$, and note the values at the intersection of these lines with the outer wavelength scale. (Example: 0.176 wavelength at A, and 0.324 wavelength at B.) These are the two locations "toward the generator" from a voltage maximum position where a matching stub can be located.

3. The intersections of the radial lines with the susceptance circles where they intersect the unit conductance circle is then followed out to the rim where the inner wavelengths scale values are noted. (Example: 0.344 wavelength at A and 0.156 wavelength at B.) These are the lengths of open-circuited stubs which may be placed at the positions A and B, respectively, to provide an admittance match.

If short-circuited stubs are desired, their equivalent length is obtainable by adding or subtracting one-quarter wavelength from the length required for the open-circuited stubs. Example: short-circuited stubs should be $0.344 \pm 0.25 = 0.094$ or 0.594 wavelength long at A or $0.156 \pm 0.25 = -0.094$ or 0.406 wavelength long at B. In this latter case the negative stub length is of course impossible and must be discarded as a possible choice.

9.2.3 Mathematical Determination of Required Stub Length of Specific Characteristic Impedance

The required length of a matching stub as determined in (3) above is for a stub whose characteristic impedance is the same as that of the main line. If the stub has a different characteristic impedance its required length will be different from the values indicated above but of such length as will provide the same absolute input reactance. The mathematical determination of stub lengths of different characteristic impedance from that of the main line which will provide equivalent absolute input reactance can be shown as follows:

The absolute input reactance X of any lossless short-circuited stub line is

$$X = jZ_{0s} \tan \frac{2\pi L'}{\lambda} \tag{9-6}$$

and that of any lossless open-circuited line is

$$X = -jZ_{0s} \cot \frac{2\pi L'}{\lambda} \tag{9-7}$$

Designating:

L = length of stub of characteristic impedance Z_0
L' = length of stub of characteristic impedance Z_{0s}
X = absolute input reactance of stub of characteristic impedance Z_0
X' = absolute input reactance of stub of characteristic impedance Z_{0s}

we may write, for the case of short-circuited stubs,

$$jZ_0 \tan \frac{2\pi L}{\lambda} = jZ_{0s} \tan \frac{2\pi L'}{\lambda} \tag{9-8}$$

and for the case of open-circuited stubs,

$$-jZ_0 \cot \frac{2\pi L}{\lambda} = -jZ_{0s} \cot \frac{2\pi L'}{\lambda} \tag{9-9}$$

We also note that in any particular problem the ratio Z_0/Z_{0s} is a constant K. Thus, the ratio of the lengths of two short-circuited stubs which have the same input reactance can be obtained from

$$\frac{\tan(2\pi L/\lambda)}{\tan(2\pi L'/\lambda)} = K \tag{9-10}$$

from which

$$\frac{L'}{\lambda} = \frac{1}{2\pi} \tan^{-1} \frac{\tan(2\pi L/\lambda)}{K} \tag{9-11}$$

and similarly the ratio of the lengths of two open-circuited stubs which have the same input reactance can be obtained from

$$\frac{\cot(2\pi L/\lambda)}{\cot(2\pi L'/\lambda)} = K \tag{9-12}$$

from which

$$\frac{L'}{\lambda} = \frac{1}{2\pi} \cot^{-1} \frac{\cot(2\pi L/\lambda)}{K} \tag{9-13}$$

As an example, the open-circuited stub required at B in Fig. 9.1 has the same characteristic impedance as that of the main line and was determined from this chart to be 0.156 wavelength long. If the characteristic impedance of another stub which it may be desired to use is 1.25 times that of the main line, that is, $K = 1.25$, we obtain the required new length L' by substitution of the above values in Eq. (9-13); thus

$$L' = 0.172\lambda \tag{9-14}$$

9.2.4 Determination with a SMITH CHART of Required Stub Length of Specified Characteristic Impedance

A method of finding stub lengths of different characteristic impedance having equivalent input reactance by means of the SMITH CHART can be shown by use of the above example. The required length of an open-circuited stub of characteristic impedance equal to that of the main line was found to be 0.156 wavelength, and it was desired to find the length of a stub whose characteristic impedance was 1.25 times that of the main line which would have the same absolute input reactance. Proceed as follows:

1. Enter the SMITH CHART (Fig. 8.6) at the open-circuit position on the impedance coordinates, where $R/Z_0 = \infty$.

2. Move "toward the generator" 0.156 wavelength (to the outer wavelengths scale position, 0.156 + 0.25 = 0.406 wavelength) and observe at the perimeter of the impedance coordinates that the input reactance of the stub is $-j\,0.67\ Z_0$.

3. Divide $-j\ 0.67\ Z_0$ by 1.25 to obtain $-j\ 0.536\ Z_{0s}$, which is the input reactance of a stub of the desired characteristic impedance having this same length.

4. Move to new position at 0.536 at the periphery of the coordinates and observe a new required stub length of characteristic impedance Z_{0s} to be $0.172\lambda (0.422\lambda - 0.25\lambda)$, which checks the value obtained in Eq. (9-14).

A summary of the several relationships described above between stub position D, stub length L, standing wave ratio S, and type of stub (short-circuited or open-circuited) is provided in Fig. 9.2. Figure 9.2 can be plotted point by point from data obtained from the SMITH CHART of Fig. 9.1 by the method described above. For convenience in plotting, the standing wave ratio scale in Fig. 9.2 corresponds to that on the SMITH CHART in Fig. 9.1.

9.3 MAPPING OF STUB LENGTHS AND POSITIONS ON A SMITH CHART

Figures 9.3 and 9.4 are overlays for the SMITH CHART impedance or admittance coordinates. The curves thereon show loci of constant required stub lengths and distances of attachment of the stub, toward the generator, from the load terminals. Figure 9.3 applies to short-circuited and Fig. 9.4 to open-circuited matching stubs, respectively. For the lengths indicated, the stubs must have the same characteristic impedance or characteristic admittance as that of the main line. If this is different, the stub length must be such as will provide the same absolute input reactance as that of the specified stub, in accordance with methods previously outlined.

If the normalized complex load impedance or load admittance is known, these curves make it possible to directly determine the required point of attachment of the stub, without reference to the voltage minimum position along the main line.

Figures 9.3 and 9.4 can be overlaid directly on the impedance coordinates of Charts A, B, or C in the cover envelope. When overlayed on the admittance coordinates of these latter charts they must first be rotated through an angle of 180°.

9.4 IMPEDANCE MATCHING WITH TWO STUBS

Two stubs, each of which is adjustable in length and fixed in position along a transmission line, may be used in lieu of a single stub which is adjustable in length and position, to provide a general-purpose impedance matching device. If each of these two stubs is permitted to be either open-circuited or short-circuited at its far end and separately adjustable in length over a range of one-half wavelength, the combination will provide the means for matching the characteristic impedance of the main line to the impedance of the load within a specified range of load impedance values on the SMITH CHART.

The matching capability of such a pair of stubs is a function of the stub spacing

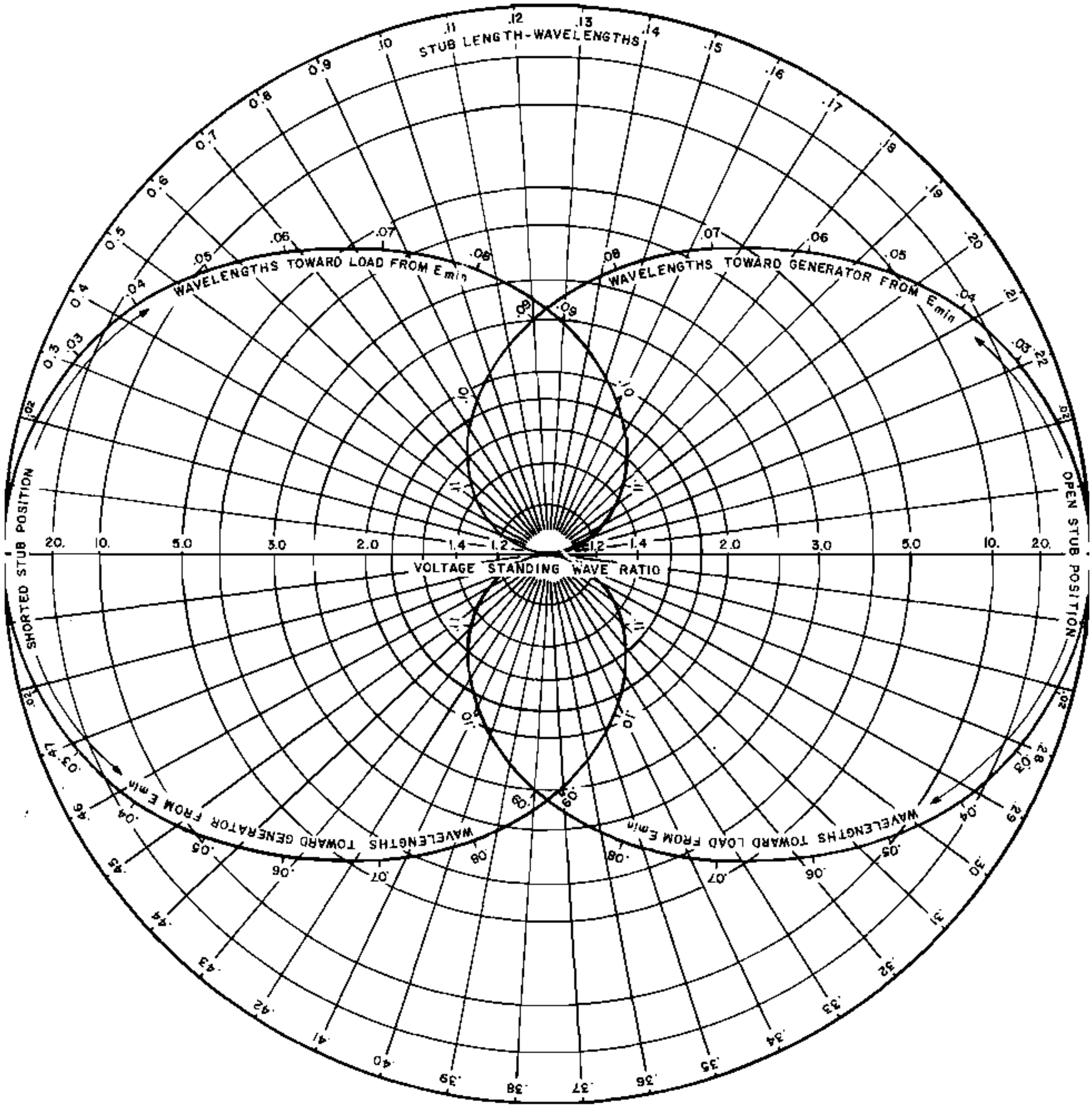


Fig. 9.2. Allowable locations and lengths of a single short-circuited, or open-circuited, matching stub.

only and can be represented on the SMITH CHART of Fig. 9.5 as a family of "forbidden" areas each of which corresponds to a specific spacing. In Fig. 9.5 two such families are shown, one representing forbidden admittances and the other equivalent forbidden impedances.

Should the load normalized impedance Z_ℓ/Z_0 or admittance Y_ℓ/Y_0 of a two-stub matching circuit fall within their respective forbidden areas no adjustment of the two stubs is possible which will provide a match. However, if the two stubs are moved as a pair along the same transmission line in either direction a

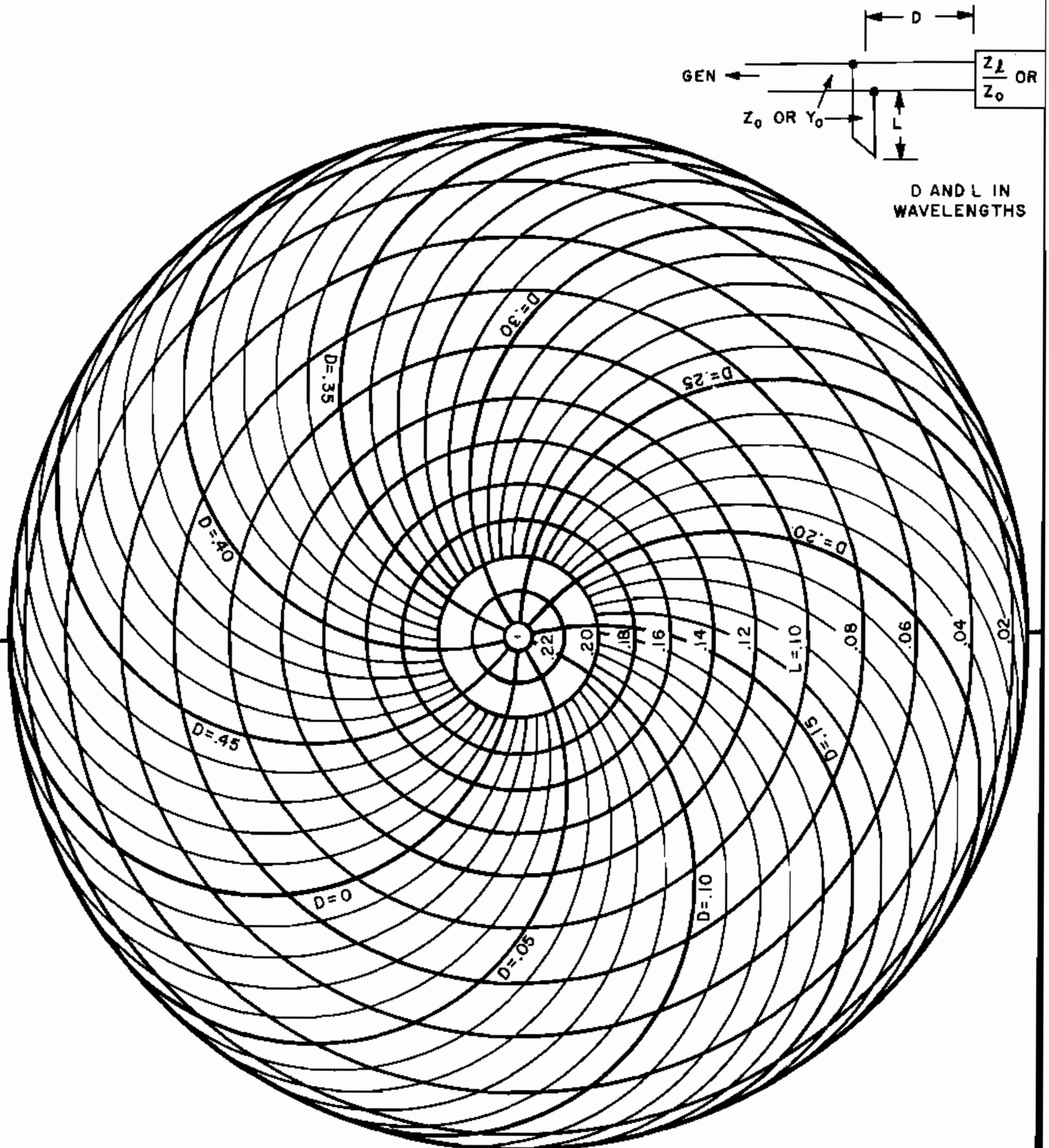


Fig. 9.3. Required length and position for a short-circuited matching stub (overlay for Chart A, B, or C in cover envelope).

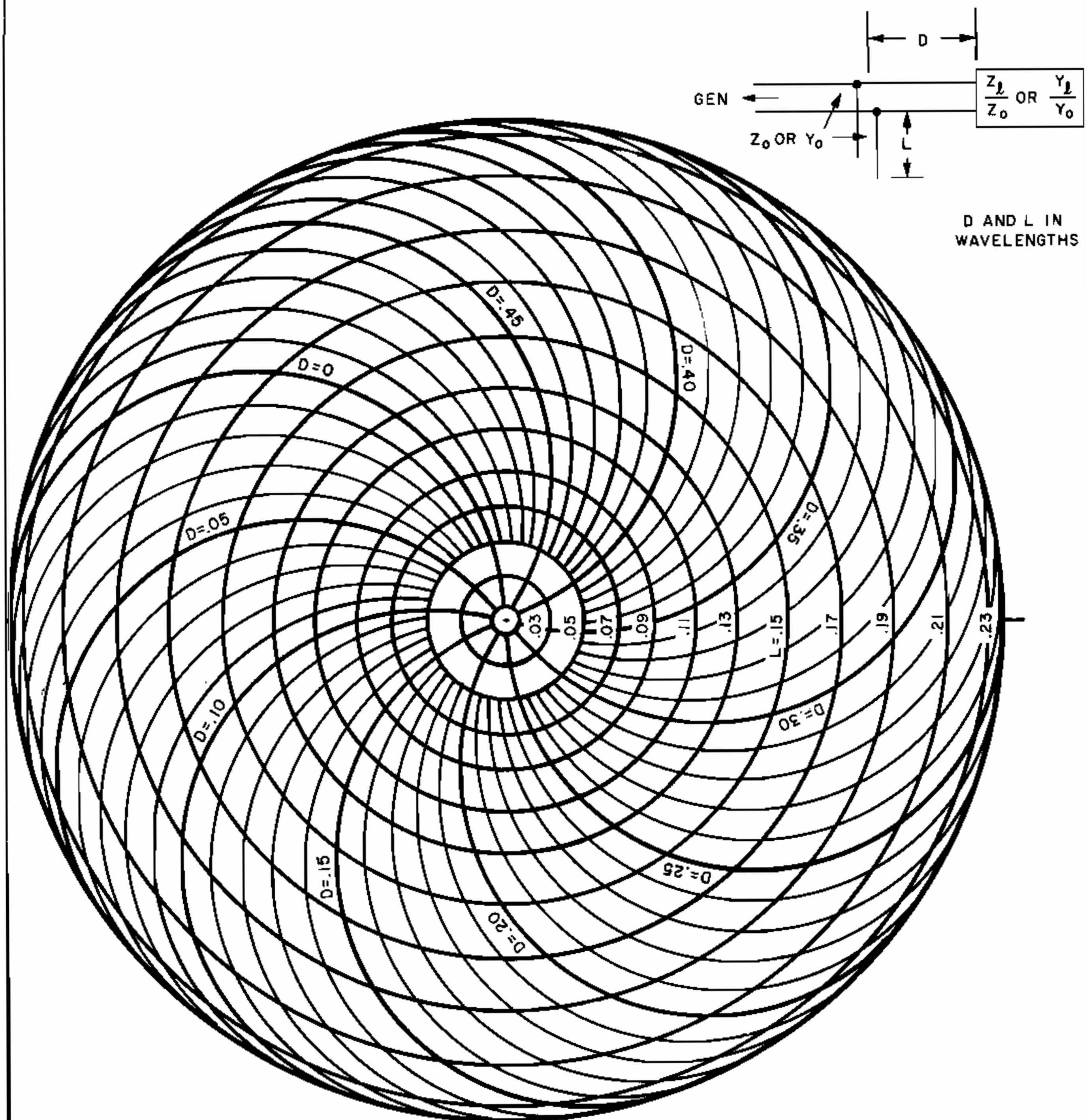


Fig. 9.4. Required length and position for an open-circuited matching stub (overlay for Chart A, B, or C in cover envelope).

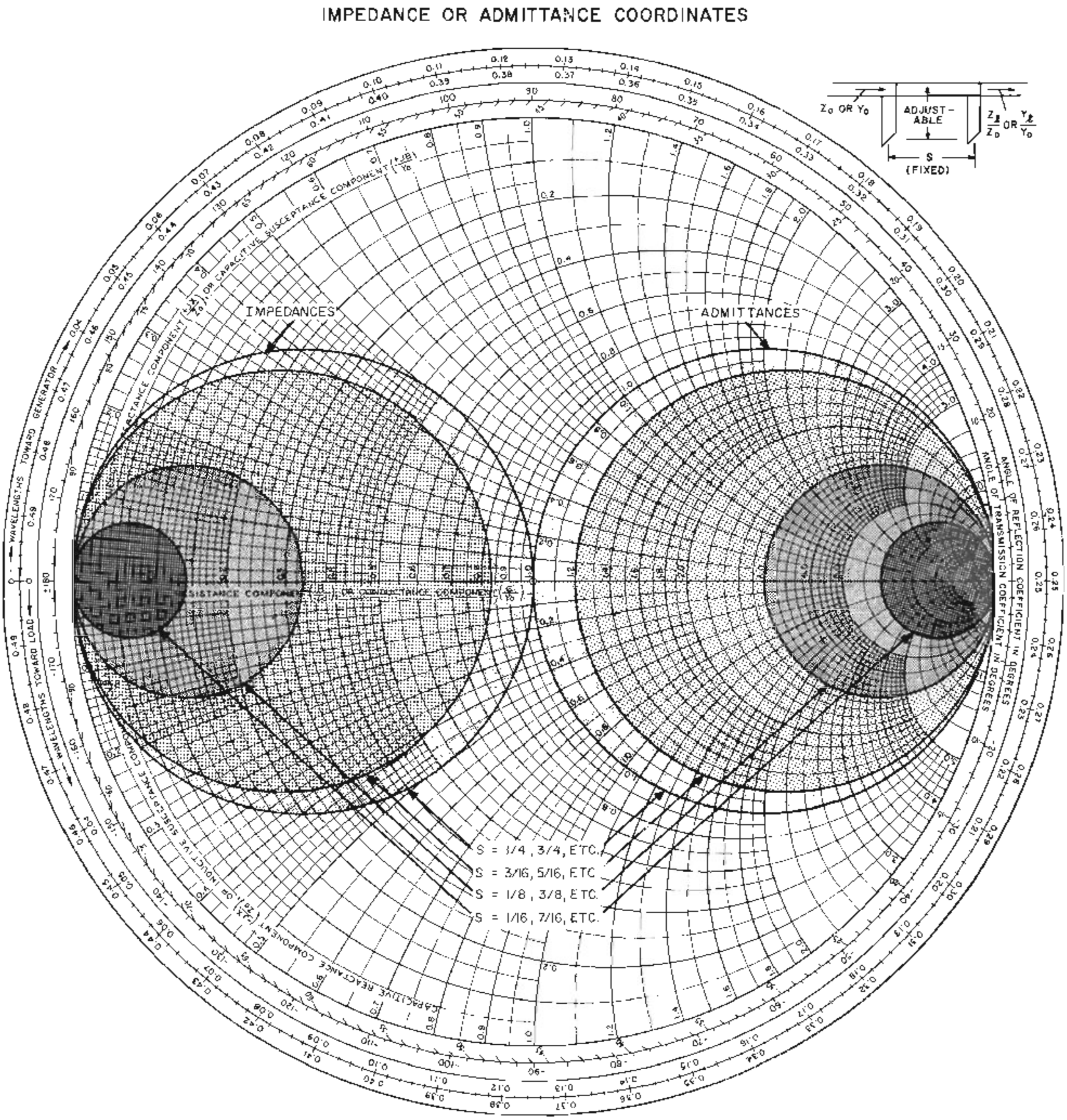


Fig. 9.5. Normalized waveguide impedances or admittances, except those in "forbidden areas," are transformable to Z_0 or to Y_0, with two adjustable-length stubs at fixed spacing S.

distance of one-quarter wavelength, the load impedance will then fall outside of the forbidden area and an adjustment of the stub lengths will be possible which will provide the match.

An example of a typical problem is indicated in Fig. 9.6 for a stub spacing of one quarter wavelength.

The point of entry to the SMITH CHART in Fig. 9.6 is the normalized input admittance of the transmission line at $Y_\ell/Y_0 = 0.4 - j0.18$, and it can be seen that this occurs 0.216 wavelength "toward the generator" from a voltage minimum point on the line. The transforming effect of shunting Y_ℓ/Y_0 (see insert in Fig. 9.6) with an inductive susceptance B_1 whose normalized value is $-jB_1/Y_0 = 0.31$ is to move Y_ℓ/Y_0 to the position Y_1/Y_0. The additional transforming effect of the main line section between the stubs $S/\lambda = 0.25$ is to move Y_1/Y_0 to the position Y_2/Y_0 on the unit conductance circle. Finally, the transforming effect of shunting $Y_2 = 1.0 + j1.22$ with an inductive susceptance B_2, whose normalized value $-jB_2/Y_0 = 1.22$, is to move Y_2/Y_0 to $Y_0 = 1.0 \pm j0$, at which point the desired match is accomplsihed.

In all cases, Y_2/Y_0 must be on the unit conductance circle, and the distance between Y_1/Y_0 and Y_2/Y_0, as measured on the peripheral scales to which these points are projected radially, must correspond to the spacing of the stubs.

9.5 SINGLE-SLUG TRANSFORMER OPERATION AND DESIGN

An elementary type of transmission line impedance transformer consists of a one-quarter wavelength section of transmission line of appropriate characteristic impedance. In operation such a transformer is always inserted in the main line between a voltage maximum and adjacent minimum as these positions exist prior to its insertion. Such positions along the line will provide input and output impedances which are representable on the SMITH CHART along the real axis.

The characteristic impedance Z_t of the quarter-wave "slug" transformer must in all cases be made equal to the geometric mean between the characteristic impedance Z_0 of the main line and the real impedance R_ℓ at the load terminals of the slug. Thus

$$\frac{R_\ell}{Z_t} = \frac{Z_t}{Z_0} \tag{9-15}$$

From Eq. (9-15) it is seen that for a fixed value of Z_t only a single load resistance R_ℓ can be transformed to Z_0.

If the ratio Z_t/Z_0 is between zero and unity the load terminals of the transformer must be positioned at the voltage (and consequently impedance) minimum point along the line, in which case the transformer will step up the low impedance of its load to match the characteristic impedance of the line. Conversely, if it is between unity and infinity its load terminals must be positioned at a voltage (and consequently impedance) maximum point, and it will then step down the load impedance to match the line characteristic impedance.

For a general-purpose matching device the position of the quarter-wave slug transformer must be continuously adjustable over a range of one-half wavelength in order to ensure that it can always be properly positioned with respect to the standing wave which it is required to eliminate.

The design and operation of a single quarter-wave transformer can be shown on the SMITH CHART in Fig. 9.7 as follows:

1. Assume that a mismatched transmission line has a standing wave whose ratio S is 3.333. Draw the circle for $S = 3.333$ centered on the chart coordinates, and note that this

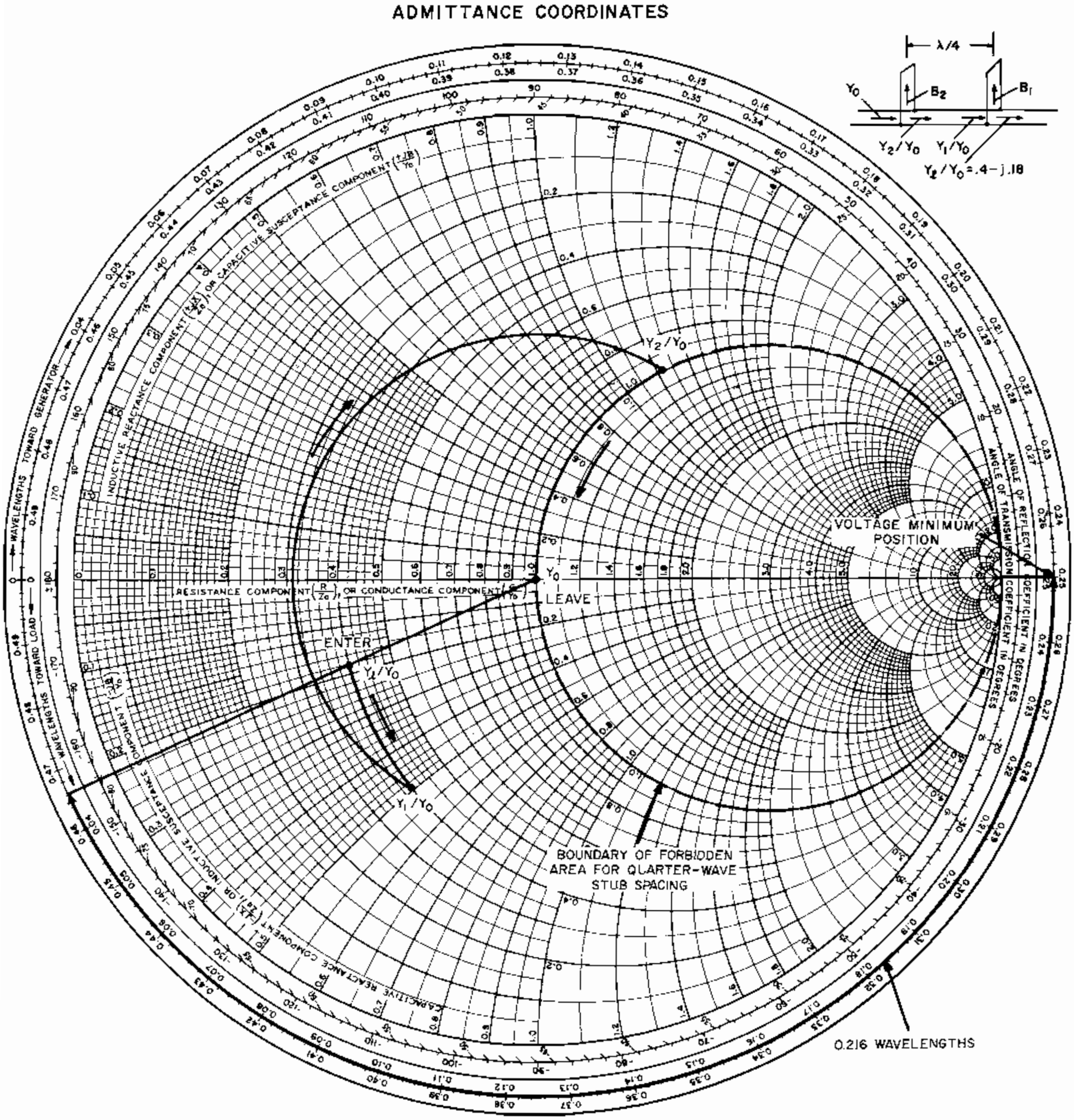


Fig. 9.6. Representation of an admittance transformation from $Y\ell/Y_0$ to Y_0 with two shunt matching stubs spaced one-quarter wavelength.

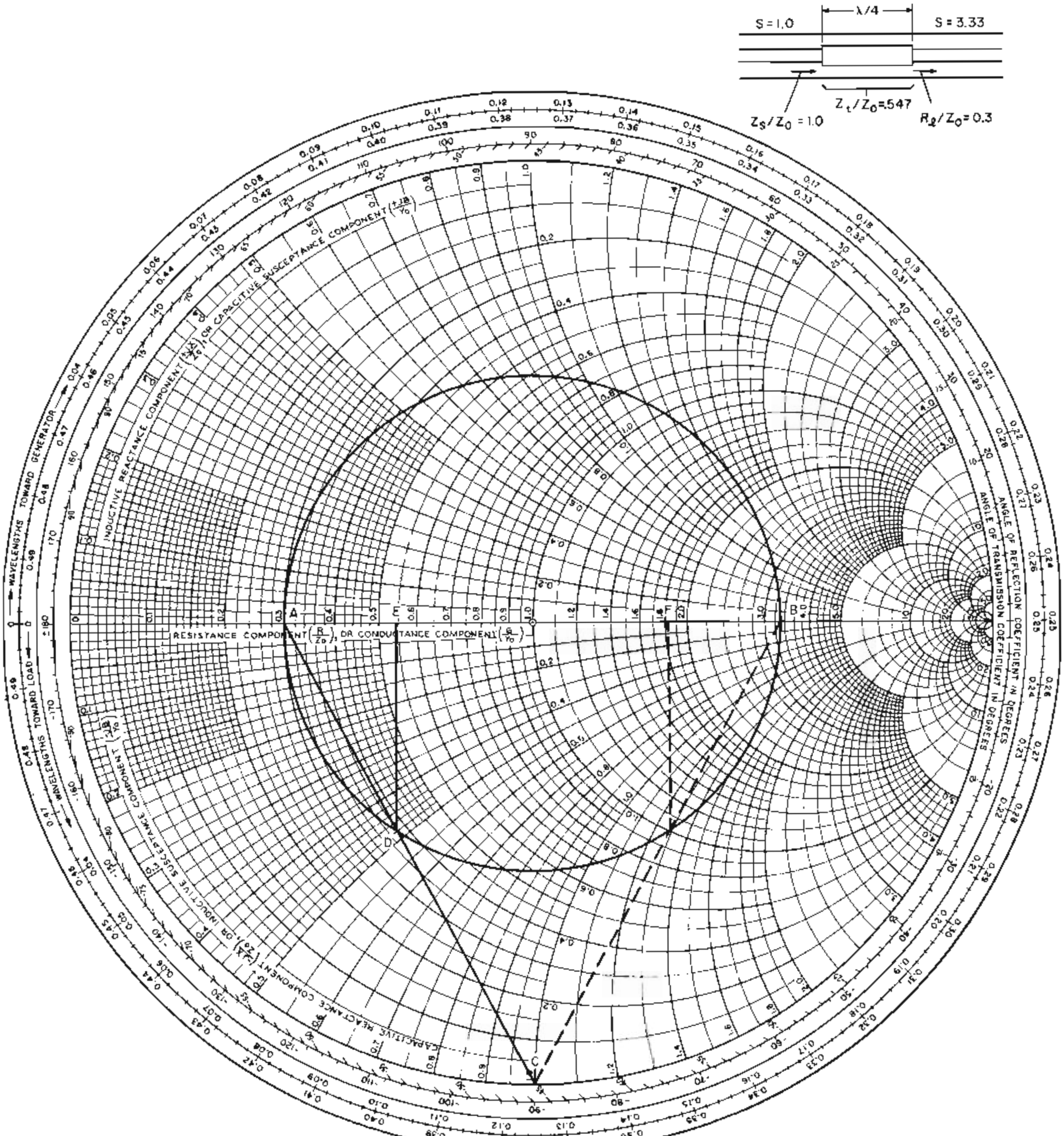


Fig. 9.7. Impedance matching with a single quarter-wave slug transformer.

circle intersects the real axis at points A and B where $R_\ell/Z_0 = 0.3$ and 3.333, respectively. Assuming that it is desired to use a transformer which steps up the load resistance, select the lower of these two possible values as the load impedance for the transformer, that is, $R/Z_0 = 0.3$ at point A.

2. Draw a construction line connecting point C on the perimeter of the chart with point A, and at its intersection with the S = 3.333 circle (point D) erect a perpendicular to intersect the real axis of the chart at point E. The required normalized characteristic impedance Z_t/Z_0 of the slug transformer is found, in this example, to be 0.547. (Had a step-down transformer been selected the normalized value of its characteristic impedance would have been 1.83, as indicated at the extremity of the dotted construction line at point F. The results thus obtained will be seen by inspection to satisfy Eq. (9-15).

9.6 ANALYSIS OF TWO-SLUG TRANSFORMER WITH A SMITH CHART

A more versatile matching circuit whose design and performance characteristics are described herein is the two-slug transformer. The analysis of this transformer is carried out in a specific example on the SMITH CHART of Fig. 9.8.

The two slugs shown schematically in Fig. 9.8 are generally constructed in the form of longitudinally adjustable enlargements of the inner conductor diameter of a coaxial line, as shown, or sleeve-type reductions of its outer conductor diameter. Either case results in a reduction of the characteristic impedance of the line section occupied by the slug. Alternatively, the slugs may be constructed of a dielectric material.

Given sufficient space along the line for adjustment of slug position, this type of transformer is capable of eliminating standing waves of any amplitude ratio which does not exceed its maximum design value, which value depends only upon the fixed value of characteristic impedance for the slugs.

Individual conducting slugs are frequently made one-quarter wavelength long, and dielectric slugs, which completely fill the space between inner and outer conductor, are usually made mechanically shorter by the factor $1/\sqrt{\epsilon}$ to take into account the reduction in velocity of a wave in a dielectric material whose dielectric constant ϵ is greater than unity.

The two slugs are usually fixed in length and in characteristic impedance for a particular application in accordance with a predetermined maximum standing wave matching requirement. The slugs are adjustable along the inner conducter (1) with respect to each other, and (2) with respect to a fixed reference position along the line.

If minimum overall length of a two-slug transformer is important in a given application, one-quarter wavelength is not necessarily the most desirable length for the individual slugs. For this reason the analysis which follows will consider slugs whose length ranges between zero and one-quarter wavelength, and whose characteristic impedance ranges between zero and one times that of the main line, to determine how the overall matching capability is related to these design variables. From this data the range of transformable load impedances for a given slug geometry as a function of frequency may readily be found.

A specific combination of two identical one-eighth wavelength-long slugs with a normalized characteristic impedance Z_t/Z_0 = 0.4 is represented by the construction in Fig. 9.8. By a successive consideration of other slug lengths and normalized characteristic impedances by the same method, the data in Fig. 9.9 is obtained. The use of identical slugs in a two-slug transformer is justified on the basis that it simplifies the

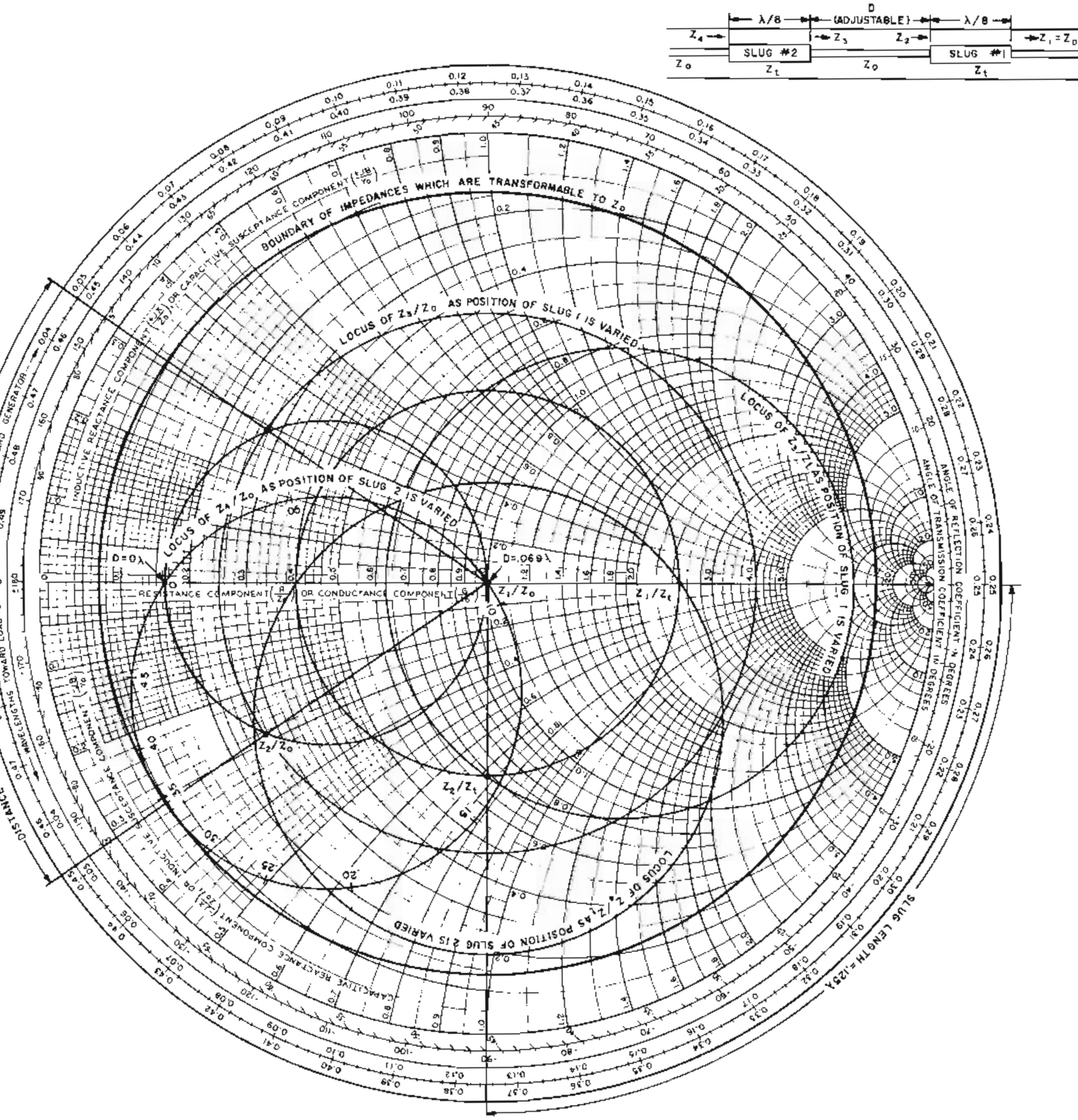


Fig. 9.8. Determination of impedance matching capability of two one-eighth-wavelength-long slugs when $Z_t/Z_0 = 0.4$.

analysis while still not restricting the method of analysis for two slugs of arbitrary lengths or characteristic impedances.

Let it be required to find the boundaries of impedance areas on the SMITH CHART which define the impedance transforming capabilities of two-slug radio frequency impedance transformers as a function of (1) the slug length in wavelengths, (2) the ratio of characteristic impedance of the slugs to the characteristic impedance of the main line, that is, Z_t/Z_0, (3) the spacing D between slugs in wavelengths, and (4) the maximum required overall length of slugs plus position adjustments.

For the purpose of analysis the conditions of the problem may be reversed and it may then be assumed that the load is a constant impedance which perfectly matches the normalized characteristic impedance of the line. The boundary of all possible input impedance vectors for all possible slug positions is then found. The principle of impedance conjugates may then be applied to the matching problem, that is, for example, if a normalized input impedance of $2.0 + j\,4.0$ falls within this boundary of impedances which are available at the input terminals, then this transformer adjustment will provide a match for a normalized load impedance of $2.0 - j4.0$. The input SWR for the conditions calculated will be the same as the output SWR in the actual case. It will be found that in all cases the matchable impedance area is bounded by a circle centered at Z_0.

9.7 DETERMINATION OF MATCHABLE IMPEDANCE BOUNDARY

1. On the circular transmission line impedance chart (Fig. 9.8) draw a circle centered at Z_0 cutting the R axis through the value assigned to Z_t/Z_0, assumed in this example to be 0.4. This is the locus of impedances along that slug which is nearest to the load, normalized with respect to Z_t. Label the load (maximum) impedance point Z_1/Z_t.

2. Lay off the point Z_2/Z_t on this circle at a distance of 0.125 wavelength, corresponding to the length of a single transformer slug in wavelengths towards generator, i.e., clockwise from point Z_1/Z_t.

3. Normalize Z_2 with respect to Z_0, by multiplying each component of the complex impedance Z_2/Z_t, that is, $0.62 - j\ 0.72$, by the numerical ratio, 0.4, which was assigned to Z_t/Z_0. Plot the resulting point, that is, $0.276 - j\,0.288$, at Z_2/Z_0.

4. Draw a circle centered at Z_0 and passing through Z_2/Z_0. This is the locus of impedances Z_3/Z_0 as the separation of the slugs is changed.

5. Normalize the above locus circle Z_3/Z_0 with respect to Z_t, that is, find the locus of Z_3/Z_t, by dividing each component of each complex impedance value Z_3/Z_0 on the periphery of this locus circle by the numerical ratio assigned to Z_t/Z_0.

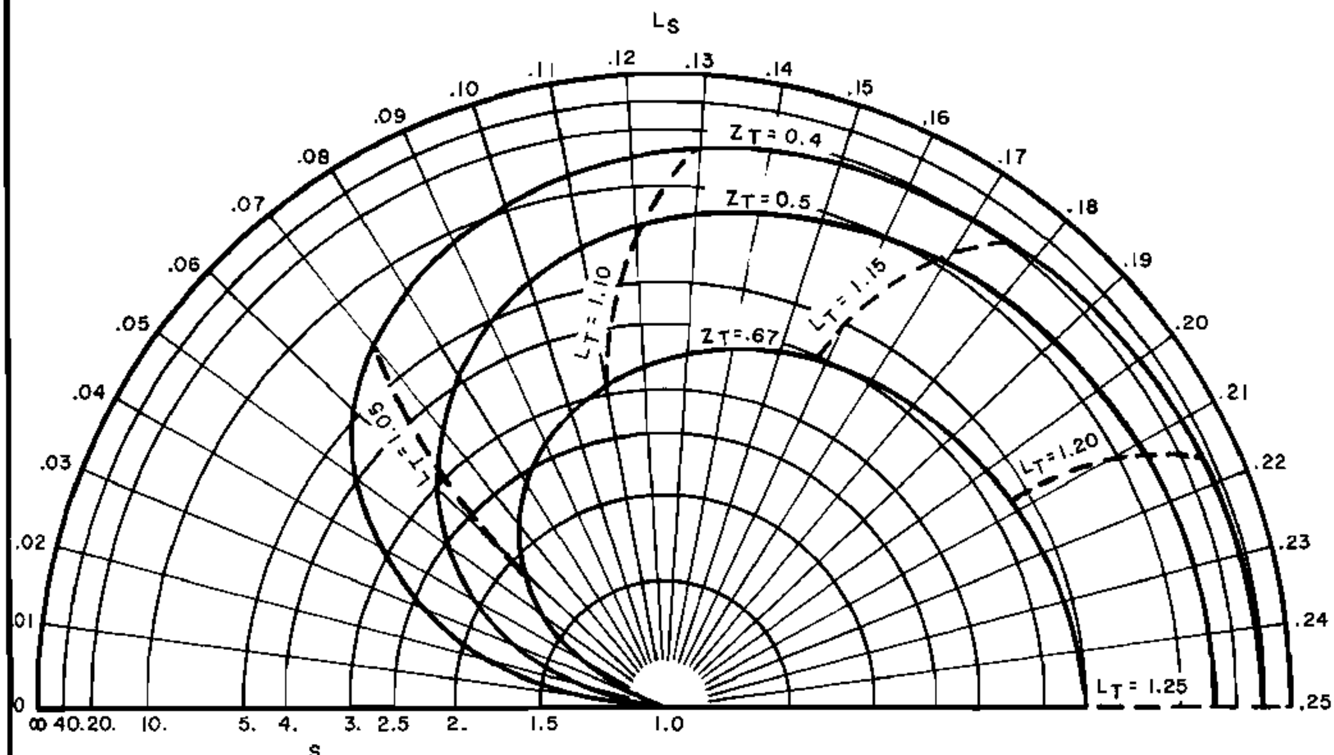
6. Rotate the locus circle Z_3/Z_t clockwise about point Z_0 by an amount corresponding to the length of a single transformer slug, that is, 0.125 wavelength, to obtain the locus of Z_4/Z_t.

7. Normalize and replot the above locus circle Z_4/Z_t with respect to Z_0, that is, find the locus of Z_4/Z_0, by multiplying each component of each complex impedance value Z_4/Z_t on the periphery of this locus circle by the numerical ratio assigned to Z_t/Z_0, that is, 0.4.

8. Draw a circle representing the Z_4/Z_0 boundary. This circle is centered at Z_0 and is tangent to the outermost point of locus circle Z_4/Z_0. It will be observed from Fig. 9.8 that the locus of Z_4/Z_0 is a circle passing through unity (the center point of the SMITH CHART).

This, in effect, states that regardless of the slug length or slug characteristic impedance, it is always possible to find a slug spacing such that the reflection introduced by one slug just cancels that from the other. The result is an overall 1/1 impedance transformation and the slug spacing required is such that Z_2/Z_0 and Z_3/Z_0 are conjugate. Any departure in slug spacing in either direction from the spacing which results in a 1/1 impedance transformation introduces a standing wave of some amplitude ratio other than unity. The maximum amplitude of this standing wave is reached when the slug spacing departs from the above setting by a quarter wavelength. The position of the standing wave introduced by the slugs is adjusted with respect to any arbitrary fixed reference point along the line by sliding the slugs as a pair (maintaining constant spacing). For a general purpose two-slug transformer it is necessary that the position of the standing wave be adjustable throughout a full one-half wavelength. Thus the minimum required overall length of a general purpose two-slug transformer is seen to be composed of the sum of the following lengths:

1. One-half wavelength required for phase adjustment.
2. The spacing required between slugs.
3. The combined lengths of the slugs.



L_S = INDIVIDUAL SLUG LENGTHS IN WAVELENGTHS

L_T = OVERALL TRANSFORMER LENGTH IN WAVELENGTHS

Z_T = RATIO OF SLUG TO MAIN LINE CHARACTERISTIC IMPEDANCE

S = MAXIMUM SWR WHICH CAN BE REDUCED TO UNITY

Fig. 9.9. Impedance matching capability of two-slug impedance transformer.

Figure 9.9 is a summary of data obtained as described from a SMITH CHART, showing the overall impedance matching capability of a two-slug transformer. In this figure, L_S is the slug length and L_T is the overall transformer length in wavelengths, and Z_T is the characteristic impedance of the slugs normalized with respect to the characteristic impedance of the main line. The chart shows the maximum value of the standing wave ratio S which can be reduced to unity.

The analysis on the SMITH CHART of the two-slug transformer, as described, reveals the possibility of using slugs much shorter than a quarter wavelength in applications where it is desirable to minimize the overall transformer length, or to limit the maximum value of the SWR which the two-slug combination can reduce to unity, thereby improving the operating bandwidth of the transformer. For example, it can be seen from Figs. 9.8 and 9.9 that a pair of one-eighth-wavelength-long slugs whose characteristic impedance is 0.4 times that of the main line will match any possible load impedance which could result in standing wave ratios between unity and 14/1. Furthermore, such a combination of slugs will provide a transformer whose overall length never exceeds 1.096 wavelengths ($0.5 + 2 \times 0.125 + 0.096 + 0.25$).

CHAPTER 10

Network Impedance Transformations

10.1 *L*-TYPE MATCHING CIRCUITS

The advantages of matching impedances at a junction between two circuits or media have been pointed out in Chap. 9 and elsewhere in this book and need no further explanation here.

One of the most commonly used and generally satisfactory impedance transforming networks for radio frequency applications is the half-section *L*-type circuit employing two essentially pure reactance elements [19, 149]. At a single frequency and, for most practical purposes, embracing at least the sideband frequencies of a radio telephone transmitter, the simple *L*-type circuit may be used effectively to transform any load impedance to any desired pure input resistance value. Conversely, the *L*-type circuit may be employed to accomplish the reverse transformation, that is, to transform any load resistance to any desired complex input impedance value. It is necessary to consider only the former type of transformation, however, since the circuit can always be reversed to make the transformation in the opposite direction. This simplifies the presentation of design information.

It will be seen by referring to Fig. 10.1 that there is a total of eight possible combinations of reactance types, i.e., inductive and capacitive, in an *L*-type circuit. Each of these eight circuits is capable of transforming a restricted range of complex load impedance values to a given pure resistance value. The transformable impedance values associated with each circuit can conveniently be represented by the impedances within a bounded area on a SMITH CHART. A set of eight such representations will therefore completely outline the capability and limitations of the eight possible reactance combinations, and will furnish a comprehensive outline of the impedance transforming capability of each reactance combination.

For radio-frequency applications, the losses in an *L*-type circuit are usually small in comparison to the power which is being conducted through the circuit. Thus, the circuit losses generally will not limit to any serious extent the range of load impedance values which can be transformed to a desired

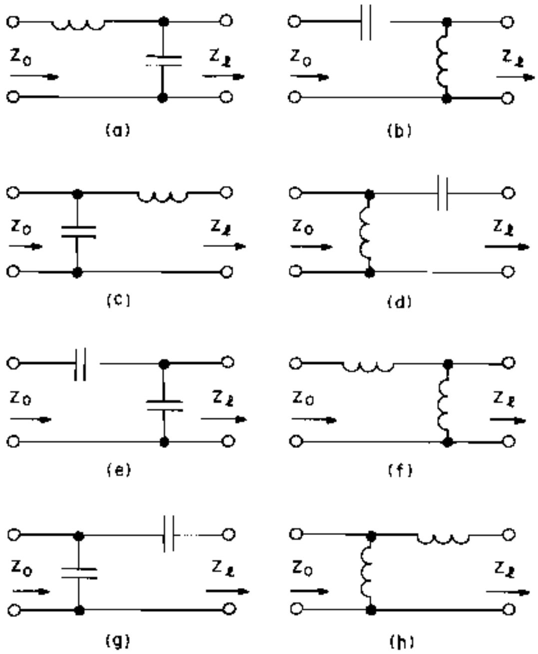


Fig. 10.1. Eight possible L-type circuits for transforming a complex load impedance Z_{ℓ} to a pure resistance Z_0.

resistance, nor will they ordinarily have a major effect upon the reactance values for the circuit elements which are theoretically required on the assumption that they are lossless. The design charts to be described are, therefore, plotted for the idealized case of lossless circuits. Having selected a suitable lossless circuit and having obtained the reactance values required in such a circuit from the charts, the probable resistance of the circuit elements which must be used and the resulting losses will be more readily determinable.

10.1.1 Choice of Reactance Combinations

The eight SMITH CHART overlays in Fig. 10.2 summarize the matching capabilities of the eight possible L-type circuit combinations of Fig. 10.1, and serve as a guide to the selection of a suitable L-type matching circuit for any particular impedance transformation.

A shaded area is shown on each of the eight diagrams in Fig. 10.2. This is to indicate that any load impedance vector whose extremity falls anywhere within this "forbidden" area cannot be transformed to Z_0 (the desired input resistance value) with the specific circuit to which the diagram applies, and that in this case one of the seven other L-type circuits must be selected. If the extremity of the load impedance vector falls anywhere inside of the unshaded area, the circuit is capable of performing the desired impedance transformation.

In cases where the impedance transforming capabilities of two or more L-type circuits overlap, the particular circuit which calls for the more practical circuit constants should, of course, be chosen. It is of interest to note that the circuits shown in Fig. 10.2(e) and (g) are each capable of transforming the same range of load impedances, although each accomplishes a given transformation with different reactance values. The circuits in Fig. 10.2(f) and (h) are also both capable of making the same impedance transformations.

10.1.2 SMITH CHART Representation of Circuit Element Variations

On each of the eight diagrams shown in Fig. 10.2 an example of the function of each element of the circuit is illustrated using an assumed load impedance vector Z_{ℓ}. The influence of each of the circuit elements upon Z_{ℓ} may be regarded as forcing the latter to move along an "impedance path" on a SMITH CHART from its initial position to a position along the R axis, with its extremity at position Z_0. This impedance path followed by a single vector is illustrated on each of the eight diagrams of Fig. 10.2 by a heavy line and an accompanying arrow.

For example, refer to Fig. 10.2(a). Here, any load impedance (such as Z_{ℓ}) whose

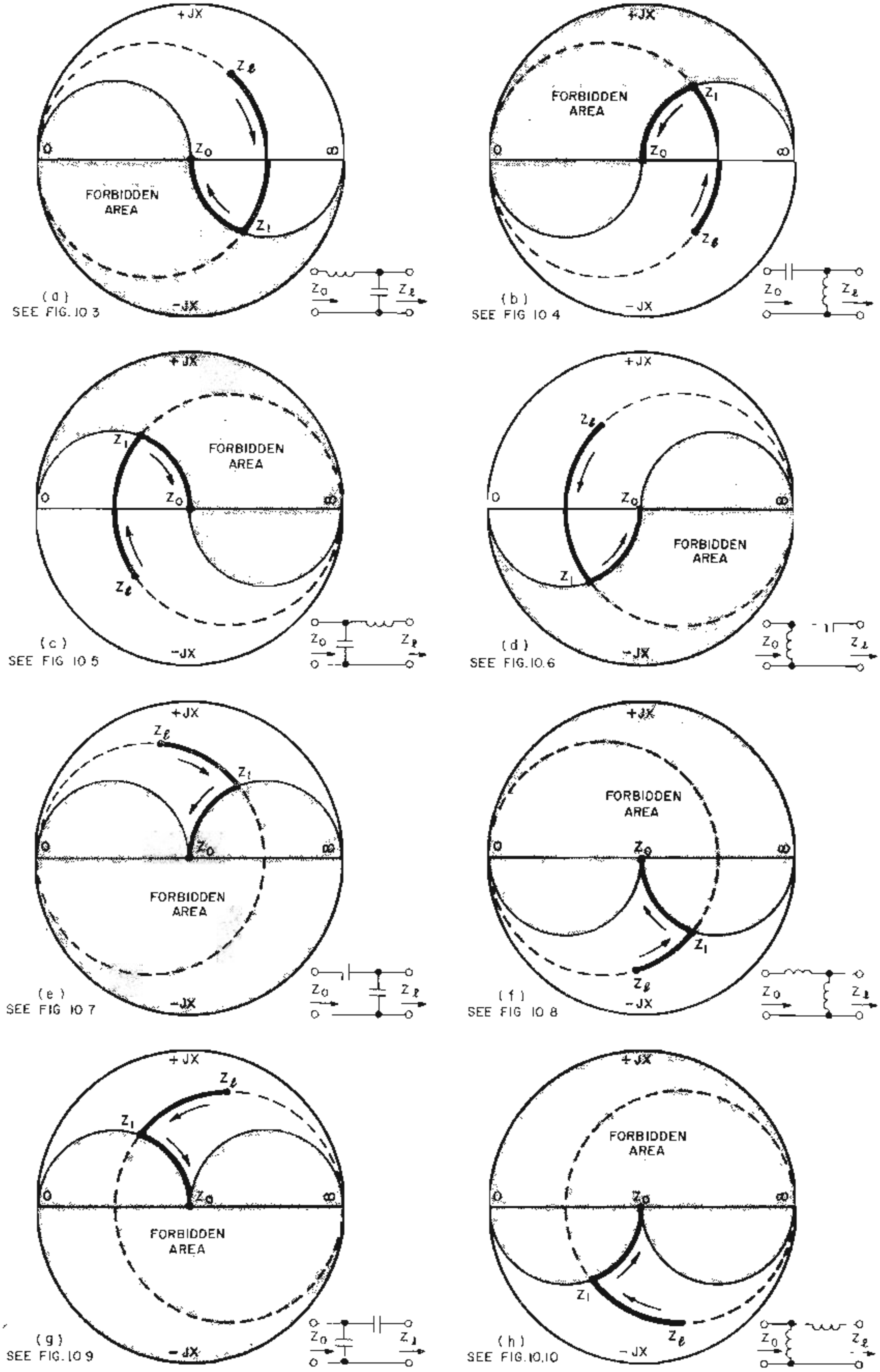


Fig. 10.2. Impedances in unshaded areas of SMITH CHARTS, represented by eight circular boundaries, are transformable to a pure resistance Z_0 with specific L-type circuits indicated (transforming effect of each circuit element is indicated by a heavy line with arrows).

extremity falls in the unshaded area may be selected to be transformed with an L-type circuit of the type indicated on this diagram to a chosen value of pure resistance Z_0. In this case it will be noted that the effect of the shunt capacitive reactance on the impedance vector Z_ℓ is to rotate its extremity clockwise around a circular path leading to the point Z_1. This path is always along a circle tangent to the X axis at $X = 0$, and centered on the R axis of the SMITH CHART. Z_1 represents the extremity of a second impedance vector, the resistance component of which is equal to Z_0. (To simplify the diagrams, only the extremities of the vectors are indicated.) The capacitive reactance component of the impedance vector Z_1 is then canceled by the reactance of the series inductance element of the L-type circuit, which moves the vector along the path to position Z_0, thus completing the transformation.

The required inductive and capacitive reactances of the L-type circuit elements are not shown in Fig. 10.2, which, as explained, is useful primarily to compare the matching capabilities of the various circuits.

10.1.3 Determination of L-type Circuit Constants with a SMITH CHART

To obtain the proper value of the inductive and capacitive reactances required in a given L-type circuit to transform a given complex load impedance Z_ℓ to a given pure resistance Z_0, select the design curves which have been plotted for the particular circuit chosen. These are plotted in Figs. 10.3 through 10.10 for each reactance element of each of the eight possible circuits. The applicable circuit can be identified by referring to the small schematic diagram associated with each. Each of these families of design curves is used as an overlay for SMITH CHART A, in the cover envelope, with which it must be accurately aligned.

The load impedance, indicated at the extremity of the load impedance vector, should be spotted on the SMITH CHART coordinates which are superimposed on the appropriate design chart. The required circuit reactance values are then obtained from the design chart by interpolating between the indicated values on the nearest reactance curves plotted thereon.

Problem 10-1, which follows, further illustrates this use of the design charts.

10-1 (a) Select an L-type circuit which will transform a load impedance of 140 + j 60 ohms to a pure resistance of 50 ohms.

Solution:
From the foregoing we may write:

$$Z_0 = 50$$

$$Z_\ell = 140 + j60 = 2.8\,Z_0 + j1.2\,Z_0$$

Refer to SMITH CHART A and Fig. 10.2 and observe that the above load impedance vector Z_ℓ falls within the unshaded (transformable) area of diagrams a and b, and within the shaded or "forbidden" area of diagrams c to h inclusive. A choice of two circuits is therefore available for this transformation. Select one—for example, that of diagram b.

(b) Determine the reactance value of each element of the circuit selected.

Solution:
On SMITH CHART A locate the above impedance value; then superimpose this chart on the design curves of Fig. 10.4 (as directed on diagram b of Fig. 10.2). Next determine the correct

reactance values for X_L and X_C by noting that the extremity of this load impedance vector Z_ℓ, as plotted on the SMITH CHART, falls at the intersection of the design curves labeled $X_L/Z_0 = 3.0$ and $X_C/Z_0 = 1.5$. Since $Z_0 = 50$ ohms, $X_L = 3.0 \times 50 = 150$ ohms, and $X_C = 1.5 \times 50 = 75$ ohms.

If a complex load impedance is not known exactly but can be estimated within certain limits, these limits may be mapped directly on the SMITH CHART and the range of circuit reactances required can thus be completely bracketed.

This feature will be most appreciated when an L-type circuit must be designed to accommodate any one of a range of possible load impedance values. The design of a circuit to match the input impedance of a radio antenna, which is usually not definitely known in advance of its construction, to the characteristic impedance of a transmission line is readily accomplished with this type of diagram. In such cases, the limitations of a given circuit establish limiting requirements for the circuit elements. Problem 10-2, which follows, illustrates this case.

10-2 (a) Select an L-type circuit which can be adjusted to match any load impedance falling within the range 25 to 75 ohms resistance and 0 to 50 ohms positive reactance to a pure resistance of 100 ohms.

Solution:

From the foregoing we may write

$$Z_0 = 0$$

$$Z_\ell = (20 \text{ to } 80) + j(0 \text{ to } 60) \text{ ohms}$$

$$= (0.20\,Z_0 \text{ to } 0.80\,Z_0) + j(0 \text{ to } 0.60\,Z_0) \text{ ohms}$$

From Figs. 10.3 to 10.10 and SMITH CHART A select a diagram upon which the above "block" of impedance values all fall within a transformable (unshaded) area. The L-type circuit of Fig. 10.6 is found to be the only suitable circuit for this case.

(b) Determine the limiting reactance values of each of the two circuit elements.

Solution:

On SMITH CHART A outline the above range of impedance values, then superimpose this chart on the design curves of Fig. 10.6. Determine the limiting values for X_L and X_C from the curves which just touch the edges of the outlined area. The following limiting values will be observed:

$$X_L = 0.5\,Z_0 \text{ to } 2.0\,Z_0 = 50 \text{ to } 200 \text{ ohms}$$

$$X_C = 0.5\,Z_0 \text{ to } 1.1\,Z_0 = 50 \text{ to } 110 \text{ ohms}$$

10.2 *T*-TYPE MATCHING CIRCUITS

By the addition of a third reactance element in series with the chosen input resistance obtained with an L-type impedance matching circuit, thus forming a T-type circuit, any complex load impedance value can be transformed to any desired complex input impedance value. The overlay charts of Figs. 10.3 to 10.10 inclusive are applicable in this case also. The reactance required in the third element depends upon the value of input reactance desired. If a circuit is chosen which already includes a series reactance element in the input side, such as one of the circuits shown in diagrams a, b, e, and f of Fig. 10.1, the "third" reactance required would be combined algebraically with the former, resulting in a single net reactance value in this position.

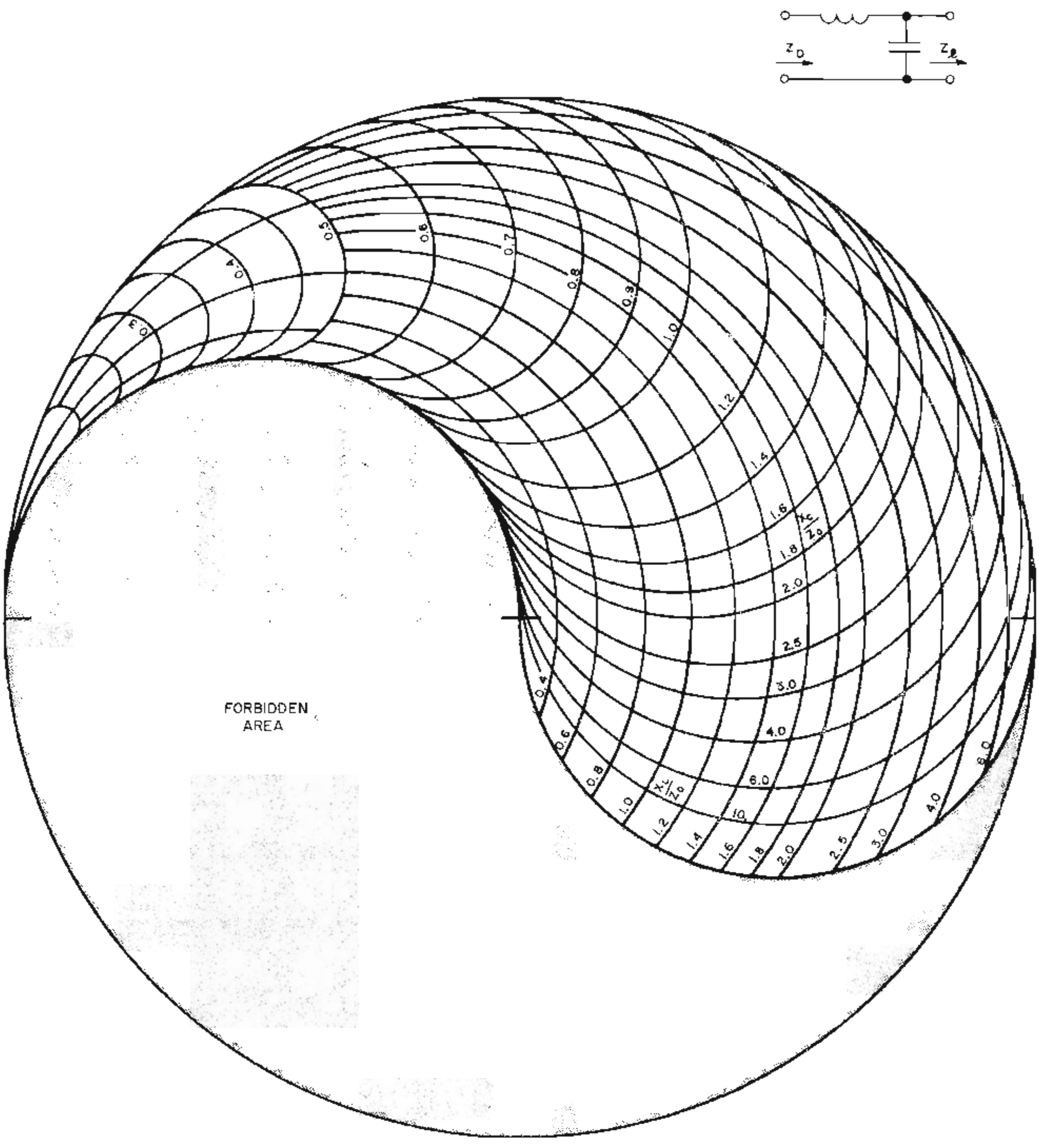


Fig. 10.3. Normalized reactances of L-circuit elements required to transform Z_ℓ to Z_0 (overlay for Charts A, B, or C in cover envelope).

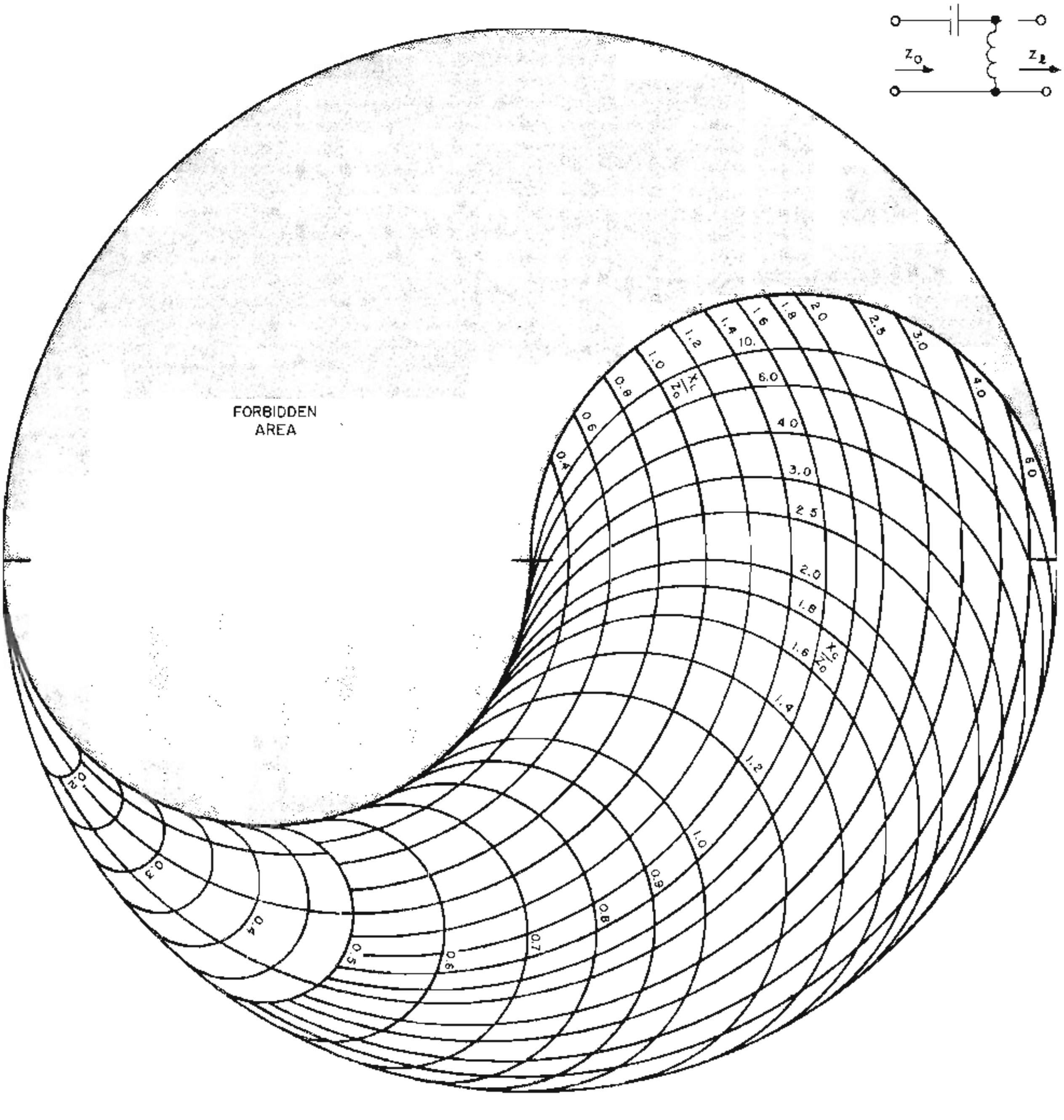


Fig. 10.4. Normalized reactances of L-circuit elements required to transform Z_ℓ to Z_0 (overlay for Charts A, B, or C in cover envelope).

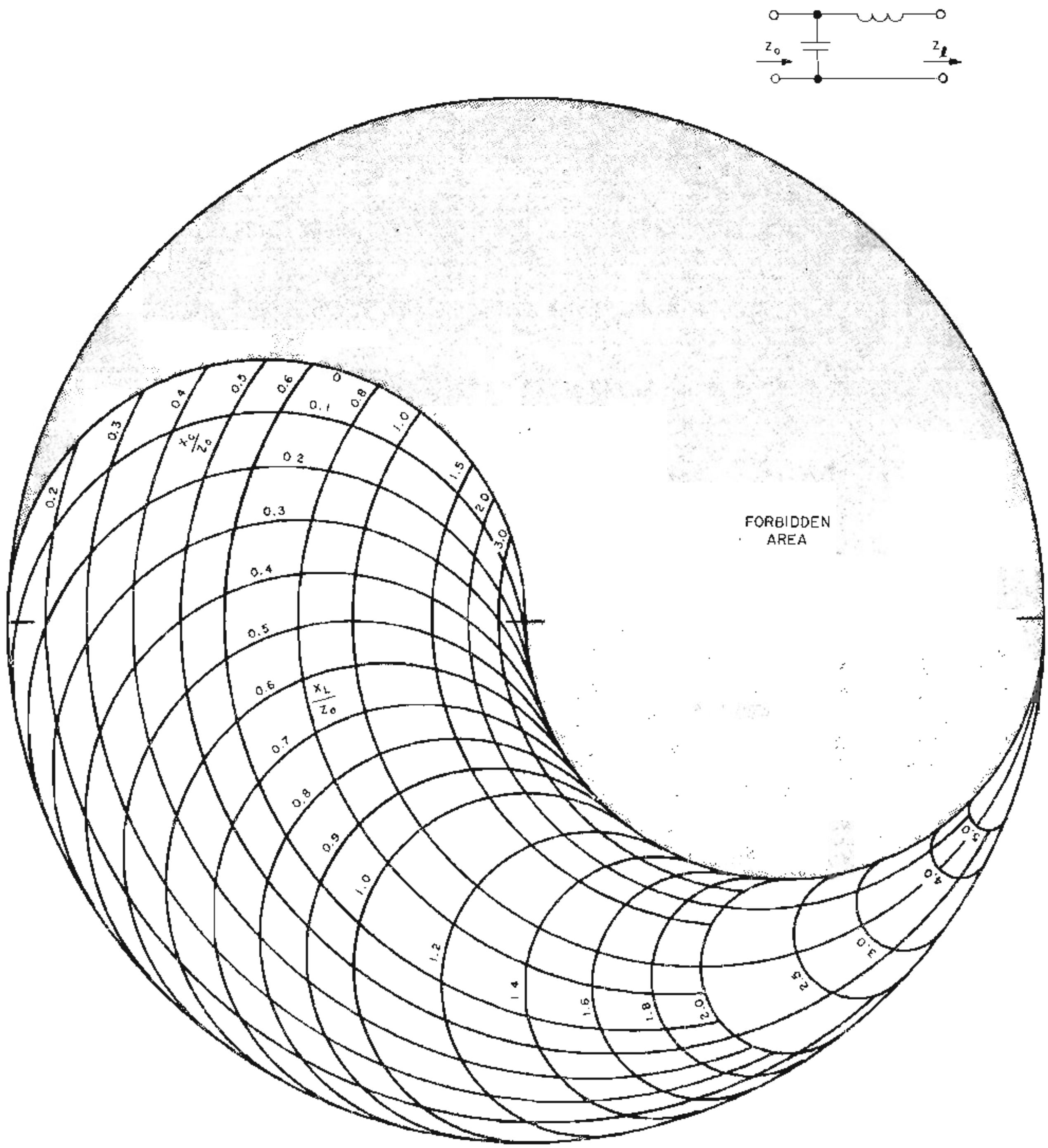


Fig. 10.5. Normalized reactances of L-circuit elements required to transform Z_ℓ to Z_0 (overlay for Charts A, B, or C in cover envelope).

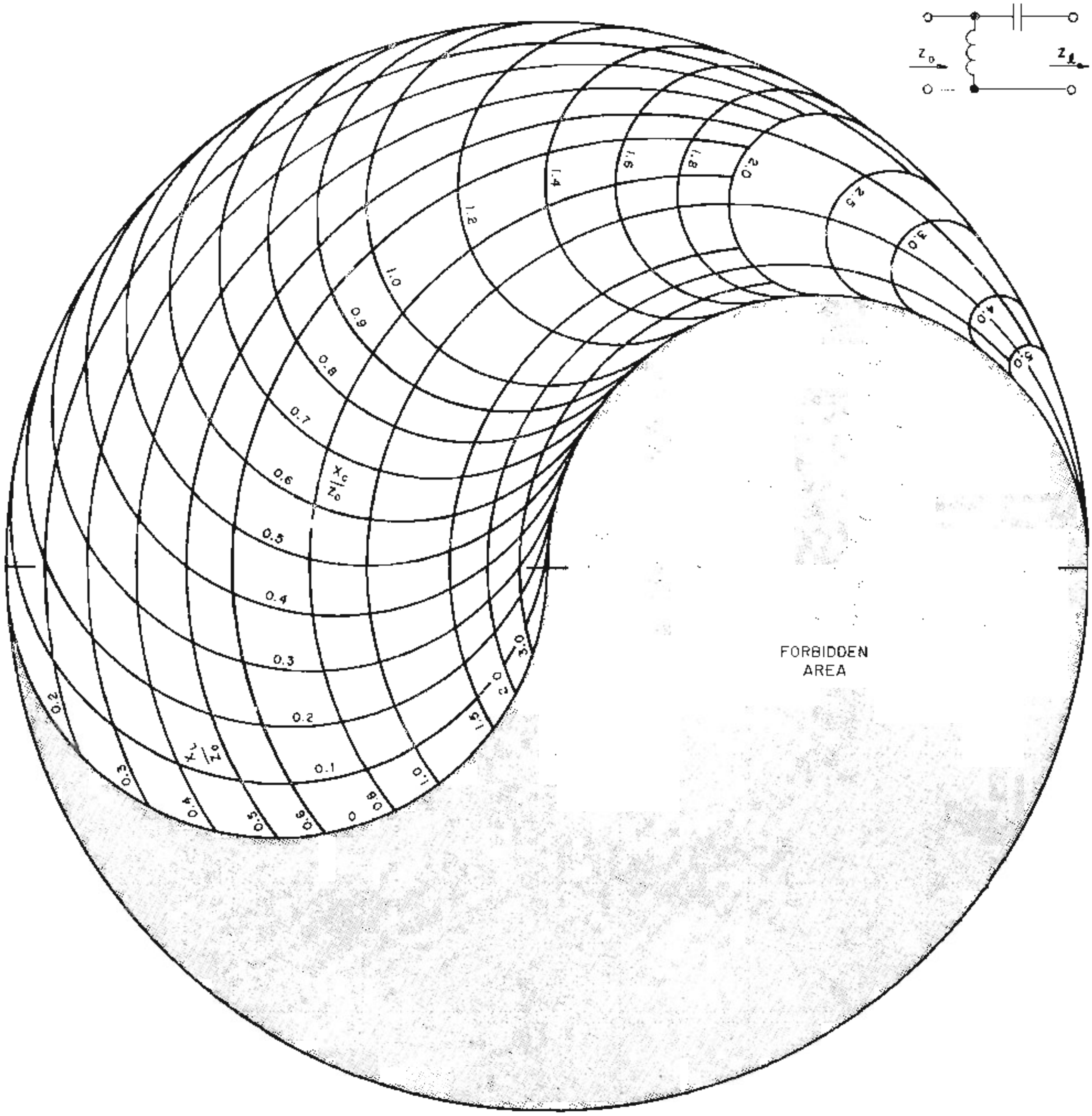


Fig. 10.6. Normalized reactances of L-circuit elements required to transform Z_ℓ to Z_0 (overlay for Charts A, B, or C in cover envelope).

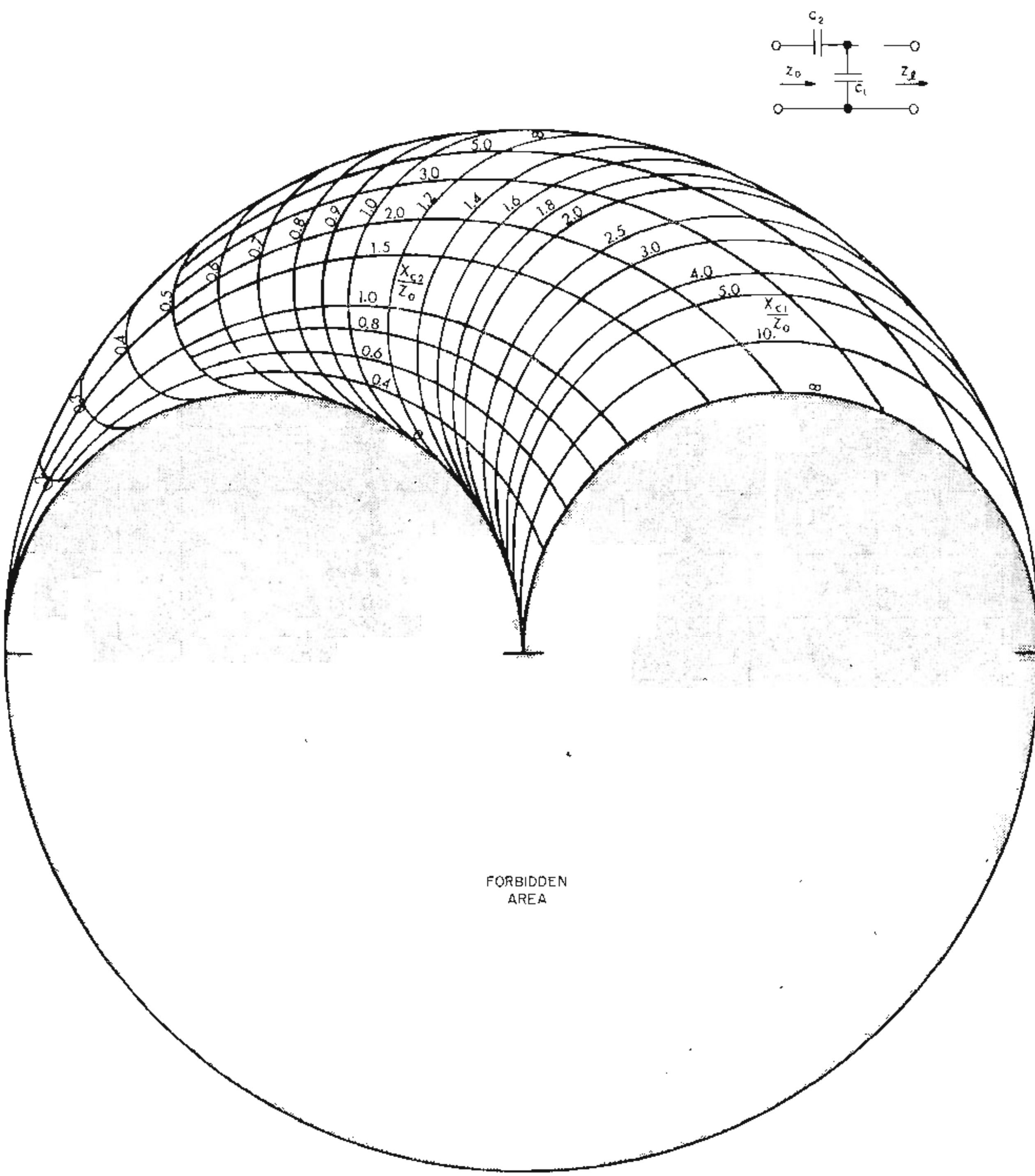


Fig. 10.7. Normalized reactances of L-circuit elements required to transform Z_l to Z_0 (overlay for Charts A, B, or C in cover envelope).

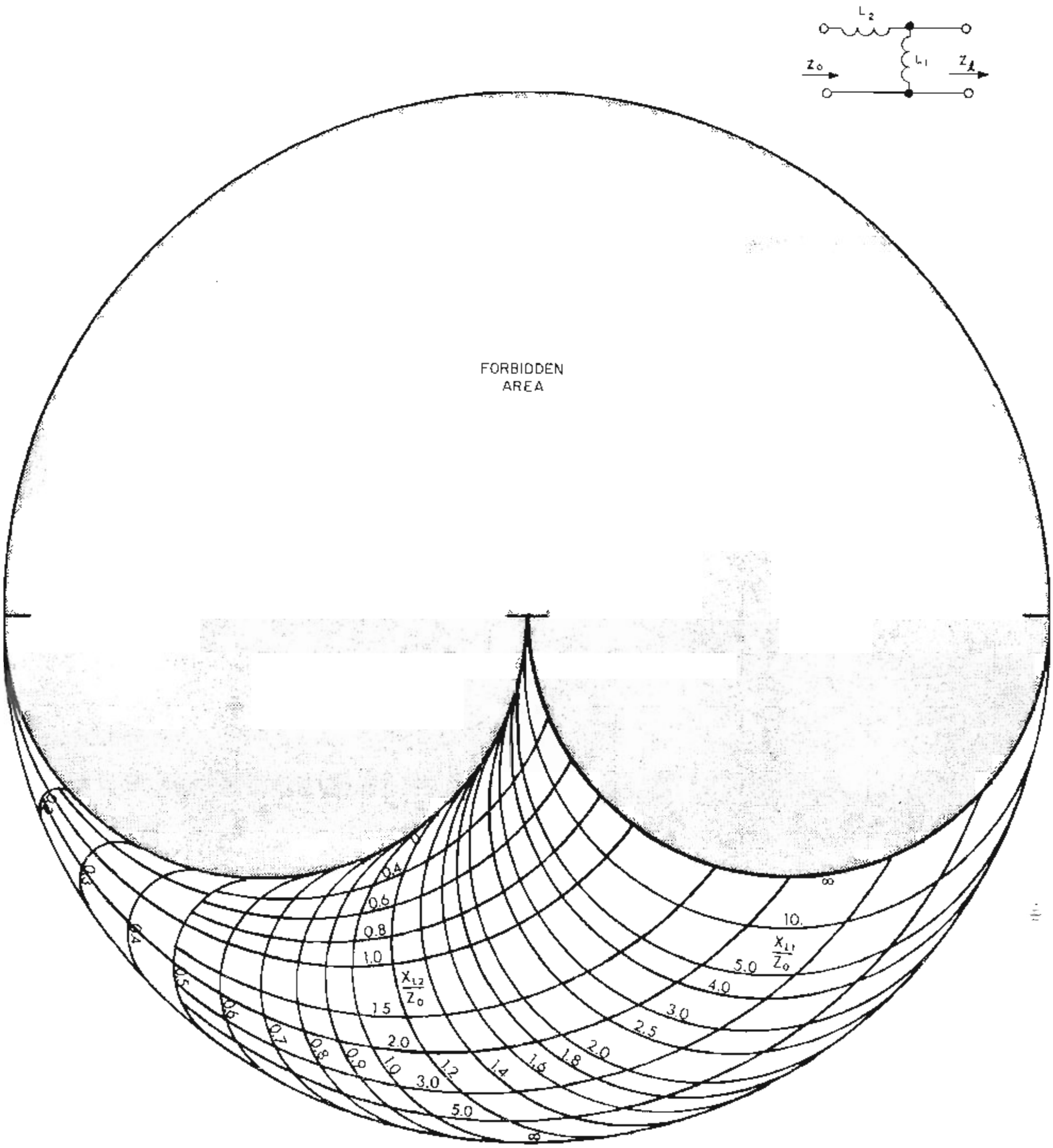


Fig. 10.8. Normalized reactances of *L*-circuit elements required to transform Z_ℓ to Z_0 (overlay for Charts A, B, or C in cover envelope).

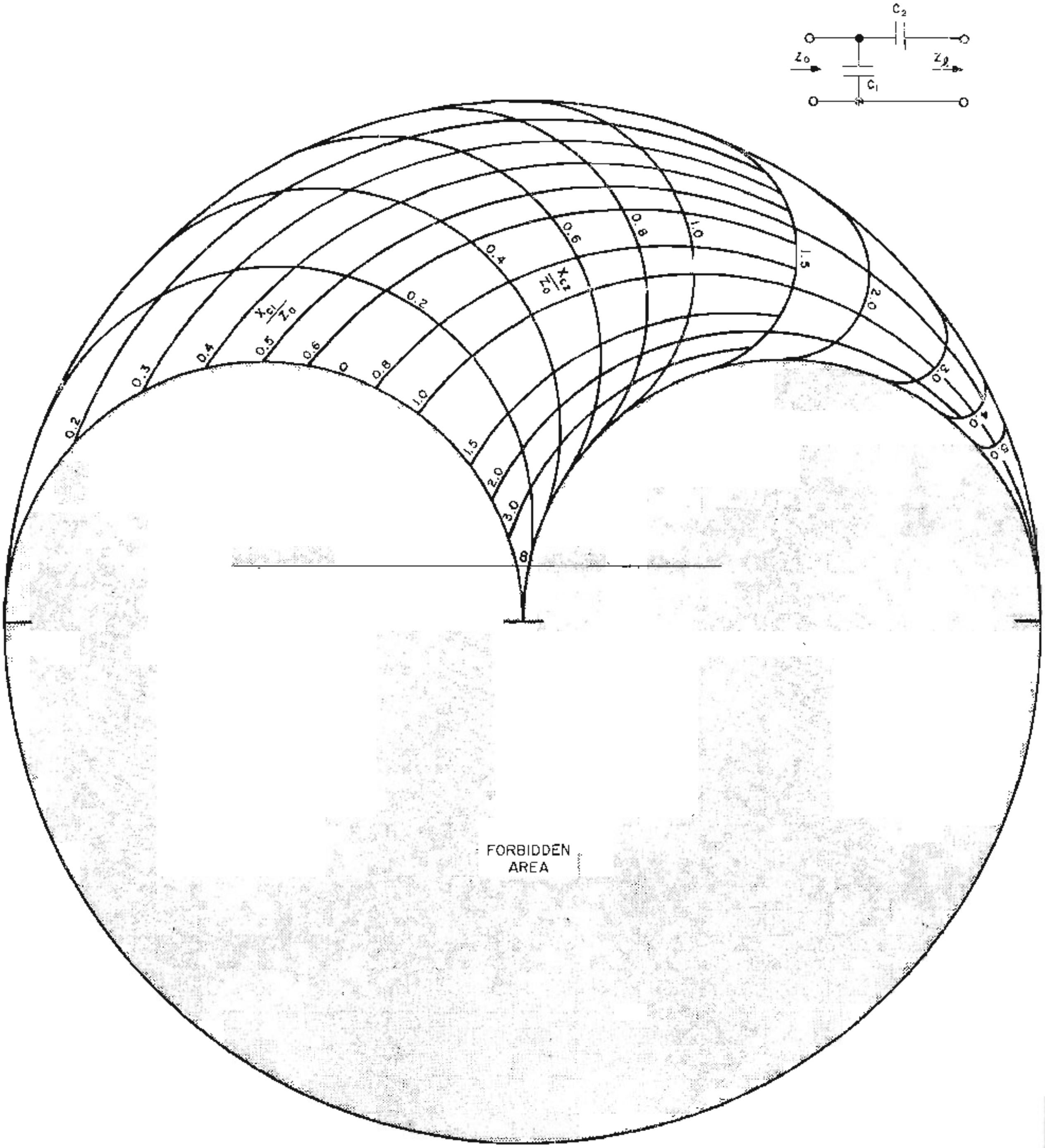


Fig. 10.9. Normalized reactances of L-circuit elements required to transform Z_ℓ to Z_0 (overlay for Charts A, B, or C in cover envelope).

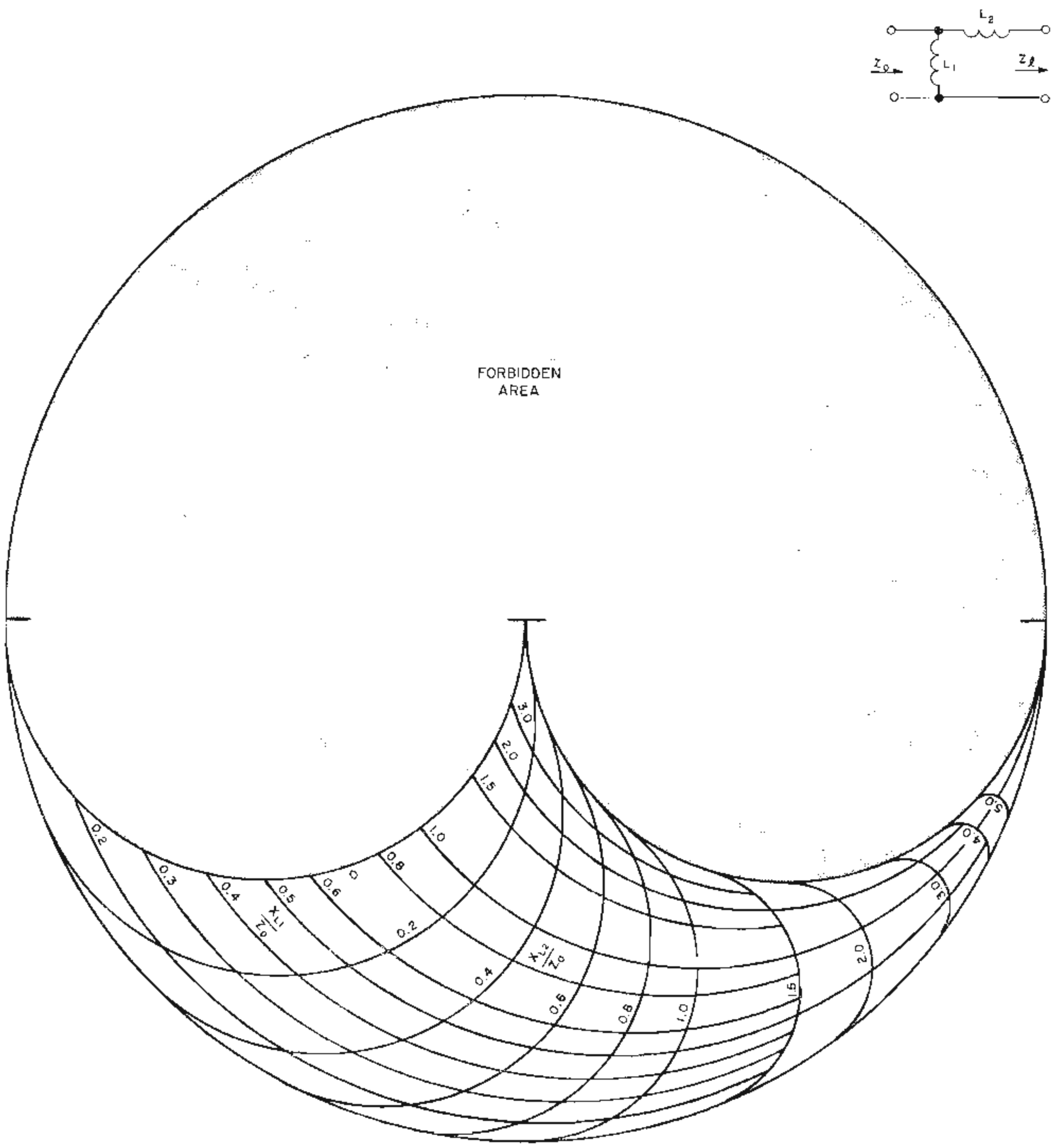


Fig. 10.10. Normalized reactances of L-circuit elements required to transform Z_ℓ to Z_0 (overlay for Charts A, B, or C in cover envelope).

10.3 BALANCED *L*- OR BALANCED *T*-TYPE CIRCUITS

If the input impedance of a matching circuit must be balanced with respect to ground, the *L*-type circuit design curves can be used to design a suitable impedance matching circuit by treating the problem as an unbalanced one. The required series reactance obtained from the diagrams is simply divided into two parts, each having one-half of the value called for on the charts. These two halves of the necessary total series reactance are then connected in series with each side of the circuit to preserve the balanced-to-ground arrangement.

CHAPTER 11

Measurements of Standing Waves

11.1 IMPEDANCE EVALUATION FROM FIXED PROBE READINGS

At radio frequencies where slotted waveguide or transmission line sections would be excessively long, probe measurements of the relative current (or relative voltage) amplitudes at discrete sampling points along the waveguide provide a convenient and practical technique for measuring the complex impedance and related parameters. The SMITH CHART is useful for interpreting and evaluating data obtained from such measurements [112, 208] as will be described herein. The principle is made use of in the SMITH CHART plotting board shown in Fig. 14.5.

In a waveguide or transmission line propagating electromagnetic wave energy in a single mode, relative amplitude measurements of either current or voltage at three fixed probe positions uniquely determine the standing wave ratio and the wave position, provided that no two of the probes are separated an exact multiple of one-half wavelength. The standing wave pattern, in turn, is related to the impedance and other waveguide parameters, as described in previous chapters. The electrical separations of the probes need not be uniform but must be known. This is generally calculable [10] from the physical construction of the waveguide or transmission line, as described in Chap. 3.

As was shown in Chap. 3, the shape of standing current and voltage waves of a given ratio are identical, and both are different and unique for each different standing wave ratio, varying from a succession of half sine waves as the amplitude ratio approaches infinity to a sinusoidal shape as the amplitude ratio approaches unity. It is possible for this reason to plot unique families of impedance locus curves, each family corresponding to a fixed probe spacing, and each curve of each family corresponding to a fixed ratio of current (or voltage) probe readings. Figures 11.1 through 11.4 show four families of such probe ratio curves, corresponding to fixed probe spacings of one-sixteenth, one-eighth, three-sixteenths, and one-quarter wavelength.

For flexibility in the application of these curves to voltage-probe as well as current-probe readings, the SMITH CHART impedance coordinates with which they are associated have been omitted. However, when required these impedance coordinates can conveniently be supplied as an overlay thereon by the use of the translucent SMITH CHART (Chart A) in the back cover of this book.

Any two families of probe ratio curves in combination with the translucent SMITH CHART coordinates are capable of determining values of all possible load impedances along a waveguide. In any given situation, however, a single value is indicated at P_ℓ on the SMITH CHART coordinates, which point occurs at the intersection of a single pair of probe ratio curves corresponding to the actual readings.

At 180° (one-half wavelength) separation two probe readings along a uniform lossless waveguide are identical and the impedance loci curves for the two families of probe ratio curves become coincident with one another. One-half wavelength spacing for any of the three sampling points must, therefore, be avoided. Sampling point spacings of less than about one-sixteenth wavelength or between seven- and nine-sixteenths wavelength, fifteen- and seventeen-sixteenths wavelength, etc., are inadvisable, since in these cases the families of ratio curves so nearly parallel each other that small errors in readings can represent large errors in the determination of the impedance.

11.1.1 Example of Use of Overlays with Current Probes

Three current probes are spaced one-eighth wavelength apart along a transmission line; the overall separation of probes is, therefore, one-quarter wavelength. Two probes, each designated P_g, are located on the generator side, and a third probe P_ℓ on the load side. The ratio of the current in the center probe P_g to the current in the probe on the load side of the group P_ℓ is assumed, for example, to be 0.5. (Note that this pair of probes is spaced one-eighth wavelengths.) Simultaneously, the ratio of currents in the probe on the generator side of the group P_g to the probe on the load side of the group P_ℓ is assumed to be 0.7. (Note that this pair of probes is spaced one-quarter wavelength and that P_ℓ is common to both probe ratios.)

Find the impedance on the SMITH CHART at P_ℓ:

1. For the pair of probes which is spaced one-eighth wavelength, select Fig. 11.2, which applies specifically to probe ratios at this spacing, and trace the locus curve P_g/P_ℓ = 0.5 onto the superposed impedance coordinates of the translucent SMITH CHART (Chart A) in the back cover of this book.

2. For the pair of probes which is spaced one-quarter wavelength, select Fig. 11.4 and similarly trace the locus curve $P_g/P_\ell = 0.7$, thereon, onto the same impedance coordinates.

3. Observe the intersection of these two traces on the SMITH CHART impedance coordinates to be at the common reference point P_ℓ where the normalized impedance of the transmission line is $(0.5 + j\,0.5)\,Z_0$.

11.2 INTERPRETATION OF VOLTAGE PROBE DATA

It is equally feasible and practical to employ Figs. 11.1 through 11.4 in combination with the SMITH CHART coordinate overlay, to determine the impedance corresponding to any three voltage probe readings. As was shown in Chap. 4, the normalized voltage at any position along a uniform lossless waveguide is exactly equivalent to the normalized current one-quarter wavelength removed

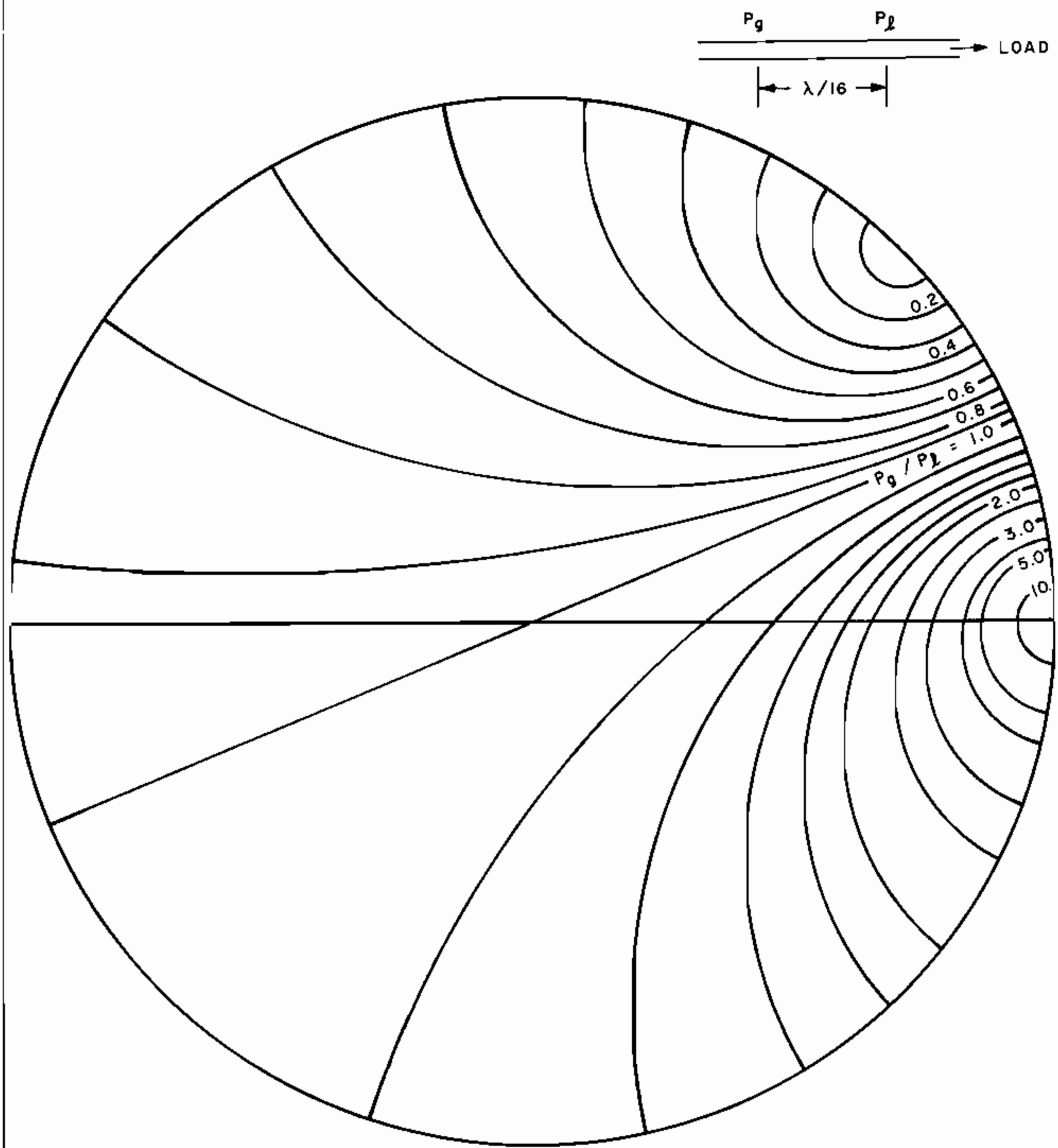


Fig. 11.1. Loci of constant ratios of current or voltage on a SMITH CHART at two probe points P_g and P_ℓ spaced λ/16 along a transmission line (overlay for Charts A, B, or C in cover envelope).

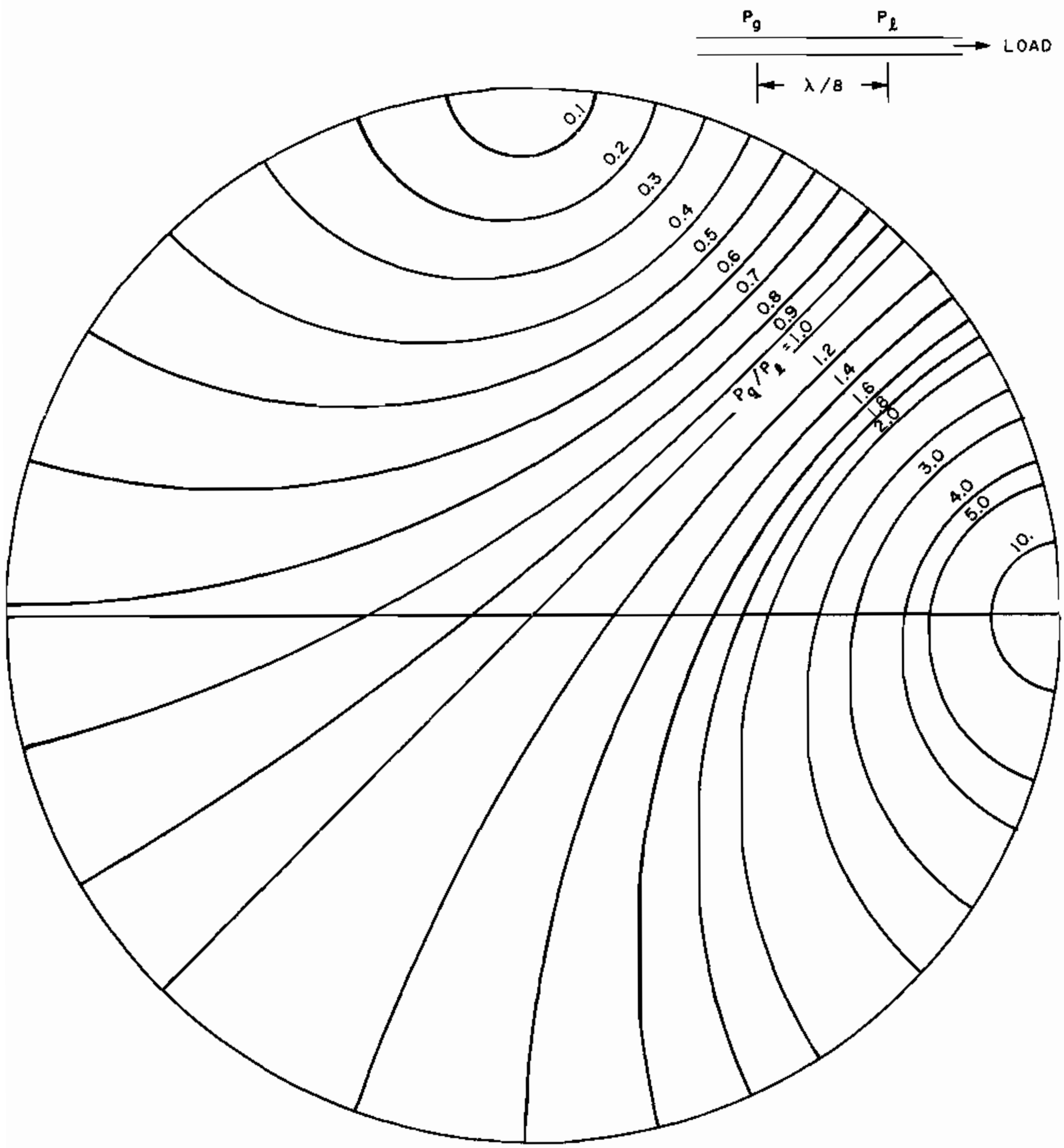


Fig. 11.2. Loci of constant ratios of current or voltage on a SMITH CHART at two probe points P_g and P_ℓ spaced λ/8 along a transmission line (overlay for Charts A, B, or C in cover envelope).

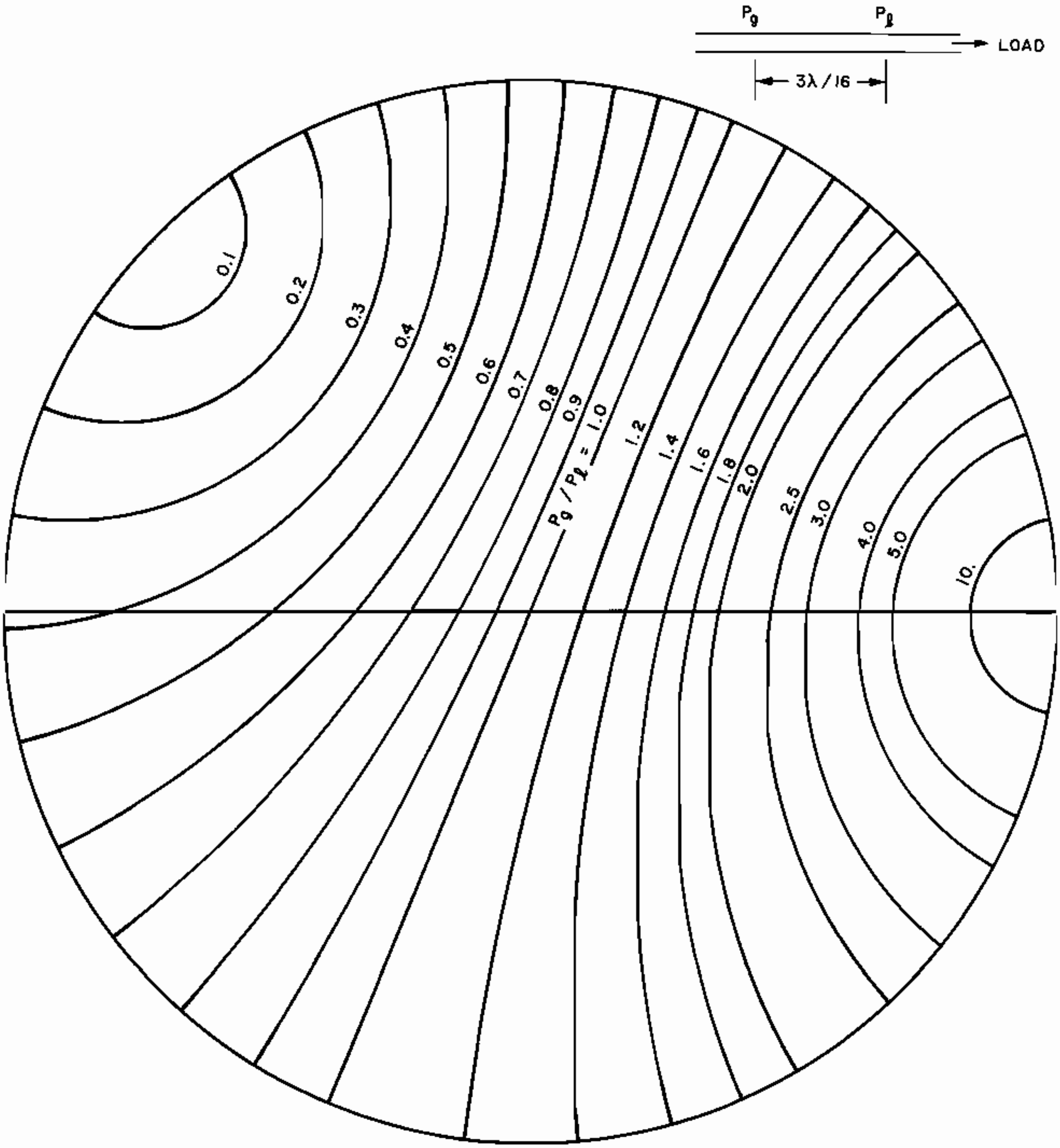


Fig. 11.3. Loci of constant ratios of current or voltage on a SMITH CHART at two probe points P_g and P_ℓ spaced 3λ/16 along a transmission line (overlay for Charts A, B, or C in cover envelope).

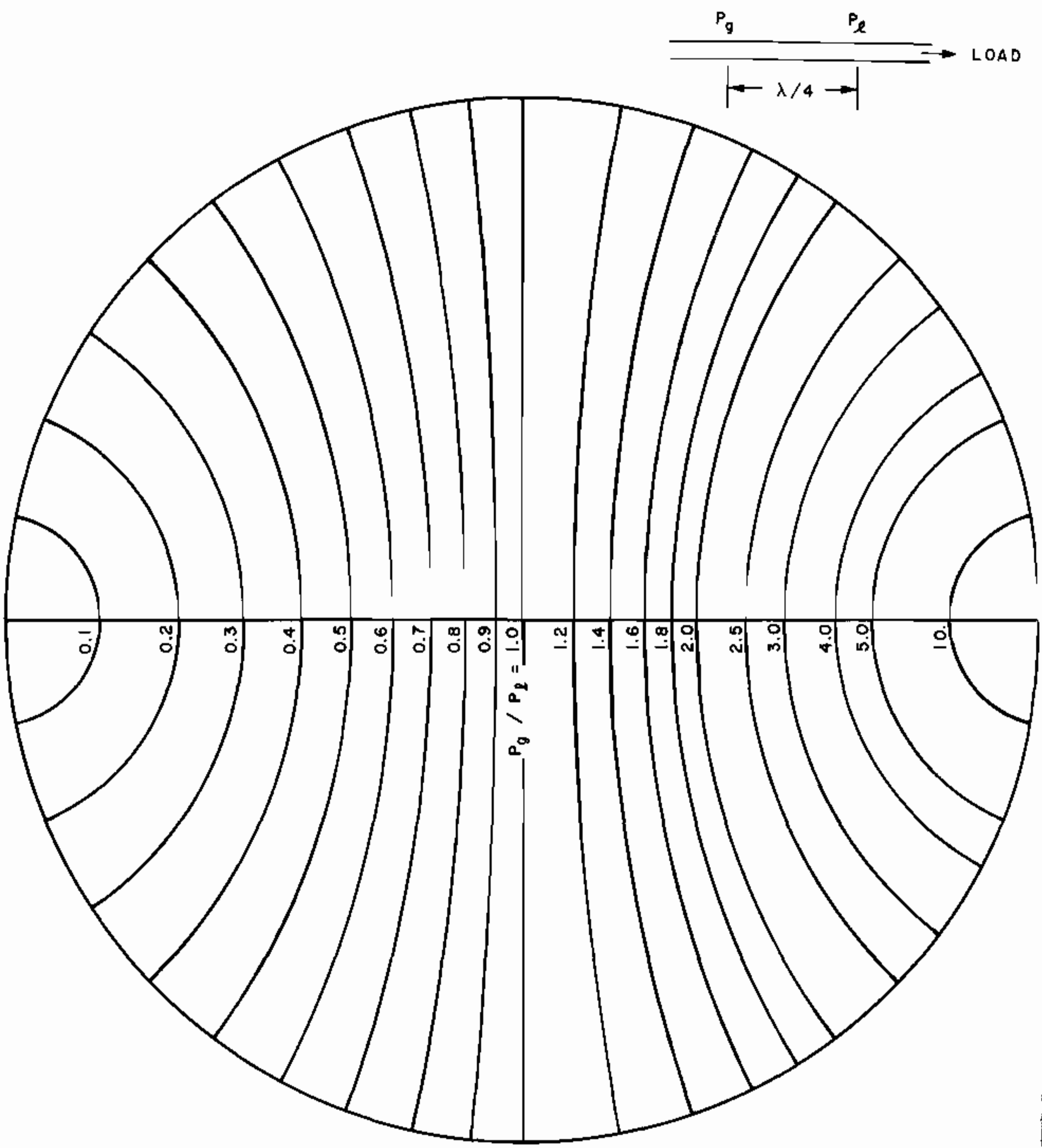


Fig. 11.4. Loci of constant ratios of current or voltage on a SMITH CHART at two probe points P_g and P_ℓ spaced λ/4 along a transmission line (overlay for Charts A, B, or C in cover envelope).

therefrom. Also, any one-quarter wavelength transfer of reference point along a waveguide is represented by a rotation around the center of the SMITH CHART coordinates an angular distance of 180°. Therefore, if voltage probes are used instead of current probes, for which the ratio curves in Figs. 11.1 through 11.4 apply directly, it is only necessary to rotate the ratio curves through an angle of 180° with respect to the SMITH CHART impedance coordinates to make them applicable. The desired impedance is then indicated at P_ℓ, as before, from any intersecting pair of voltage probe ratio curves.

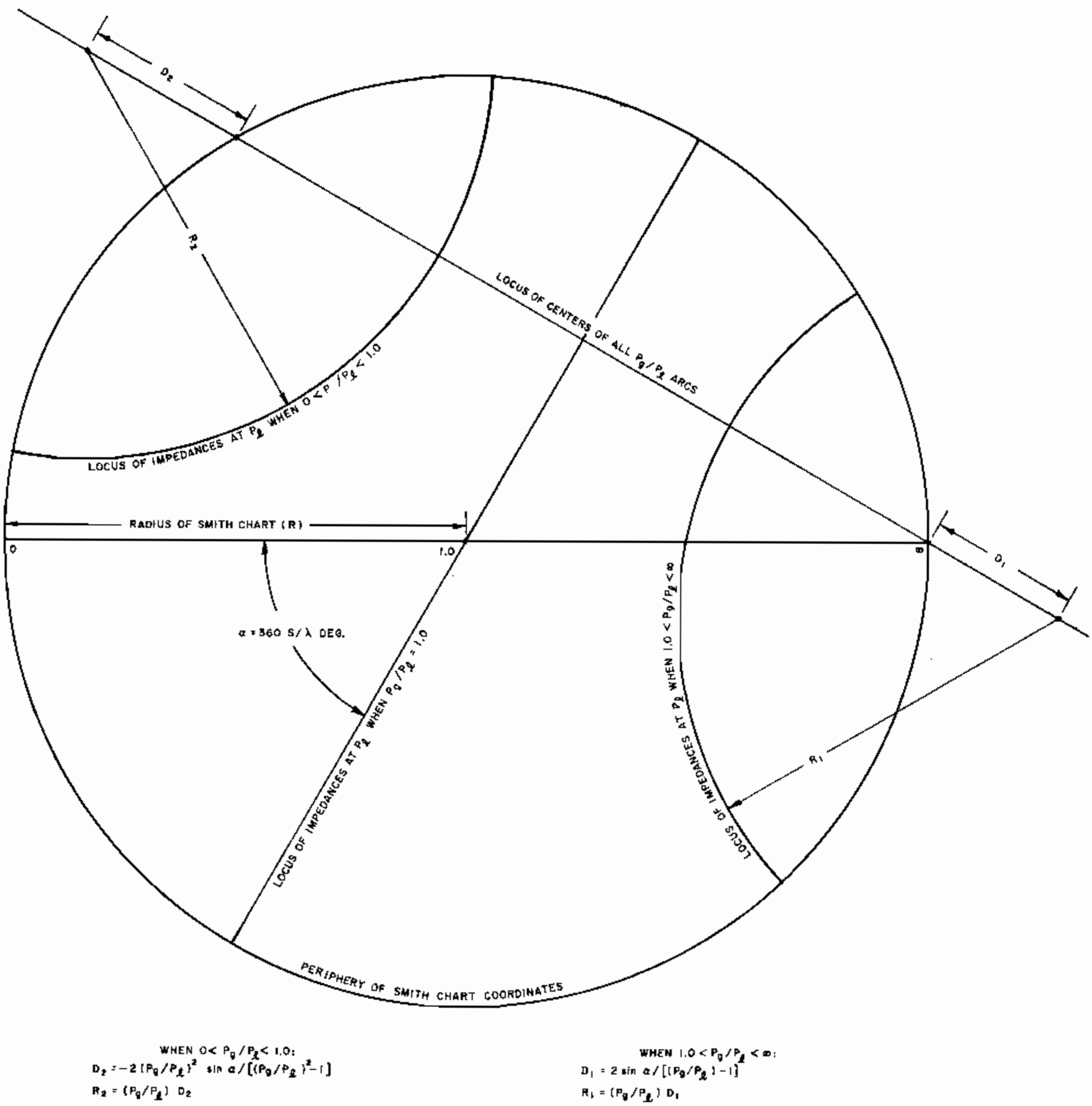


Fig. 11.5. Construction of loci of constant ratios of current on impedance coordinates of a SMITH CHART at two points spaced S/λ along a lossless transmission line.

In summary, as oriented in Figs. 11.1 through 11.4 with respect to the overlay Charts A, B, or C, in the cover envelope, the intersection point of any pair of current ratio curves gives the impedance at P_ℓ, while the intersection point of any pair of voltage ratio curves gives the admittance at P_ℓ. When rotated 180° about their axes with respect to the coordinates of the overlay charts A, B, or C, the intersection point of any pair of current ratio curves gives the admittance at P_ℓ, while the intersection point of any pair of voltage ratio curves gives the impedance at P_ℓ.

11.3 CONSTRUCTION OF PROBE RATIO OVERLAYS

Information is given in Fig. 11.5 for plotting probe ratios which correspond to any desired probe separation. All such plots require only straight lines and circles for their construction. The outer boundary of such a construction corresponds to the boundary of the SMITH CHART coordinates.

As shown in Fig. 11.5 the separation of any two sampling points S/λ determines the angle α as measured from the horizontal R/Z_0 axis. This angle establishes the position of a straight line through the center of the construction which represents the locus of impedances at the probe position P_ℓ when the current standing wave ratio is varied from unity to infinity while the wave is maintained in such a position along the transmission line with respect to the position of the two sampling points that they always read alike, that is, that $P_g/P_\ell = 1.0$.

A construction line perpendicular to the locus $P_g/P_\ell = 1$ and passing through the infinite resistance point on the R/Z_0 axis will then lie along the center of all of the P_g/P_ℓ circles which it may be desired to plot.

The ratio of each of these circular arcs (R_1 and R_2) which corresponds to a particular current ratio, and the distance of their centers from the chart rim (D_1 and D_2), is given by the formulas in Fig. 11.5 as a function of the ratio of P_g to P_ℓ and the SMITH CHART radius R.

CHAPTER 12

Negative Smith Chart

12.1 NEGATIVE RESISTANCE

In all passive waveguide structures, the primary circuit elements resistance and conductance act like absorbers of electromagnetic wave energy. As such, their mathematical sign is, by convention, positive. However, certain active electrical devices exist whose equivalent circuit can most conveniently be represented by negative resistance or negative conductance elements. Negative resistance and negative conductance circuit elements act more like sources than absorbers of electromagnetic wave energy.

Electrical devices such as parametric and tunnel diode reflection amplifiers, which can be schematically represented by negative resistance or negative conductance in combination with conventional circuit elements, are frequently used in connection with waveguides. In order to properly represent their characteristics on a SMITH CHART for graphical analysis, it is important to understand the effect of negative resistance or negative conductance upon the associated waveguide electrical characteristics. This will be discussed more fully herein. It will be found, for example, that when a waveguide is terminated in a circuit which has a resultant negative resistance or negative conductance component, the complex reflection coefficients and the complex transmission coefficients at all points along the waveguide are different from what they would have been had the sign of the resistance or conductance been positive.

Negative resistances and negative conductances do not actually produce energy, but rather act to transform and release energy from an associated source (such as a battery) into electromagnetic energy whose frequency is controlled by a primary source of smaller energy level. Negative resistance is defined [7] as the property of a two-terminal device with an internal source of energy which is controlled either by current through or by voltage across the terminals, but not by both. Negative conductance is defined as the reciprocal of negative resistance.

The mathematical distinction between positive and negative resistance (or positive and negative conductance) is portrayed by the slope at a point on a plot of the voltage-current relationship, where current is represented as the ordinate and voltage as the abscissa on a cartesian coordinate grid. Figure 12.1(a) illustrates the voltage-current characteristics of a typical negative-resistance device such as a tunnel diode. If the current through a resistance or conductance increases with increased applied voltage (positive slope), the sign of the resistance or conductance is considered to be positive in this portion of its operating range. On the other hand, if the current decreases with increased applied voltage (negative slope), the sign of the resistance or conductance is considered to be negative in this portion of its operating range.

In Fig. 12.1(a) the points P_m and P_n, where the slope is zero, illustrate conditions where the resistance is plus and minus infinity, respectively. The inflection point P, where the slope changes from a decreasing negative value to an increasing negative value, represents conditions where the negative resistance is minimum (negative conductance is maximum). The effective value of negative resistance at any point on the voltage-current plot of Fig. 12.1(a) is shown in Fig. 12.1(b).

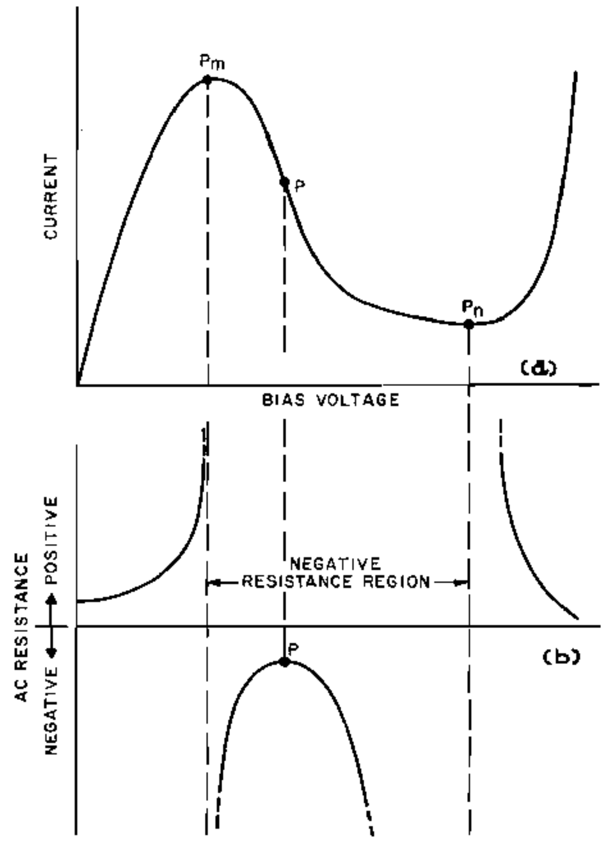


Fig. 12.1. Electrical characteristics of a typical tunnel diode.

12.2 GRAPHICAL REPRESENTATION OF NEGATIVE RESISTANCE

The graphical representation on the SMITH CHART of impedances or admittances with negative resistance or negative conductance components, respectively, presents no special problems, as may have been inferred from the various transformations of the conventional chart coordinates which have been suggested to accomplish this [12].

It will be shown in this chapter how the conventional SMITH CHART impedance coordinates and associated peripheral scales (Fig. 3.3) are applicable without any transformation to the representation of impedances with either positive or negative resistance components and, in fact, how a single representation best serves both purposes. Since the sign of a negative resistance is, by convention, identical to the sign of the equivalent conductance, it follows that no change is required in the admittance coordinates of the conventional SMITH CHART to represent admittances with either positive or negative conductance components. All *peripheral* scales of the conventional SMITH CHART *including the reflection coefficient phase angle* scale are independent of the sign of the real components of impedance or admittance.

All *radial* scales of the conventional SMITH CHART which have been described in previous chapters *except the reflection coefficient magnitude scale and the reflection loss scale* apply in all cases where the real components of impedance or admittance are designated with positive or negative values.

Interpretation of the use of these previously described radial scales in connection with negative real-component coordinates may be helpful and, accordingly, will be discussed herein. A new radial scale for reflection coefficient magnitude, to be used exclusively with negative real-component coordinates, will be fully described. The radial "reflection loss" scale described in Chap. 4 is meaningless in connection with real-component coordinates.

The reflection coefficient and transmission coefficient magnitude and phase overlays (Figs. 3.5 and 5.4, respectively) which have been described for positive real-component coordinates of impedance and admittance are not applicable to negative real-component coordinates. In their place, a new reflection coefficient magnitude and phase overlay (Fig. 12.6) and a new transmission coefficient magnitude and phase overlay (Fig. 12.7), specifically applicable to negative real-component coordinates, will be presented and discussed.

The SMITH CHART negative real-component coordinates are particularly applicable to the design and performance analysis of reflection-type oscillators and amplifiers employing, for example, tunnel diodes which function in connection with waveguides, well up into the microwave frequency range. The gain and stability characteristics of such devices may readily be determined from an input impedance or input admittance plot of operating parameters, such as frequency and bias voltage, on negative real-component SMITH CHART coordinates.

12.3 CONFORMAL MAPPING OF THE COMPLETE SMITH CHART

A set of cartesian coordinates is shown in Fig. 12.2(a) whose axes are designated with normalized values of positive and negative resistance, and positive and negative reactance. (In order to simplify this discussion the optional admittance component designations on these coordinates are omitted.) Upon this coordinate system the loci of the reflection coefficient amplitude and phase components may be plotted (Fig. 12.2(b)). These loci are observed to be orthogonal families of circles.

On the positive resistance (right) half of the plot in Fig. 12.2(b) the reflection coefficient magnitude is at all points less than unity. In terms of the normalized impedance components R/Z_0 and $+jX/Z_0$ the complex reflection coefficient on the positive half of this plot is given by

$$\rho = \frac{(R/Z_0 + jX/Z_0) - 1}{(R/Z_0 + jX/Z_0) + 1} = |\rho| e^{-j\theta} \qquad (12\text{-}1)$$

On the negative (left) half of the plot of Fig. 12.2(b) the magnitude of the reflection coefficient is the reciprocal of the magnitude of the reflection coefficient at the corresponding position on the positive resistance half, as reflected through the origin. Thus, on the negative half it is

$$\rho = \frac{(-R/Z_0 + jX/Z_0) - 1}{(-R/Z_0 + jX/Z_0) + 1} = |\rho^{-1}| e^{-j\theta} \qquad (12\text{-}2)$$

It will be observed from Fig. 12.2(b) that both the complex reflection coefficient and the related complex impedance component curves extend to infinity in all directions from the origin. This seriously limits the useful range of such a plot of this relationship.

To overcome the above difficulty a bilinear transformation of coordinates may be made, the results of which are shown in Fig. 12.2(c).

In such a transformation the circular shape of all individual curves and the angle between respective curves of the two families at all

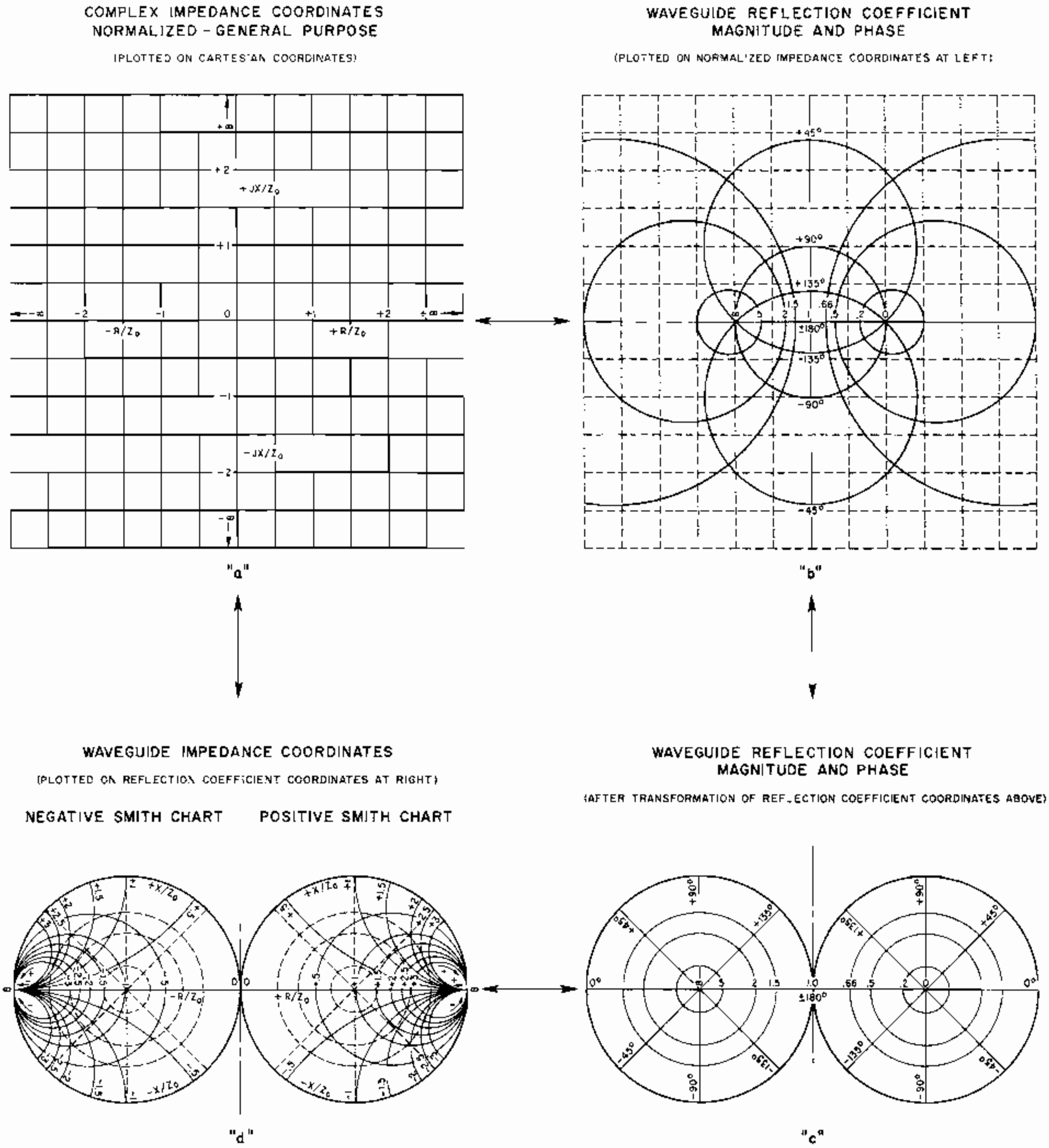


Fig. 12.2. Conformal mapping of complete SMITH CHART.

points of intersection are unchanged. It will be noted that, in contrast to Fig. 12.2(b), the finite area occupied by the transformed coordinates of Fig. 12.2(c) includes all possible values of the complex reflection coefficient.

Using Fig. 12.2(c) as a new coordinate grid, the loci of normalized waveguide impedance components can be plotted from the relationship given in Eqs. (12-1) and (12-2). This results in a complete (positive and negative real component) SMITH CHART shown on Fig. 12.2(d).

At this point it is important to recognize that any graphical representation of the interrelationship of several dependent variables which can be plotted on a flat sheet of paper is equally valid when viewed from the front, or through the paper on which the plot is made, i.e., from the back. Accordingly, one is free to choose the more convenient of these two representations. With this in mind, it will be observed that the left half of the impedance plot of Fig. 12.2(d) (shown as solid line curves) viewed through the paper from the back is identical to the right half, viewed from the front, with the single exception that the sign of the normalized resistance component values is negative. It will also be observed that when compared in this way the sign of the normalized reactance component is unchanged, i.e., the positive reactance region of the plot remains on the upper half and the negative reactance region of the plot remains on the lower half. Also, the magnitude of the positive reactance increases in a clockwise direction and the magnitude of the negative reactance increases in a counterclockwise direction on both representations.

Thus, by electing to view the two halves of Fig. 12.2(d) as indicated above, the same plot may be used to represent either positive or negative resistance components. However, in the same way that one must choose whether the SMITH CHART coordinates are to represent normalized impedance *or* normalized admittance *before entering the chart,* so must he also choose whether these same coordinates are to represent impedances (or admittances) having positive *or* negative real parts.

The advantages to be derived from a common SMITH CHART representation of impedances or admittances with either positive or negative real part components are apparent when it is appreciated that this permits the application of a common technique for the graphical evaluation of impedance employing standing wave amplitude and position measurements along a waveguide.

12.4 REFLECTION COEFFICIENT OVERLAY FOR NEGATIVE SMITH CHART

As explained above, the conventional SMITH CHART of Fig. 3.3 or Fig. 8.6 can serve for the representation of impedances or admittances with either positive or negative real parts. However, certain important differences exist in the respective complex reflection coefficient and transmission coefficient overlays. As previously stated, the phase angle of the voltage or current reflection coefficient overlay, displayed on the peripheral scale of the above chart and also as the family of radial lines on the overlay (Fig. 3.5), is unaltered in magnitude or sense of direction when the coordinates are chosen to represent negative real parts. The magnitude of the voltage or current reflection coefficient, as represented on this overlay by the family of concentric circles, applies only to SMITH CHART coordinates with positive real components. For negative real components the voltage or current reflection coefficient magnitudes are represented in the overlay of Fig. 12.6.

When SMITH CHART coordinates (Fig. 12.3) are used for displaying impedances with negative resistance components, the magnitude of the voltage or current reflection coefficient at any point on the chart, represented by the values designated on the concentric circles on the overlay of Fig. 12.6, is the reciprocal of that shown at the same chart position on the overlay (Fig. 3.5) for the positive resistance chart. This may be seen from a comparison of Eqs. (12-1) and (12-2). On the positive resistance SMITH CHART coordinates the magnitude of the voltage or current reflection coefficient (radial scale) progresses linearly from unity at the rim of the chart to zero at its center (as shown on the right half of Fig. 12.2(c)), while on the negative resistance SMITH CHART coordinates this scale is not linear (as shown on the left half of Fig. 12.2(c)). Physically, this simply means that the magnitude of the reflected voltage (or current) waves along a waveguide is greater than the magnitude of the corresponding incident waves from the generator.

The net power flow along a waveguide terminated in a negative resistance (or negative conductance) is, accordingly, towards the generator. Such a termination thus acts like a secondary source of electromagnetic energy whose amplitude is equal to the amplitude of the primary source times the magnitude of the reflection coefficient.

For a given termination mismatch ratio the standing wave ratio (SWR) accompanying a negative resistance (or negative conductance) load component is the same as the SWR accompanying a positive resistance (or positive conductance component) of equivalent amplitude. The sign of the SWR is negative in the former case, which has no physical significance, and can, therefore, be ignored. Since it is not possible to distinguish from standing wave ratio and wave position measurements whether the load resistance (or conductance) component is positive or negative, to resolve this question it may be necessary to measure the absolute magnitude of the standing wave voltage and/or current, and to determine from these measurements the actual power flow in the waveguide in relation to the maximum available power output from the primary source operating into a known positive resistance load. If the power flow in the waveguide is more than the available power from the primary source, the load impedance or admittance has a negative resistance (or conductance) component. If the waveguide has sufficient attenuation, another method for resolving this question is to observe, from standing-wave probe measurements, whether the input impedance vs. position along the waveguide spirals toward the chart center (positive resistance or conductance termination) or toward the rim (negative resistance or conductance termination) as the fields are probed along the waveguide toward the generator.

12.5 VOLTAGE OR CURRENT TRANSMISSION COEFFICIENT OVERLAY FOR NEGATIVE SMITH CHART

The voltage, current, and power transmission coefficient as applied to waveguides terminated in impedances or admittances with positive real parts was discussed in Chap. 5. The definitions for these transmission coefficients as given therein apply regardless of whether the waveguide is terminated in impedances or admittances with positive or negative real parts. The transmission coefficient is defined in all cases as the complex ratio of the resultant of an incident and reflected quantity associated with the wave (such as voltage, current, or power) to the corresponding incident quantity.

A voltage (or current) complex transmission coefficient overlay for the coordinates in Fig. 12.3 is shown in Fig. 12.7. As oriented in

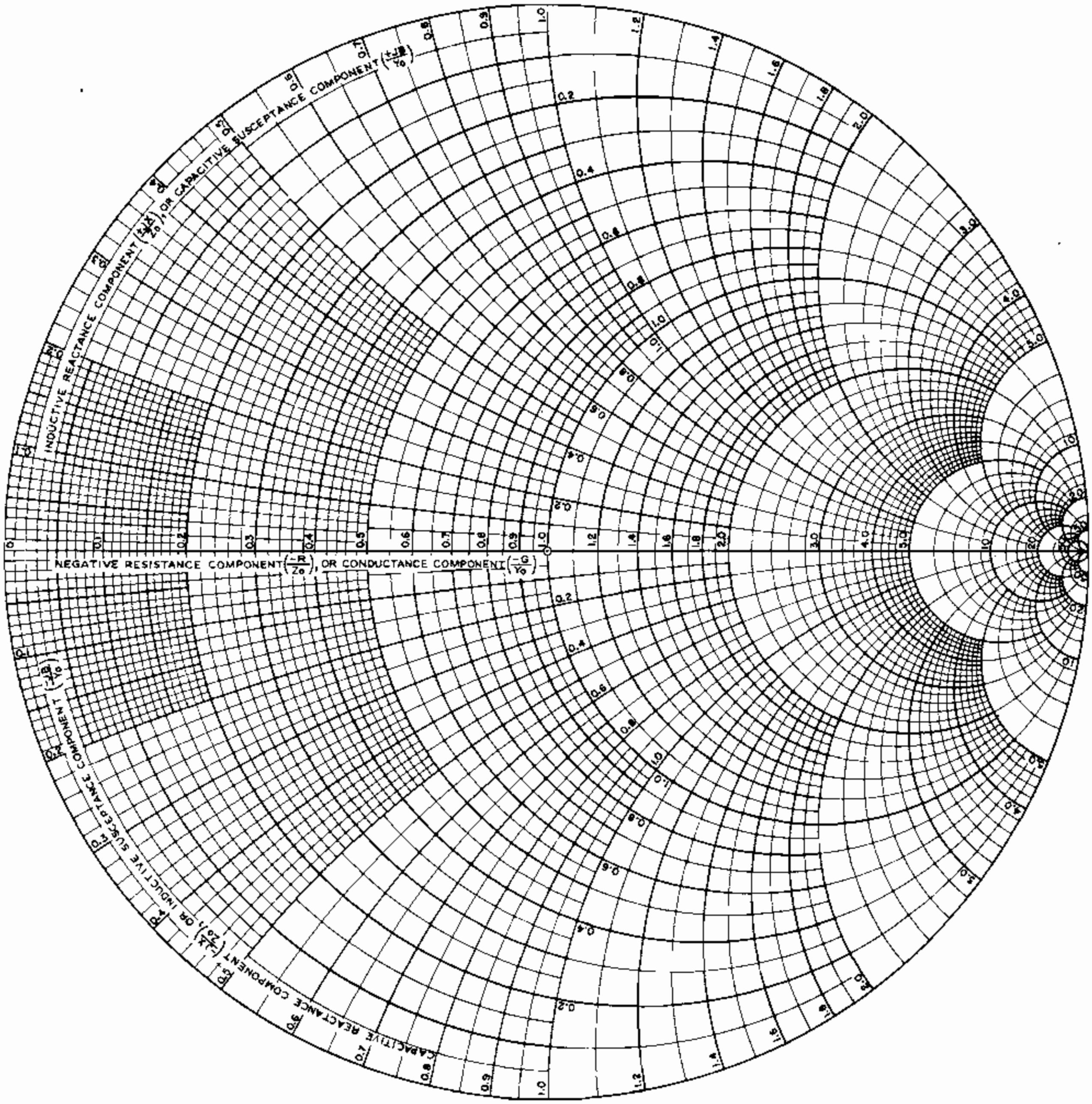


Fig. 12.3. SMITH CHART coordinates displaying rectangular component of equivalent series-circuit impedance (or of parallel-circuit admittance) with negative real parts.

Fig. 12.7 in relation to Fig. 12.3, it represents the voltage transmission coefficient on the negative resistance coordinates or the current transmission coefficient on the negative conductance coordinates. When rotated 180° from this orientation, the overlay of Fig. 12.7 represents the voltage transmission coefficient on the negative conductance components or the current transmission coefficient on the negative resistance coordinates of this same chart. The 180° phase contour in Fig. 12.7 should always be aligned with the corresponding standing wave minima loci on the negative impedance or admittance coordinates.

From the transmission coefficient overlay of Fig. 12.7 the shape and relative amplitudes of standing waves vs. SWR may be plotted *for a constant incident wave amplitude.* This is accomplished by plotting the intercepts of any desired standing wave circle with those circles in Fig. 12.7 which indicate the magnitude of the voltage or current transmission coefficient. The construction of standing wave circles on SMITH CHART coordinates is described in Chap. 3. Standing waves of three different amplitude ratios are plotted in Fig. 12.8. Note that in this case, where the real part of the waveguide termination is negative, the absolute value of voltage or current along the waveguide can approach infinity, as compared to a limiting value of twice the incident wave amplitude for the case where the real part of the termination is positive (compare Fig. 12.7 with Fig. 1.3).

The SWR is unity in the limiting case where the transmission coefficient is infinity.

Since, as previously stated, power has no "phase," the power transmission coefficient is the scalar ratio of the transmitted (resultant of incident and reflected) to the incident power; this is constant for all positions along a lossless waveguide regardless of whether the waveguide termination has positive or negative real components.

12.6 RADIAL SCALES FOR NEGATIVE SMITH CHART

A set of radial scales for negative SMITH CHART coordinates* is shown in Fig. 12.4. The overall length of these scales corresponds to the radius of the negative resistance coordinates in Fig. 12.3 and to the reflection and transmission coefficient overlays in Figs. 12.6 and 12.7, respectively. All but one of these radial scales, viz., the voltage (or current) reflection coefficient magnitude scale, are identical to those described in previous bulletins for use on positive SMITH CHART coordinates. (Since the reflection loss scale has no physical significance it is omitted from the radial scales in Fig. 12.4.)

Although the radial scales are the same, an interpretation of their special significance and use in connection with negative SMITH CHART coordinates may be helpful. This will be discussed in the following paragraphs.

12.6.1 Reflection Coefficient Magnitude

As previously shown, when used with negative resistance (or conductance) loads the voltage or current reflection coefficient at any point on SMITH CHART coordinates is the reciprocal of the corresponding value of voltage or current reflection coefficient when the load resistance (or conductance) is positive; thus, all points along this scale are greater than unity. A voltage or current reflection coefficient radial scale suitable for use with negative SMITH CHART coordinates is shown on the upper right-hand scale of Fig. 12.4. This scale is used with negative SMITH CHART coordinates in the same way that the voltage-current reflection coefficient

*The term "negative SMITH CHART coordinates" will henceforth be used to designate normalized impedance or normalized admittance coordinates of a SMITH CHART representing negative real components.

magnitude scale of Fig. 3.4 is used with positive coordinates.

A particular significance of the radial voltage-current reflection coefficient scale on negative SMITH CHART coordinates is that it provides a measure of the voltage gain of any negative resistance reflection amplifier. This is described in the last part of this chapter using a SMITH CHART impedance or admittance representation of the amplifier's input characteristics.

12.6.2 Power Reflection Coefficient

As on a conventional positive SMITH CHART, the radial power reflection coefficient scale indicates the ratio of reflected to incident power. Its value at any point on the negative SMITH CHART is, however, greater than unity. Values along this scale equal the square of the voltage-current reflection coefficient magnitude at the same point.

The power reflection coefficient scale is shown in Fig. 12.4 adjacent to the scale for voltage-current reflection coefficient magnitude described above. Unlike the voltage-current reflection coefficient the power reflection coefficient has amplitude only. Both power and voltage-current reflection coefficients are constant throughout a uniform lossless waveguide, but in a waveguide with attenuation they both increase with distance toward the load from a minimum value at the generator end.

12.6.3 Return Gain, dB

A waveguide with a negative resistance and/or negative conductance termination has more power reflected toward the generator than is incident on the load. Consequently the *return loss* is negative, i.e., the waveguide termination results in a *return gain*. The scalar values as obtained from the return loss scale for positive resistance or conductance terminations (see Chap. 4) are unchanged. However, with negative resistance or conductance terminations these values must be interpreted to be the gain (in dB) afforded by the termination. It will be noted that this scale is also the power reflection coefficient in dB.

The return gain scale is shown in Fig. 12.4 directly below the scale for power reflection coefficient.

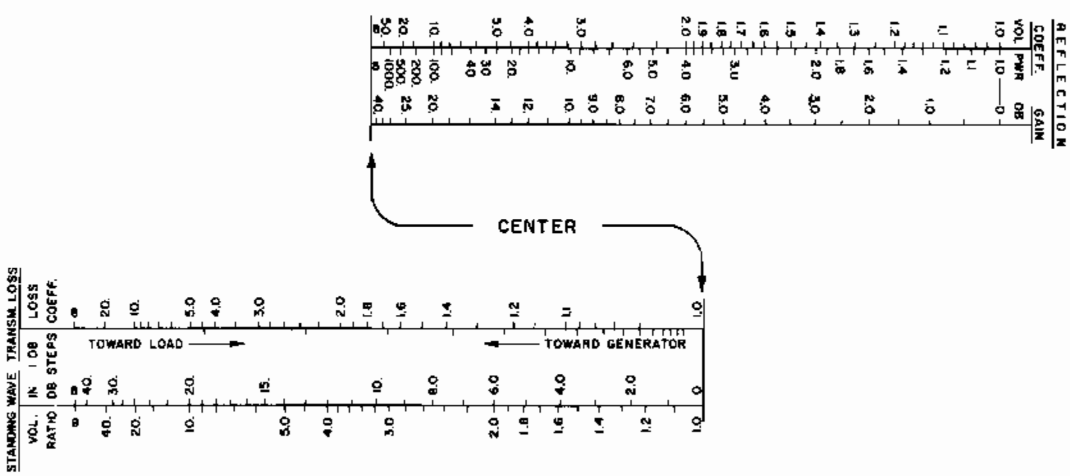


Fig. 12.4. **Radial scales for negative SMITH CHART coordinates of Fig. 12.3.**

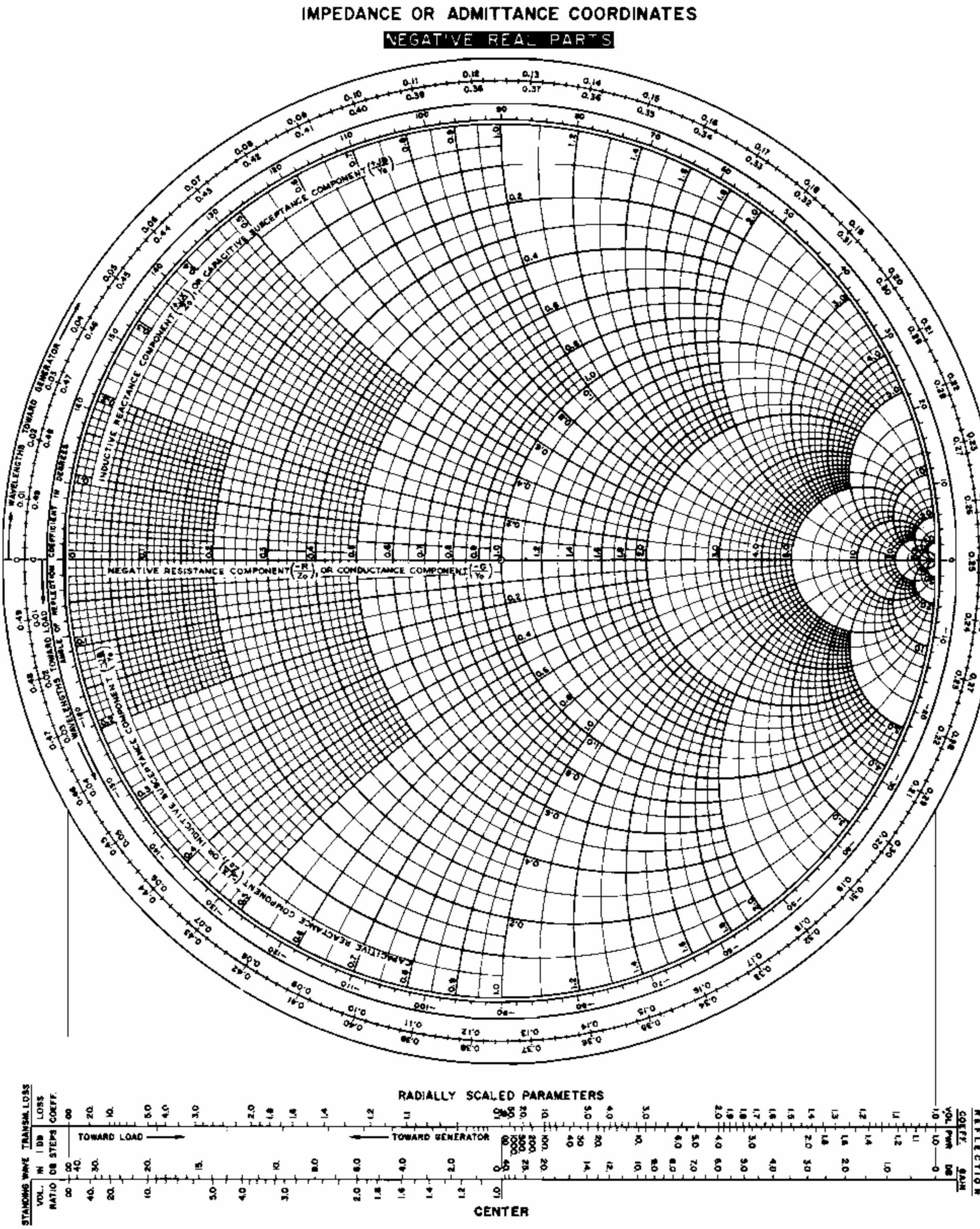


Fig. 12.5. **SMITH CHART with coordinates of Fig. 12.3 (see Chart D in cover envelope).**

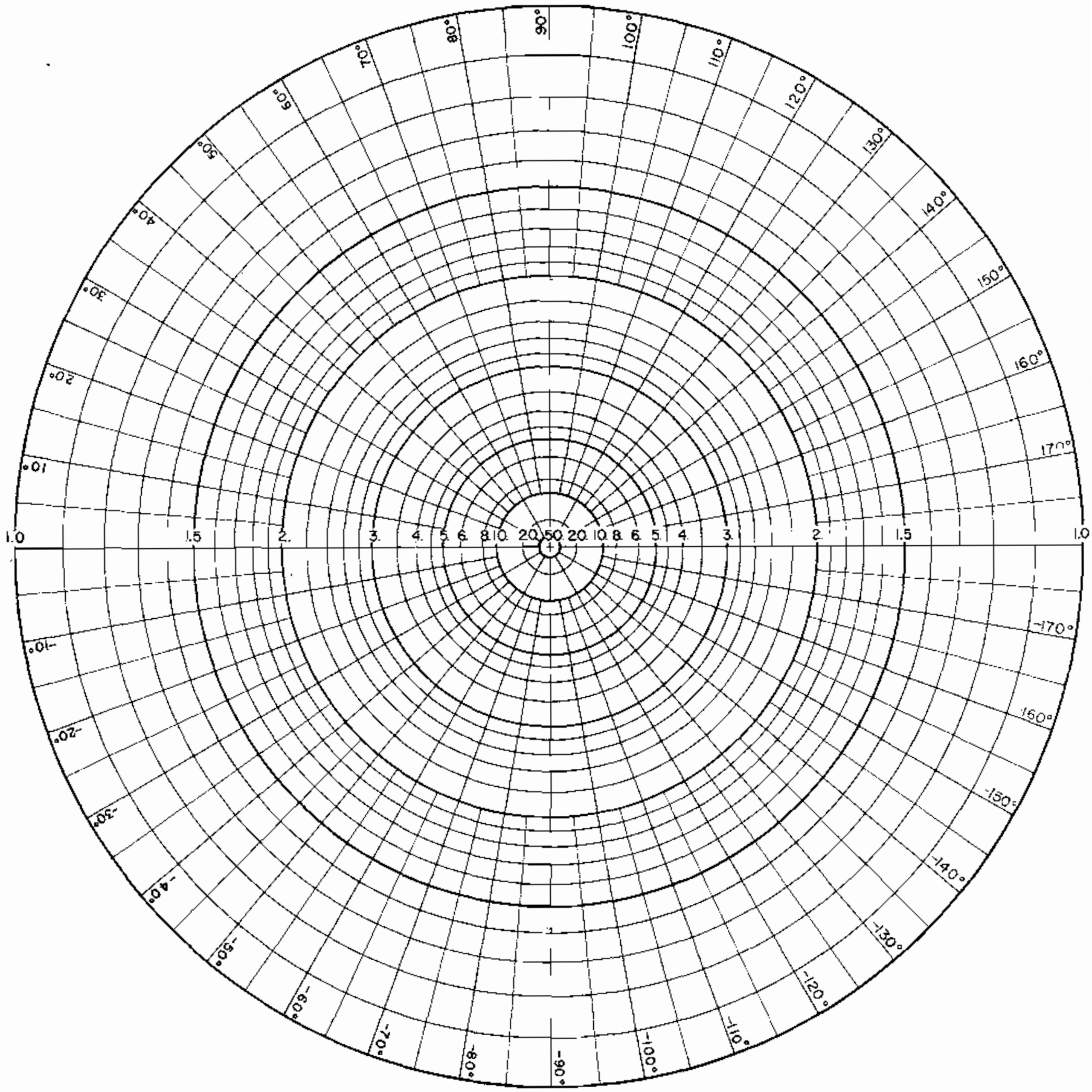


Fig. 12.6. **Complex voltage or current reflection coefficient (overlay for negative SMITH CHART D in cover envelope).**

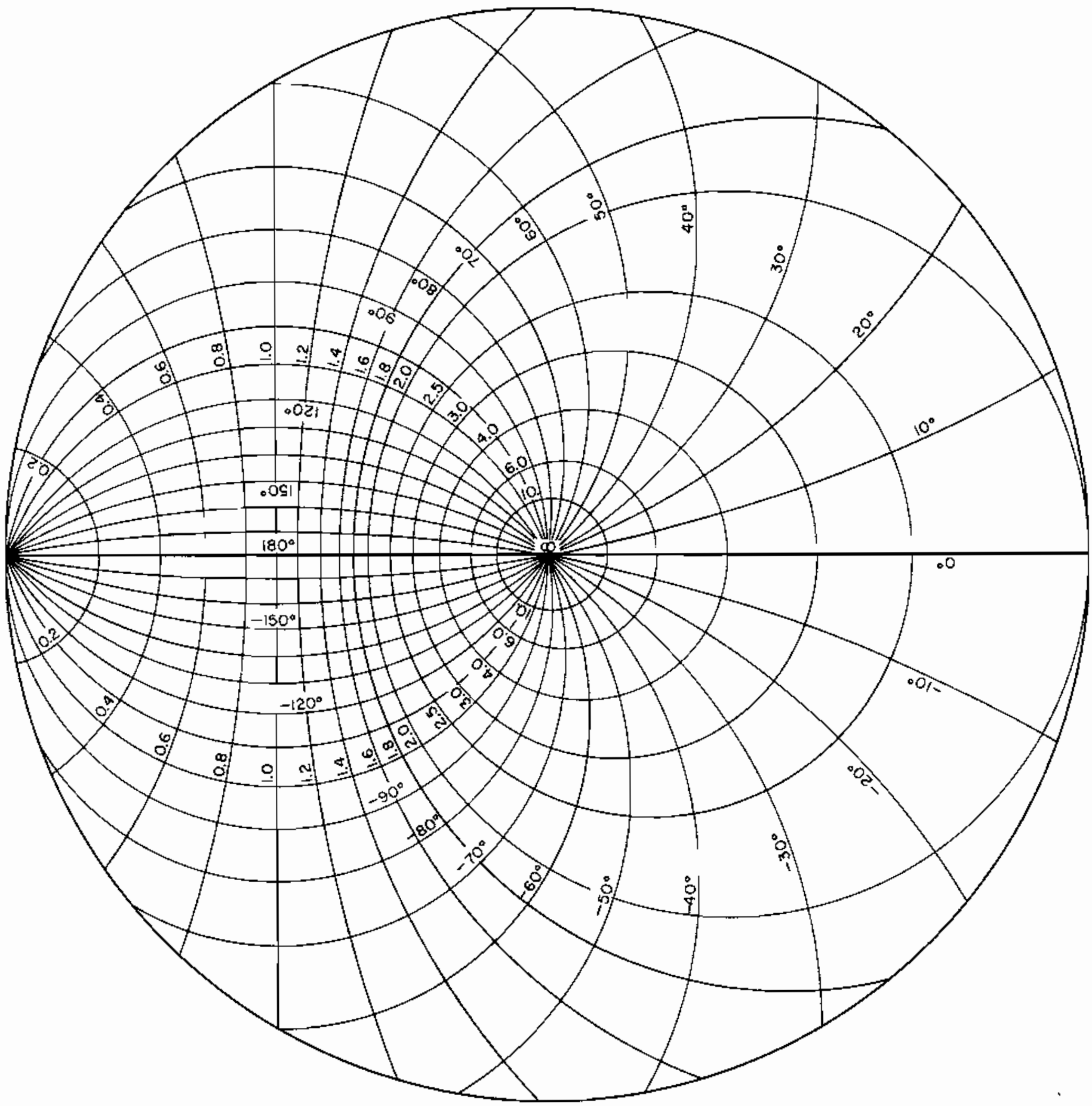


Fig. 12.7. Complex voltage or current transmission coefficient (overlay for SMITH CHART D in cover envelope).

12.6.4 Standing Wave Ratio

The radial scales expressing voltage or current standing wave ratio (both as a ratio and in dB), which are discussed in Chap. 3, are not altered by the fact that the load resistance or conductance is negative. Slotted line measuring techniques may be applied in the usual way to waveguides terminated in negative resistance or negative conductance loads. The shapes of voltage or current standing waves of several amplitude ratios are shown in Fig. 12.8. Their shape (as well as their position along the waveguide) is independent of the sign of the real component of the waveguide input impedance or admittance. The standing wave ratio scale is the lowest of the four scales on the left side of Fig. 12.4.

12.6.5 Standing Wave Ratio, dB

This scale, like the standing wave ratio scale, is unaltered by the sign of the real part of the termination. However, as discussed in Chap. 3, the use of the decibel (dB) to express standing wave ratio is not in accordance with the true meaning of this term. The standing wave ratio scale in dB is shown on the left side of Fig. 12.4 adjacent to the standing wave ratio scale.

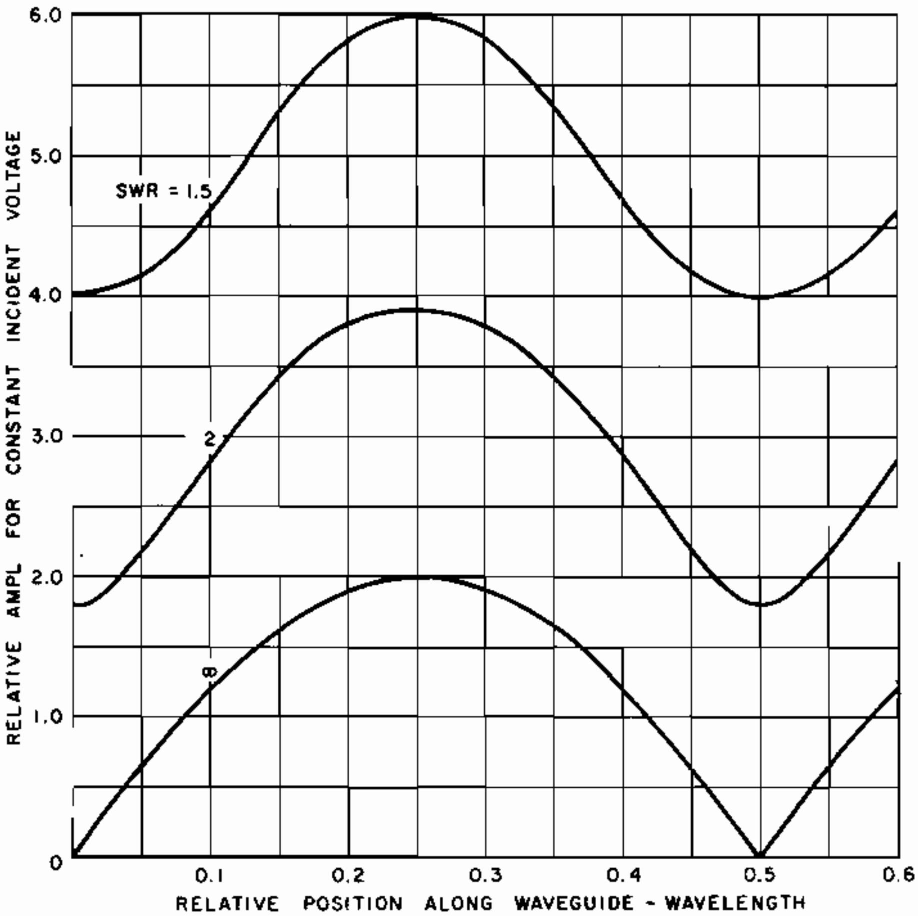


Fig. 12.8. Relative amplitudes and shapes of voltage or current standing waves along a lossless waveguide with negative resistance (or negative conductance) termination (constant input voltage).

12.6.6 Transmission Loss, 1-dB Steps

The effects of dissipative losses along a waveguide on input impedance or input admittance, when the load resistance or conductance is negative, is opposite to the usual effect encountered with loads having positive real parts. With negative resistance or negative conductance terminations, attenuation in a waveguide results in an outward spiral path from the SMITH CHART center when progressing along a waveguide toward the generator, and an inward spiral path toward the chart center when progressing along the waveguide toward the load. The one-way transmission loss scale divisions are unaltered, but the designated directions of movement along this radial scale (shown on the left side of Fig. 12.4 adjacent to the transmission loss coefficient scale) are reversed from those shown in Fig. 4.1.

12.6.7 Transmission Loss Coefficient

This scale shows the percentage increase in dissipative losses in a waveguide due to the presence of standing waves as compared to the losses in the same waveguide when transmitting the same power to a load without standing waves. It is the ratio of mismatched losses to matched losses. The transmission loss coefficient scale (shown on the left side of Fig. 12.4 adjacent to the transmission loss scale) is unaltered by the sign of the real part of the termination.

12.7 NEGATIVE SMITH CHART COORDINATES, EXAMPLE OF THEIR USE

The representation of the electrical characteristics of a device with a negative resistance (or conductance) component on a negative SMITH CHART can perhaps best be described with a practical example [13]. A tunnel diode reflection amplifier, which makes use of the increase in reflected power in an associated waveguide, will be selected for this purpose.

The dc voltage-current characteristics of a typical tunnel diode, as shown in Fig. 12-1(a), were discussed earlier in this chapter. Its negative resistance region is shown in Fig. 12.1(b). The diode is nominally biased to operate in the linear portion of its negative resistance region, near point P. Since a tunnel diode reflection amplifier depends upon negative resistance to provide amplification, only the voltage interval for which the resistance is negative is of significance.

12.7.1 Reflection Amplifier Circuit

A basic waveguide circuit for a tunnel diode reflection amplifier is shown in Fig. 12.9. An ac signal source, whose internal impedance is matched to the characteristic impedance of the waveguide to which it is connected, is indicated thereon. For this discussion it will be assumed that the internal impedance of this source is constant over the frequency band in which the amplifier is operable. In practice this is not entirely true and the extent to which the above

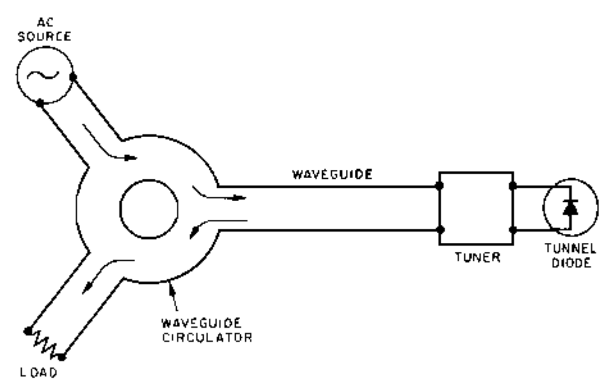


Fig. 12.9. Basic waveguide circuit for a tunnel diode reflection amplifier.

assumptions depart from true conditions will affect the analysis.

The input power from the source to the waveguide flows along the waveguide, through the circulator in the path indicated, through the tuner, and into the tunnel diode, whose bias is adjusted to provide a negative resistance. From the diode it is reflected back through the tuner into the waveguide toward the source, around another section of the circulator, and into the load resistance. The load is effectively isolated from the source by the circulator if, as will also be assumed for this discussion, its impedance matches that of the waveguide characteristic impedance over the operable band of the amplifier, and if the circulator functions without reflection over this same band.

The reflected voltage traveling wave which returns along the waveguide from the diode to the load resistance is greater than the incident wave on the diode from the source, since the diode is biased to operate in its negative resistance region. Thus, the voltage reflection coefficient along the waveguide between the circulator and the diode is greater than unity. The magnitude of the voltage reflection coefficient along this waveguide is a measure of the voltage gain of the amplifier. The usual standing wave measuring technique, wherein the standing wave ratio and wave position are plotted on a SMITH CHART, is applicable for determining the negative input impedances vs. frequency locus and the related reflection coefficient (amplifier gain). In addition to the gain, the same data yields information concerning the stability of the amplifier, and the design of a tuner which is necessary to peak the gain of the amplifier at a selected frequency, as will be seen.

12.7.2 Representation of Tunnel Diode Equivalent Circuit on Negative SMITH CHART

The first step in the analysis of the reflection amplifier characteristics is to represent the tunnel diode by its high-frequency small-signal equivalent circuit whose element values are normalized to the positive characteristic impedance (or characteristic admittance where more convenient) of the waveguide with which it is used. This representation, with typical values for the equivalent circuit elements, is shown in Fig. 12.10. As with any high-frequency negative-resistance device, there is an unavoidable shunt capacitance C and series inductance L. These reactances will, of course, be frequency dependent, which effect must be taken into account in the analysis. The small inherent capacitance

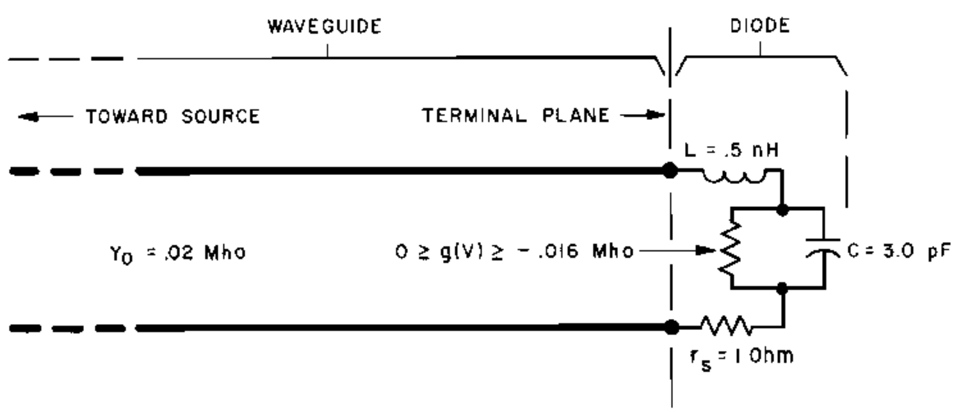


Fig. 12.10. Small-signal equivalent circuit of a tunnel diode.

C of the diode varies slightly with bias; however, this variation may generally be ignored. The diode conductance $g(V)$ is a function of its bias voltage V. The characteristic admittance Y_0 of the waveguide (coaxial transmission line in this case) is assumed to be 0.02 mho. Let it be required to plot the above diode admittance characteristics on a SMITH CHART, at a frequency of 2.0 GHz.

The normalized negative conductance of the diode is $-0.16/0.20 = -0.8$ mho. This is plotted at point A on Fig. 12.11.

The normalized shunt capacitive susceptance $+jB_c/Y_0$ at 2.0 GHz is $(2\pi fC)/Y_0 = +j\,1.9$ mho. The normalized admittance of this parallel combination is $-0.8 + j1.9$ mho, which value is located at point B. The effect of adding 1 ohm of positive series resistance r_s is next represented; however, before this is done point B, which is on negative conductance coordinates, is transferred to equivalent negative resistance coordinates at point C. Since the normalized series resistance $r_s/Z_0 = 0.02$ ohm is positive, the effect of adding this to the negative resistance at point C is to move it to point D (a less negative point on the negative resistance coordinates). The addition of normalized series inductance of 0.5 nH, whose inductive reactance $+jX_L/Z_0$ at 2.0 GHz is $(2\pi fL)/Z_0 = 1.26$ ohm, to the impedance at point D is to move it to point E, which is the desired normalized impedance of the diode at its input terminals. The equivalent normalized input admittance $Y_i/Y_0 = -1.25 + j\,2.5$ is located at point F.

12.7.3 Representation of Operating Parameters of Tunnel Diode

The effect on the diode's input admittance of changing two operating parameters, namely, the frequency and the negative conductance (bias adjustment), can be determined by standing wave measuring techniques, and the results can be represented on the negative SMITH CHART by two intersecting families of curves, shown for a typical untuned diode reflection amplifier on Fig. 12.12. The dashed curves represent loci of input admittances resulting from changes in diode negative conductance at various operating frequencies. The solid curves represent loci of input admittances resulting from changes in operating frequency at various negative conductance values established by the bias voltage. All points within the shaded area occupied by these two families of curves represent a specific combination of operating frequency and negative conductance. Point F is transferred directly from Fig. 12.11 to Fig. 12.12. Point g_0 is the diode's negative conductance at zero operating frequency (point A on Fig. 12.11). Point g_r is the conductance at the self-resonant frequency, at the diode terminals. Point g_{co} is the diode admittance at the resistance cutoff frequency. This is the frequency above which any negative-resistance device cannot amplify (or oscillate) because its total positive resistance equals its negative resistance, and consequently the real part of its input impedance equals zero.

From Fig. 12.12 it is possible to observe several significant facts concerning the stability and gain of this reflection amplifier. As previously discussed, the center point of the negative SMITH CHART coordinates is the point where the reflection coefficient is infinity. An infinite reflection coefficient corresponds to infinite gain, and infinite gain in a reflection amplifier is the criterion for oscillation. If it is not possible, by means of adjustment of the operating frequency or the conductance, to include the center point of the negative SMITH CHART coordinates (infinite reflection coefficient point) within the shaded operating area of Fig. 12.12, the amplifier will be stable. If the amplifier is

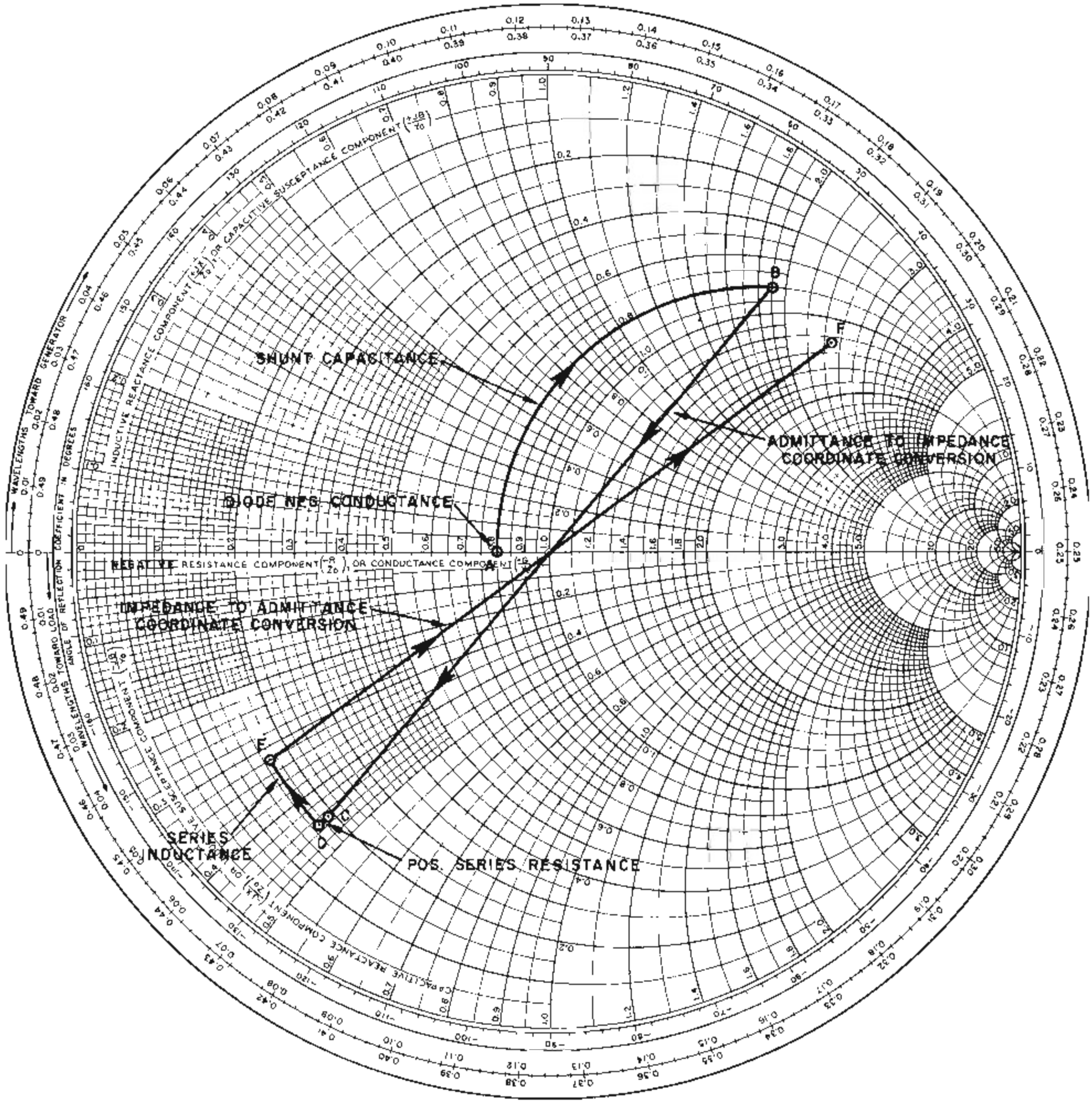


Fig. 12.11. Representation of circuit of Fig. 12.10 on negative SMITH CHART at $F = 2.0$ GHz.

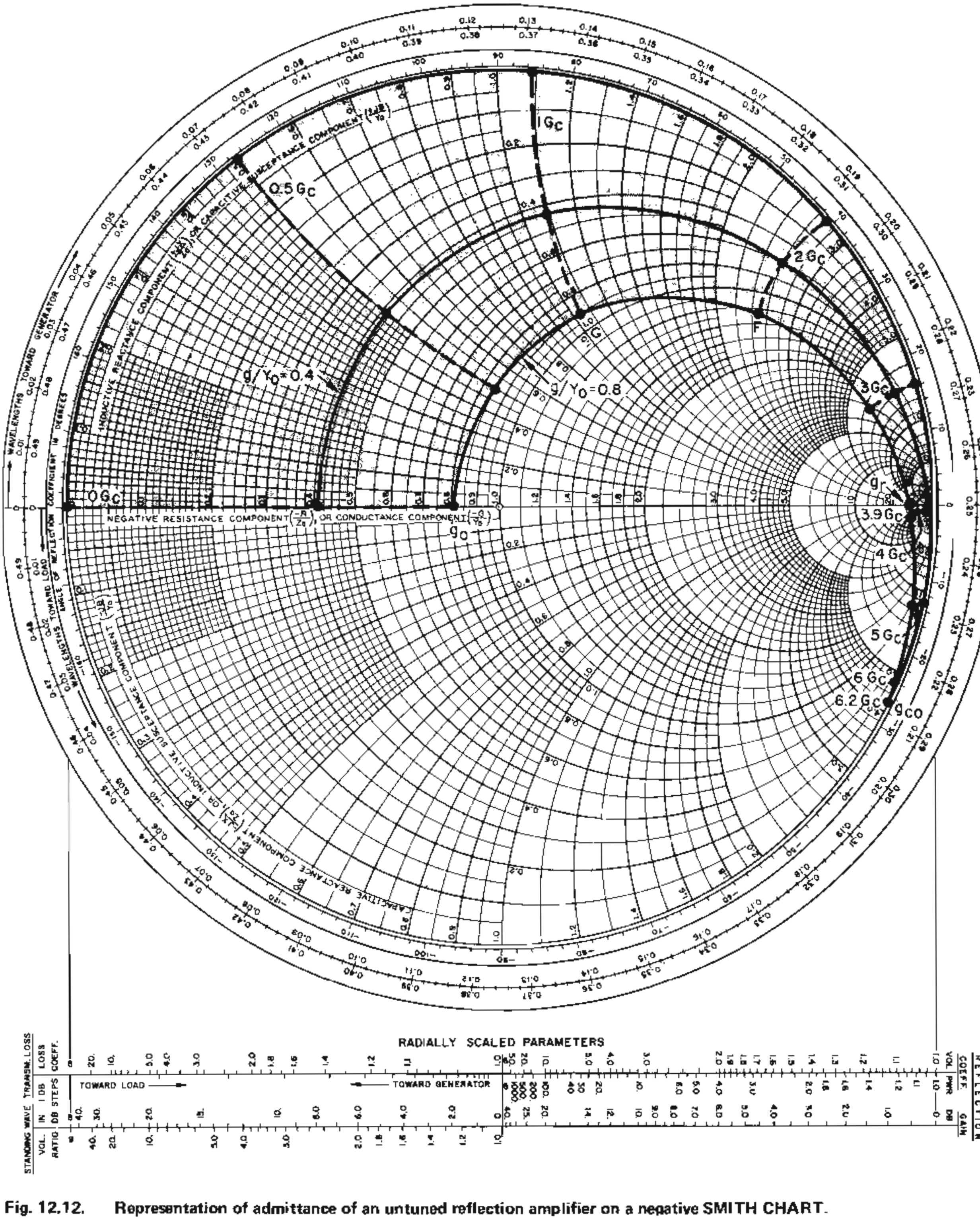


Fig. 12.12. Representation of admittance of an untuned reflection amplifier on a negative SMITH CHART.

operated at a low frequency and the bias is adjusted so that the diode admittance is essentially equivalent to a normalized negative conductance of 0.8 mho (which condition prevails at point g_0 on Fig. 12.12) the amplifier gain, as measured on the radial power reflection coefficient scale, is a maximum and, in this example, is 19.7 dB. At all other operating points (specific combination of frequency and bias settings) within the shaded area the gain is less than 19.7 dB. At point G, for example, where the frequency is 1 GHz the gain is 6.4 dB.

12.7.4 Shunt-tuned Reflection Amplifier

If it is required to peak the above amplifier gain at a particular frequency such as 1.0 GHz, this may be accomplished by employing a transforming circuit which causes point G on Fig. 12.12 to move closest to the center of the negative SMITH CHART. By adding a shunt inductance across the diode's equivalent circuit of Fig. 12.10, for example, point G may be shifted downward along a line of constant conductance, to the zero susceptance axis of the chart. The required value of normalized inductive susceptance is seen from the chart to be $-j$ 1.06 mho. The inductance corresponding to this susceptance at 1 GHz is readily calculable from the simple relationship $L = 1/j\omega BY_0$ to be 7.5 nH. All points on the plot of Fig. 12.12 are similarly shifted by the effect of this shunt inductance to their respective positions indicated on Fig. 12.13. The admittance at point G on Fig. 12.12 is thus transformed to a normalized negative conductance of −0.92 mho on Fig. 12.13. From the power reflection coefficient scale, the amplifier gain at 1.0 GHz is found to be 27 dB. It is peaked at this frequency since no other point within the shaded area of Fig. 12.13 is closer to the center of the chart.

Since the reflection gain is less than infinity, the amplifier will not oscillate at 1.0 GHz with any other bias adjustment. However, in the frequency range between 1.2 GHz and 5.0 GHz, as seen from Fig. 12.12, the normalized negative conductance component of the diode admittance is greater than minus one. If, for example, the diode's admittance was tuned to zero susceptance with a shunt inductance in this frequency range the resultant conductance would place the center of the SMITH CHART coordinates inside an area where the diode is operable. Thus, over a limited range of bias settings in this frequency range the diode reflection amplifier would oscillate.

There are, of course, other more complicated circuits, including distributed circuits composed of waveguide sections, which will permit stable operation in the higher frequency range over which the diode exhibits negative resistance or negative conductance characteristics. In one such circuit a shunt capacitance is located at a position along the waveguide toward the source from the diode's terminals, where the conductance is of the proper value to provide the desired amplifier gain (by control of reflection coefficient magnitude). The value of capacitance is adjusted to tune out the inductive input susceptance of the waveguide at the chosen frequency, and to thereby peak the amplifier gain at this frequency. Thus, this circuit allows some independent control of gain and operating frequency.

12.8 NEGATIVE SMITH CHART

The general-purpose negative SMITH CHART shown in Fig. 12.5 is reproduced as the fourth of four translucent plastic charts whose function is described in the Preface and which is contained with the other three in an envelope in the back cover of this book (Chart D).

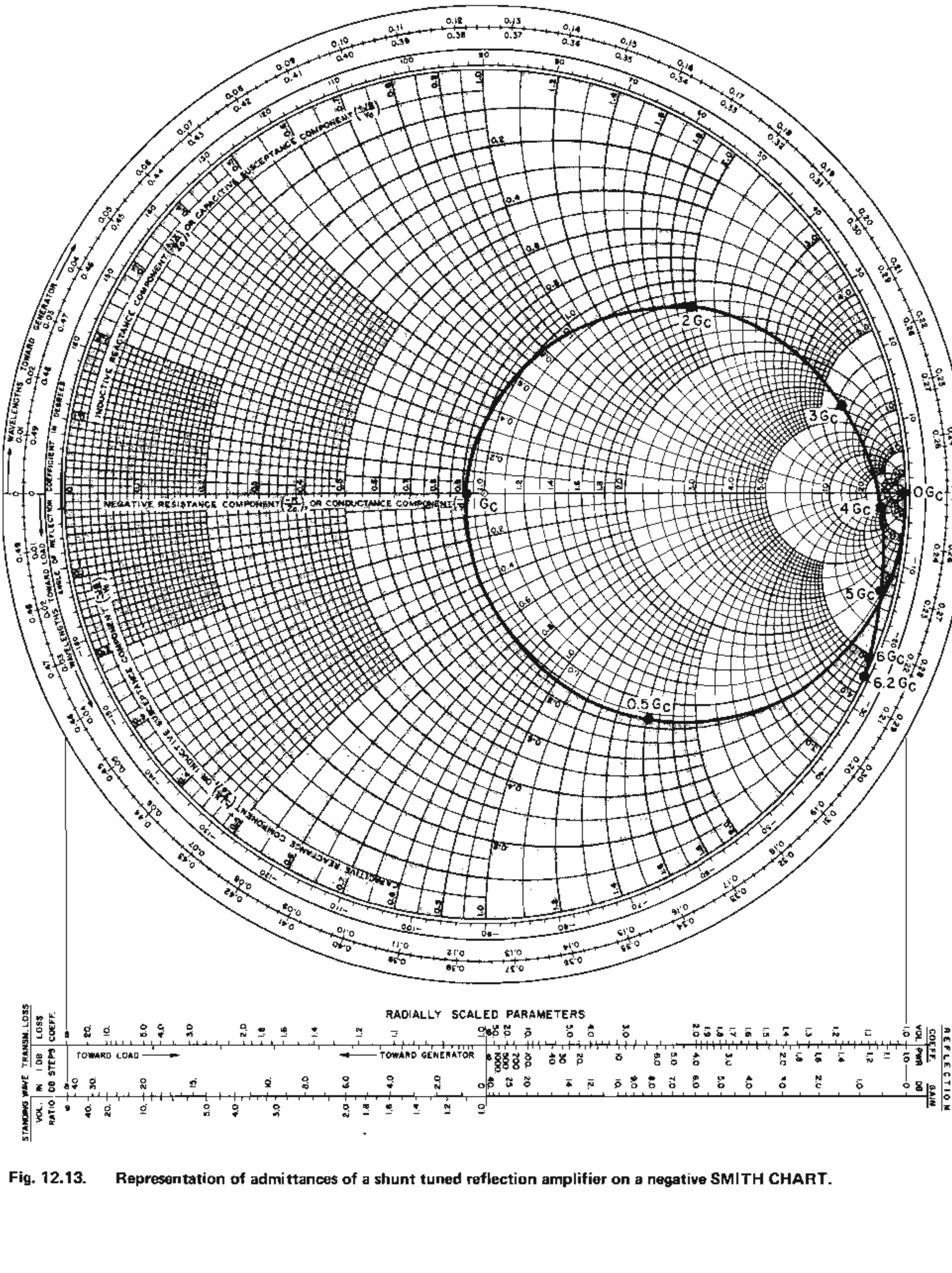


Fig. 12.13. Representation of admittances of a shunt tuned reflection amplifier on a negative SMITH CHART.

CHAPTER 13

Special Uses of Smith Charts

13.1 USE CATEGORIES

The many uses of the SMITH CHART may conveniently be classified as "basic," "specific," and "special." The more common uses in the first two categories, in which procedures for solving problems will be evident from the material which has been presented in previous chapters, are listed below. These lists are followed by descriptions of some of the more important "special" uses of this chart.

13.1.1 Basic Uses

1. For evaluating the rectangular components, or the magnitude and angle of the input impedance or admittance, voltage, current, and related transmission-reflection functions, at all positions along a uniform waveguide. These related transmission-reflection functions include:
   a. Complex voltage and current reflection coefficients.
   b. Complex voltage and current transmission coefficients.
   c. Power reflection and transmission coefficients.
   d. Reflection loss.
   e. Return loss.
   f. Standing wave loss factor.
   g. Amplitude of maximum and minimum of voltage and current standing wave, and standing wave ratio.
   h. Shape, position, and phase distribution along voltage and current standing waves.
2. For evaluating the effects of waveguide attenuation on each of the above parameters and on related transmission-reflection functions at all positions along a waveguide.
3. For evaluating input-output transfer functions.

13.1.2 Specific Uses

1. For evaluating the input susceptance or reactance of open- or short-circuited waveguide stubs.
2. For evaluating the effects of shunt susceptances or conductances, or series reactances

or resistances, on the input admittance or impedance, respectively, of waveguides.

3. For displaying or evaluating the input impedance or input admittance characteristics of resonant or antiresonant waveguide stubs, including bandwidth and Q, or inversely, for determining bandwidth or Q of waveguide resonators.

4. For design of impedance matching circuits employing single or multiple open- or short-circuited stubs.

5. For design of impedance matching circuit employing single or multiple slugs.

6. For design of impedance matching circuits employing single or multiple quarter-wave line sections.

7. For displaying loci of passive lumped circuit impedance or admittance variations attending changes in the circuit constants.

8. For displaying loci of waveguide input impedance or admittance variations attending changes in operating parameters of active terminating circuits.

9. For converting impedances to equivalent admittances.

10. For converting series to equivalent parallel-circuit representations of impedance or admittance.

11. For converting complex numbers to their equivalent polar form.

12. For converting the series circuit representation of impedance to its equivalent parallel circuit and the parallel circuit representation of admittance to its equivalent series circuit.

13. For obtaining the reciprocal of a complex number of the geometric mean between two complex numbers.

13.2 NETWORK APPLICATIONS

It is generally known that the image impedance operation of any four-terminal passive symmetrical network can be related to the behavior of a uniform transmission line insofar as the terminals are concerned [10]. The SMITH CHART of Fig. 8.6 is well suited to determining the input impedance characteristics of any four-terminal symmetrical passive network, filter, attenuator, etc.

The three constants which completely determine the operation of any such network are (1) the *image impedances* Z_{11} and Z_{12} at each end and (2) the *image transfer constant* θ. The image impedances of a symmetrical network may be thought of as corresponding to the characteristic impedance of a uniform line. When the network is not symmetrical, it has a different image impedance as viewed from each end. In this case the equivalent line is unsymmetrical and has a transforming action upon the load impedances. The image transfer constant, which is the same in either direction through the network, is analogous to the hyperbolic angle of the equivalent transmission line, the real part corresponding to the attenuation constant and the imaginary part to the phase constant of the transmission line.

All three of these parameters can be evaluated from the open- and short-circuited impedance of the network according to the following relations:

$$Z_{11} = (Z_{oc} Z_{sc})^{1/2} \tag{13-1}$$

$$Z_{12} = (Z'_{oc} Z'_{sc})^{1/2} \tag{13-2}$$

and

$$\theta = \tanh^{-1}\left(\frac{Z_{sc}}{Z_{oc}}\right)^{1/2} \tag{13-3}$$

where Z_{oc} and Z_{sc} are the impedances at the input terminals of the network with the output terminals open- and short-circuited, respectively, and Z'_{oc} and Z'_{sc} are the impedances at the output terminals with the input terminals open- and short-circuited, respectively.

To obtain the input impedance, for example, of a four-terminal symmetrical passive network using SMITH CHART A, proceed as follows:

1. Normalize the load impedance with respect to the image impedance at the load terminals of the network Z_{12} and enter the chart at this point.
2. Move radially toward the center of the chart an amount corresponding to the attenuation constant of the network in dB. (Use radial "Atten. 1 dB Maj. Div." scale at upper right, transversing the proper number of dB steps.)
3. Move clockwise an amount equal to the phase angle of the image transfer constant using the outermost peripheral scale labeled "Wavelengths Toward Generator" (one degree equals 1/360 wavelength) and obtain the normalized input impedance of the network by multiplying the normalized input impedance by the image impedance at the input terminals Z_{11}.

13.3 DATA PLOTTING

Measured or computed data on waveguide components is frequently plotted directly on the coordinates of the SMITH CHART for the purpose of analysis and evaluation of the characteristics of the device. For example, the envelope of the plotted data may be important for evaluation of the overall capabilities of impedance matching devices such as stubs, slugs, etc. A single asymmetrically pointed tuning plug which is screwed into the broad wall of a uniconductor waveguide in an off-center position is one type of matching device which will serve to illustrate this use of the chart.

If an asymmetrically pointed plug is allowed to protrude into a match-terminated waveguide it will produce a reflection whose magnitude and phase will change in some systematic way as the plug is screwed progressively deeper into the waveguide. A plot of this changing impedance on a SMITH CHART will reveal a scanned pattern of the input impedance locus at some reference position along the waveguide.

In order to determine the envelope of impedances at a given frequency within which any load impedance on the waveguide may be matched to its characteristic impedance with the asymmetrical tuning plug the problem may be turned around, for purposes of gathering data, to one of determining what resultant impedances can be obtained at a fixed reference position along the waveguide as the plug is continuously screwed in. The "reference position" may be at the center of the plug or at half-wavelength intervals toward the generator therefrom. The trace on the SMITH CHART taken by the resultant impedance vector is plotted in Fig. 13.1 for ten turns of the asymmetrical tuning plug configuration illustrated therein.

It can then be assumed that input impedances which are capable of being matched to the characteristic impedance of the waveguide by this plug will have conjugate values to those plotted in Fig. 13.1. Thus, when the plug protrudes into the waveguide its capacitive susceptance cancels the inductive input susceptance of a conjugate admittance. In its most outward position the plug is seen to be equivalent to an inductive susceptance which accounts for part of the area scanned lying in this region of the chart.

The effective diameter of the plug in guide wavelengths will determine the angle of the sector of the impedance envelope on the SMITH CHART which is scanned. This is seen to be roughly 200° in Fig. 13.1 which corresponds to an effective plug diameter of

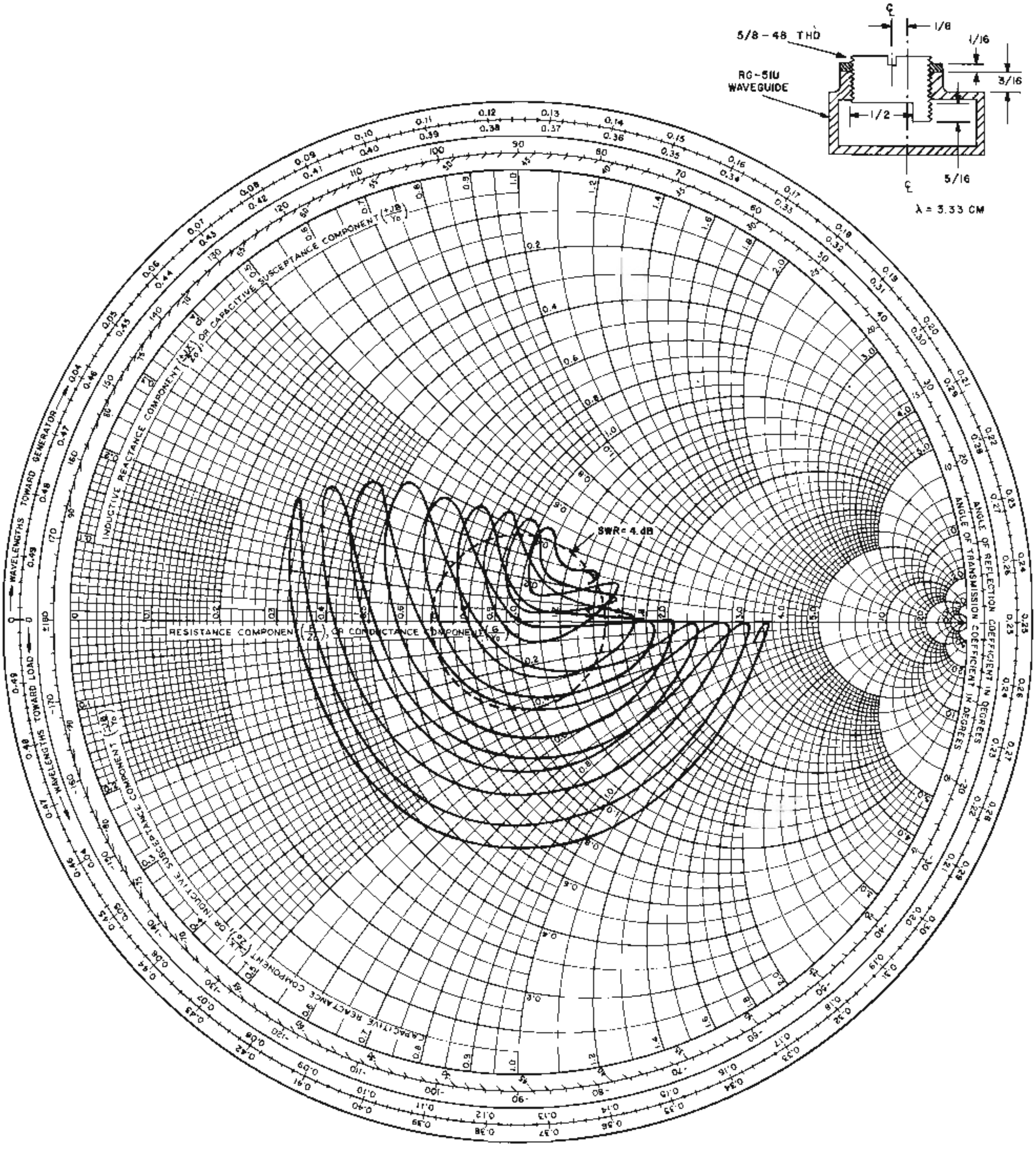


Fig. 13.1. Locus of impedances resulting from insertion of asymmetrical tuning screw into match-terminated waveguide, ten turns (conjugate impedances may be matched to Z_0).

200/360 times one-half wavelength, or approximately 0.28 wavelength. The largest circle which can be inscribed within the scanned impedance envelope, and which is centered on the SMITH CHART coordinates, in Fig. 13.1 corresponds to a standing wave ratio of 1.58 (4 dB), indicating that this tuning plug is capable of eliminating reflections of any phase which would produce standing waves between unity and this maximum value.

The plot in Fig. 13.1 also shows the effect of the pitch of the thread. A finer thread would cause the plug to advance more slowly into the waveguide as it is rotated, and will produce a larger number of scan lines within the same scanned area.

Thus, one example is seen of how the plotting of measured data reveals information concerning the design of a particular waveguide device; specifically, in the case illustrated, this includes the plug diameter, depth of penetration, and pitch of the threads.

13.4 RIEKE DIAGRAMS

One of the early uses of the SMITH CHART was for plotting constant-power and constant-frequency contours of magnetron oscillators used in World War II radar equipments. Subsequently, the chart has been widely used for displaying these same load properties of electron tube oscillators. The reason for using the SMITH CHART for this purpose is that the load characteristics of electron tube oscillators are in general a function of the complex load impedance, which is conveniently represented over its entire range of possible values by these coordinates.

A plot of the load characteristic of oscillators on a SMITH CHART is called a *Rieke diagram.* A typical Rieke diagram [24,33] consists of two families of curves, one representing contours of constant power and the other contours of constant frequency as shown in Fig. 13.2 for an *X*-band Klystron oscillator tube which operates in the frequency range from about 8,500 to 9,650 GHz. From a Rieke diagram one may select a load impedance which represents the best compromise between power output and frequency stability. For example, in Fig. 13.2 such a point is evidently near a normalized load impedance of $2 - j\,0.9$ ohms.

In general, on a Rieke diagram the constant power contours roughly parallel the contours of constant conductance on a SMITH CHART while the constant frequency contours roughly parallel constant susceptance contours. The degree to which they depart from this generalization is representative of the degree to which the equivalent circuit of the oscillator departs from a parallel circuit combination of G and jB, G being the parallel combination of the negative conductance of the oscillator, the positive conductance of the load, and the electronic conductance associated with energy conversion within the tube, the summation of which under stable operating conditions is zero. This summation is also assumed to be constant with frequency.

13.5 SCATTER PLOTS

Another application of the SMITH CHART which is useful for determining differences between intentionally alike circuit components is the *scatter plot.* Such a plot shows the effects of these differences on any or all of the parameters which the SMITH CHART interrelates. One example is a plot of the measured distribution of the input and output admittances of an *L*-band microwave transistor, as shown in Fig. 13.3. The area of the scatter plot can be made proportionately larger by showing only that portion of the chart which is of interest.

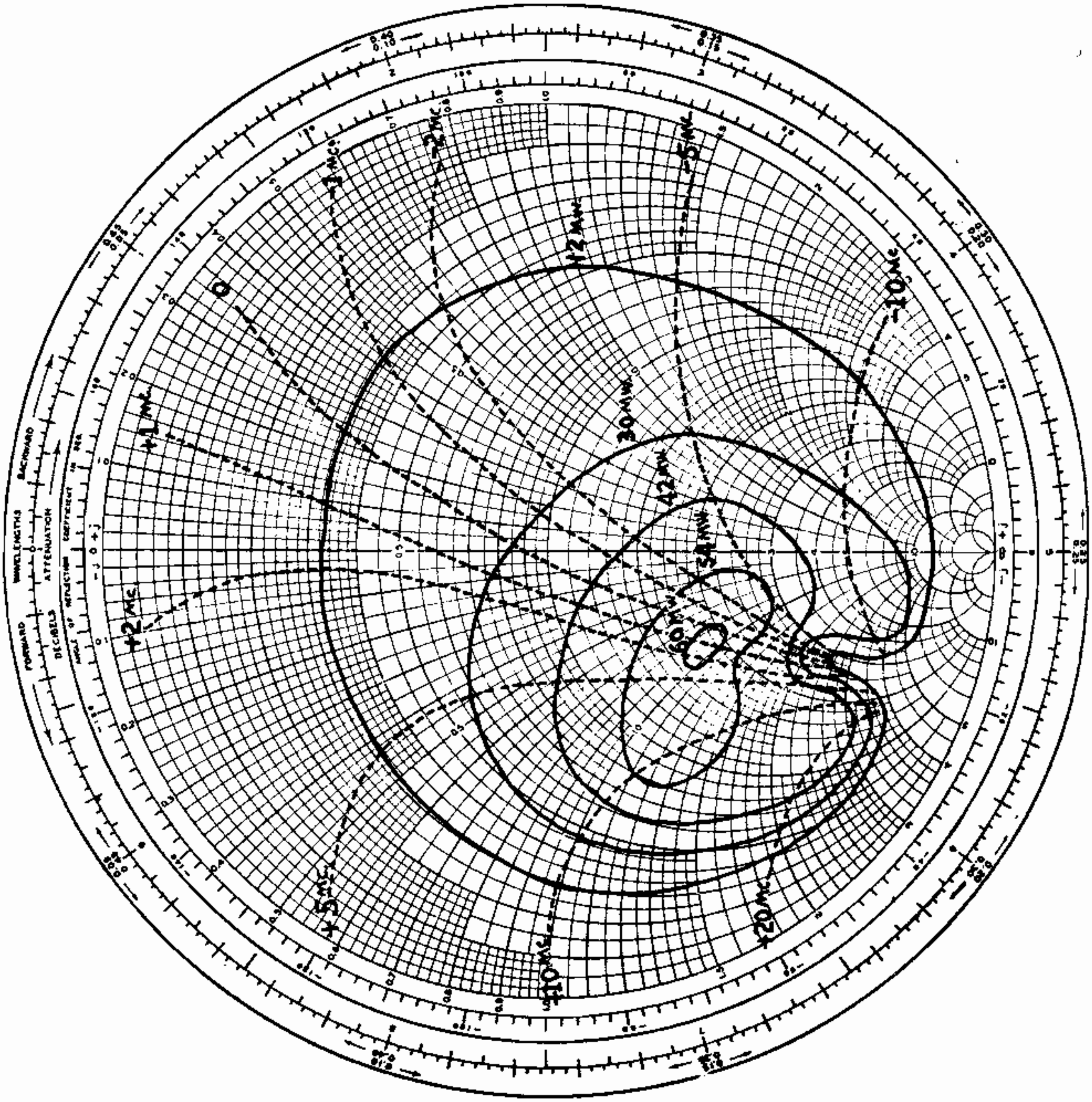


Fig. 13.2. Rieke diagram on a SMITH CHART for an UHF electron tube oscillator.

13.6 EQUALIZER CIRCUIT DESIGN

The transmission coefficient magnitude and angle scales for the SMITH CHART, as shown in Fig. 8.6, make it practical to design shunt-impedance or series-admittance equalizer circuits therefrom. A rotation of the transmission coefficient scales with respect to the chart coordinates (which can readily be accomplished by use of the transmission coefficient overlay of Fig. 5.4) extends the design possibilities of the SMITH CHART to include shunt-admittance or series-impedance equalizer circuit design. The method has been described in the

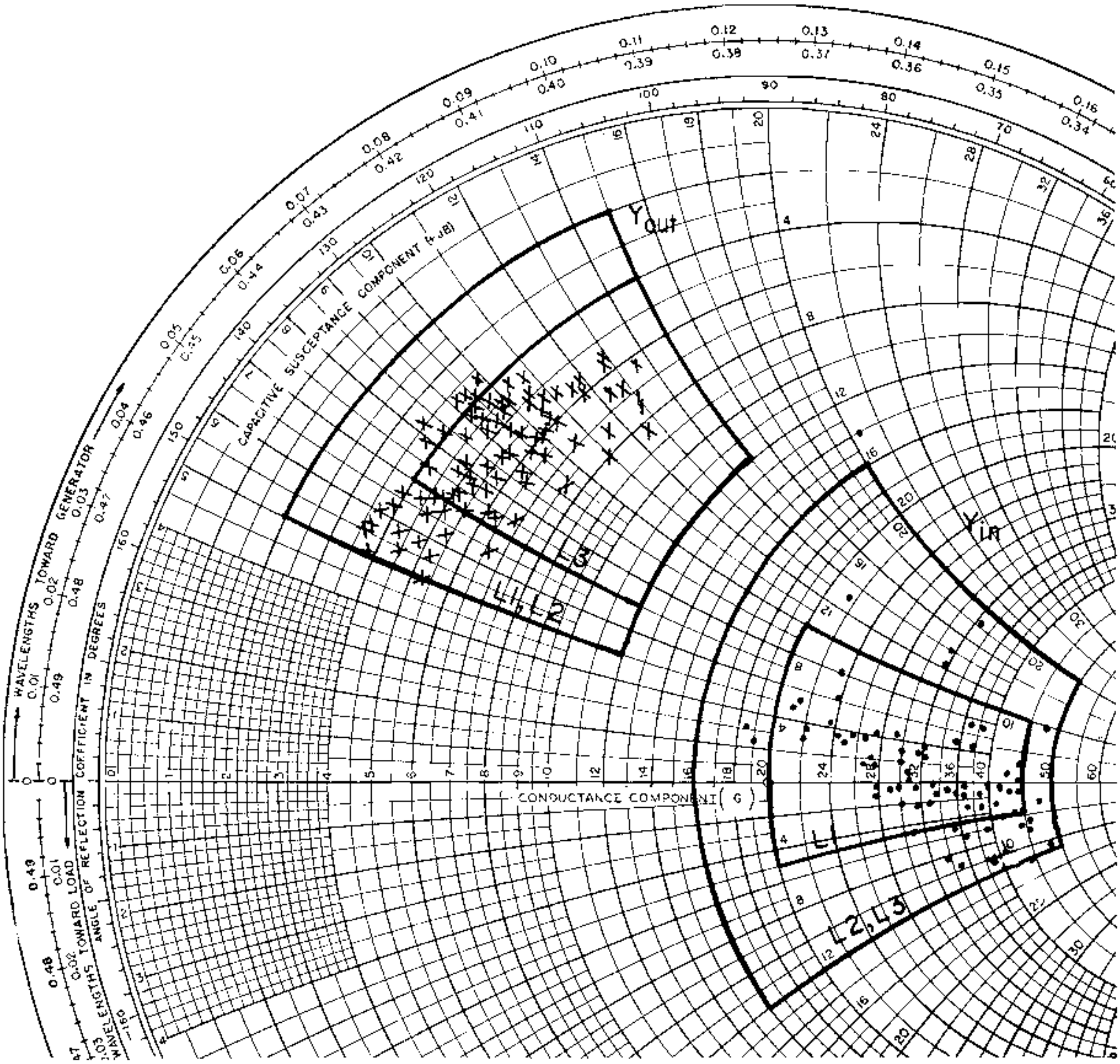


Fig. 13.3. Scatter plot on a SMITH CHART of input and output distribution of admittances for a microwave transistor (conductance coordinates, Y_0 = 20 $m\mho$).

literature [111]. It will suffice here to illustrate one such example of the use of the SMITH CHART.

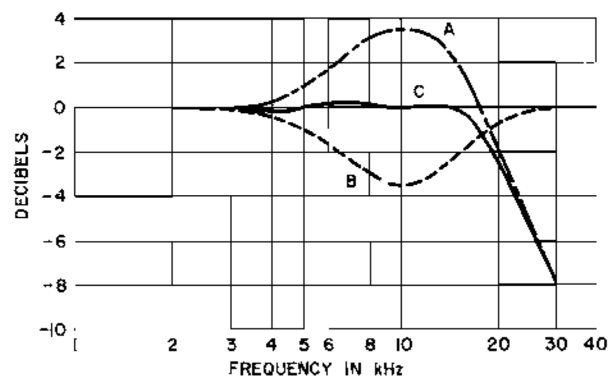


Fig. 13.4. Typical response characteristics of an electron tube audio amplifier and equalizer circuit; before equalization (A), equalizer response (B), and after equalization (C).

13.6.1 Example for Shunt-tuned Equalizer

The response curve of an electron tube audio amplifier whose gain-vs.-frequency characteristic it is desired to flatten or "equalize" is shown in Fig. 13.4 (curve A). This particular amplifier tube has an internal plate resistance R_p of 3,000 ohms and works into a load resistance R_l of 1,000 ohms. An equalizer circuit consisting of a parallel combination of resistance, inductance, and capacitance is to be inserted in series with R_p and R_l as shown within the dotted rectangle in Fig. 13.5.

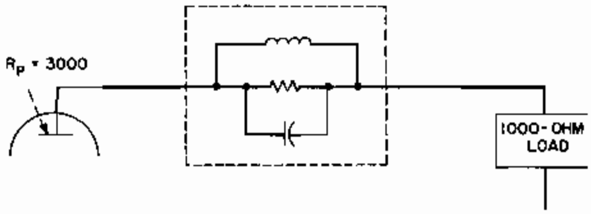


Fig. 13.5. Shunt tuned equalizer circuit (in dotted rectangle).

Independent control of these three equalizer circuit elements makes it possible to accomplish three independent results, viz., (1) to tune the equalizer circuit to resonance at any desired frequency, (2) to add any specified loss at the resonant frequency, and (3) to add some other specified loss at a frequency which is at a specified frequency off resonance.

In the example under consideration, it would be desirable (1) to tune this circuit to resonance at 10 kHz, that is, the frequency where the gain peaks, and to introduce 3.5 dB loss at this frequency, (2) to introduce a loss of 1.0 dB at a frequency of 5 kHz. This will then result in an overall loss characteristic for the equalizer circuit as shown in Fig. 13.4 (curve B), which when combined with curve A in this figure will result in the flattened characteristic shown in curve C.

The procedure for establishing the correct values for each of the three shunt circuit elements from the SMITH CHART is (with reference to Fig. 13.6) as follows:

1. Obtain the required value of normalized conductance in the equalizer circuit to produce 3.5-dB loss. To do this, enter the chart on its admittance coordinates at the infinite admittance point A, and move to the left along the zero susceptance axis a distance corresponding to 3.5 dB (the loss needed at 10 kHz) on the voltage (or current) transmission coefficient (dB) scale in Fig. 5.2.

Note: A count of 3.5 dB from the extreme right-hand end of this scale (6-dB gain point) locates the proper position along this scale at its 2.5-dB point, corresponding to point B in Fig. 13.6.

Observe that point B falls on the 2.0 normalized conductance circle. Thus the normalized conductance of the equalizer circuit must be 2.0.

2. Obtain the required value of susceptance in the equalizer circuit which will provide a loss of 1 dB at a frequency of 5 kHz. To do this, proceed from point B clockwise around the 2.0 conductance circle to the point C where this circle intersects the 1-dB contour of the aforementioned transmission coefficient scale. Observe that point C falls on the 4.0 normalized susceptance curve. Thus, the normalized susceptance of the equalizer circuit must be 4.0.

Finally, the following steps must be taken to evaluate the required resistances R, inductance L, and capacitance C of the equalizer circuit elements.:

1. Determine the normalizing value R_0 which is the total loop resistance of the circuit, namely,

$$R_0 = R_p + R_l \tag{13-4}$$

and which in the above example is 4,000 ohms.

2. Observe that at resonance (10 kHz in the above example),

$$\omega_0 C R_0 - \frac{1}{\omega_0 (L/R_0)} = 0 \tag{13-5}$$

and off resonance (5 kHz in the above example)

IMPEDANCE OR ADMITTANCE COORDINATES

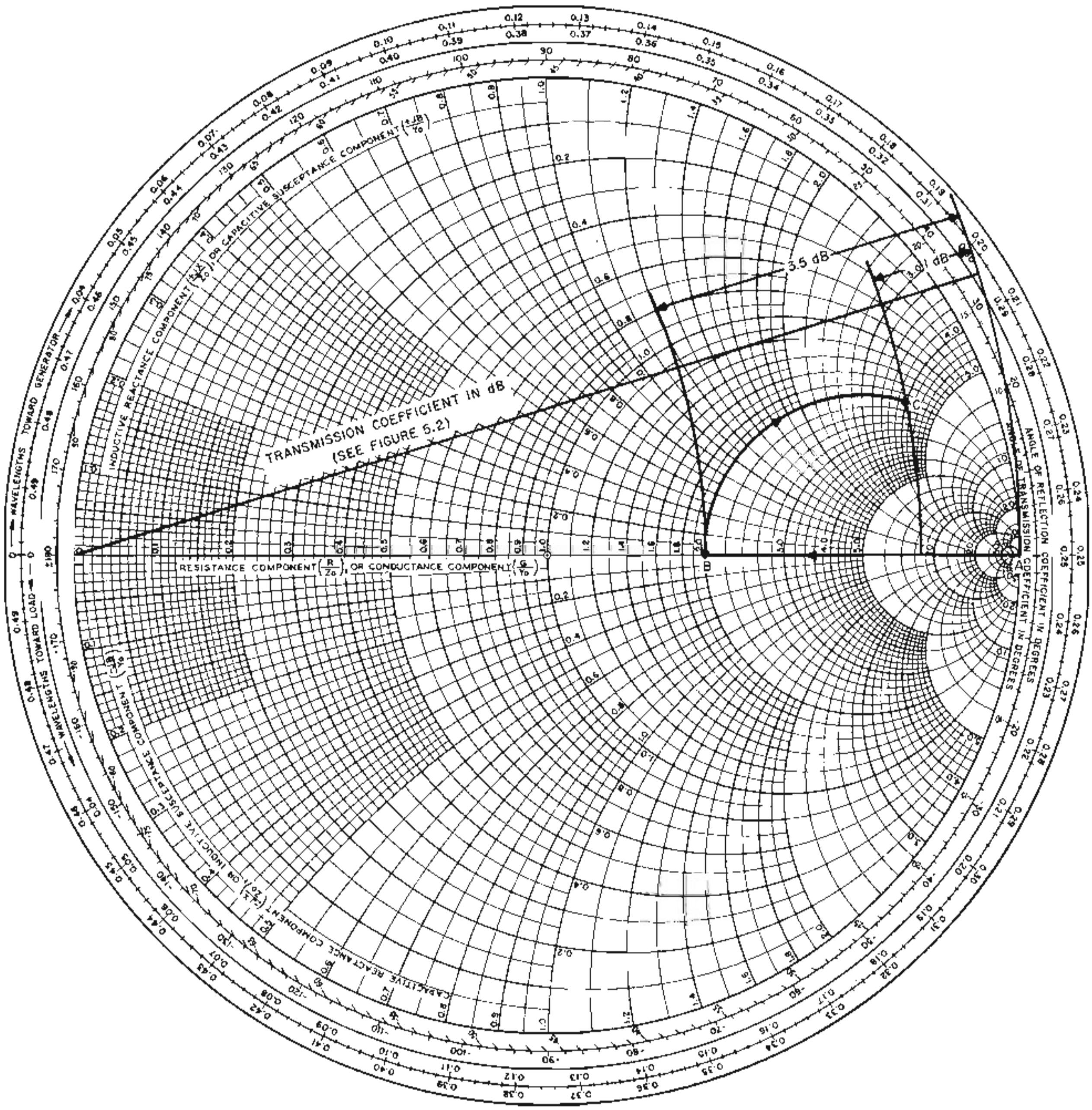


Fig. 13.6. Evaluation on a SMITH CHART of conductance and susceptance of elements of shunt-tuned equalizer circuit.

$$\omega_1 CR_0 - \frac{1}{\omega_1 (L/R_0)} = 4.0 \qquad (13\text{-}6)$$

$$CR_0 = \frac{4}{\omega_1 - \left(\omega_0^{\,2}/\omega_1\right)} \qquad (13\text{-}7)$$

from which we obtain

$$= 4.25 \times 10^{-5} \qquad (13\text{-}8)$$

and

$$\frac{L}{R_0} = 5.97 \times 10^{-6} \tag{13-9}$$

Thus, in the example given,

$R_0 = 4{,}000$ ohms and $GR_0 = 2.0$

from which

$\frac{1}{G}$ or $R = 2{,}000$ ohms

Also, from Eq. (13-8),

$C = 0.0106\ \mu\text{F}$

and from Eq. (13-9),

$L = 23.9$ mH

13.7 NUMERICAL ALIGNMENT CHART

Although not specifically intended for general-purpose arithmetic and trigonometric calculations, the outer circular boundary line of the coordinates of the SMITH CHART, and the resistance axis (with the normalized scale designations along it), can conveniently be used as an alignment chart to perform many of the numerical operations of an ordinary slide rule.

The more conventional of the mathematical operations are listed in Table 13.1 along with corresponding geometrical constructions and numerical results of specific examples as obtained from Fig. 13.7. As with the slide rule, the decimal point can be moved to accommodate a wide range of numerical representations, for example, 4.0 on the chart scales also can represent .04, .40, 40., 400., etc., provided that the decimal point is correspondingly moved in the solution.

The angle α for the tangent and cotangent is projected to the chart coordinate boundary line (at point L on Fig. 13.7) along a straight line from the peripheral transmission coefficient angle scale value to its origin at point 0.0. The sine and cosine functions of α are not directly obtainable from the SMITH CHART but should the need arise their values can be computed from corresponding values of the tangent function from the following relationships:

$$\sin\alpha = \frac{\tan\alpha}{(1 + \tan^2\alpha)^{1/2}} \tag{13-10}$$

$$\cos\alpha = \frac{1}{(1 + \tan^2\alpha)^{1/2}} \tag{13-11}$$

The symbol (⏋) at the intersection of a construction line with the resistance axis indicates that the line must always be drawn mutually at right angles to the resistance axis.

13.8 SOLUTION OF VECTOR TRIANGLES

If the magnitude and either the resistance or reactance component of any impedance is known, the remaining component can readily be determined on the SMITH CHART. This is possible because of a unique property of a one-eighth wavelength section of lossless transmission line. If such a line is terminated in any pure resistance whatsoever the magnitude of its input impedance will equal its characteristic impedance.

To make use of this property of an eighth-wavelength line in the solution of vector triangles, a vertical line connecting points $+j1.0$ and $-j1.0$ at the periphery of the impedance coordinates is first drawn across the coordinates of the SMITH CHART, such as Chart A in the cover envelope. This line will trace the locus of all input resistance and reactance components of an eighth-wavelength line which is terminated in any load resistance from zero to infinity.

For example, suppose that the magnitude of an impedance vector is known to be 275 ohms and its resistance component is 165 ohms. The corresponding reactance component is desired. The procedure is to first normalize the known input impedance component with respect to the known impedance magnitude; thus,

$$\frac{R}{|Z_0|} = \frac{165}{275} = 0.6$$

The next step is to find X/Z_0 by entering the SMITH CHART at the point along its resistance axis where $R/Z_0 = 0.6$ and then by moving upward along the 0.6 normalized resistance circle to the point where it intersects the vertical line representing the one-eighth wavelength position. At this intersection, the normalized value of the desired reactance component X/Z_0 is found to be

$$\frac{X}{Z_0} = 0.8$$

If the magnitude of the reactance component had been known and the resistance component was desired, the procedure is quite similar. In this case the SMITH CHART is entered at its periphery where $X/Z_0 = 0.8$,

Table 13.1. Results of Numerical Operations from Fig. 13.7

OPERATION	CONSTRUCTION	EXAMPLE
PRODUCT	$A \cdot B = C$	$4. \cdot .5 = 2.$
QUOTIENT	$C/A = B$ $C/B = A$	$2./4. = .5$ $2./.5 = 4.$
RECIPROCAL	$1/D = E$ $1/E = D$	$1./.33 = 3.$ $1./3. = .33$
SQUARE	$F^2 = G$	$2.^2 = 4.$
SQUARE ROOT	$G^{1/2} = F$	$4.^{1/2} = 2.$
GEOMETRIC MEAN	$(IJ)^{1/2} = H$ $I/H = H/J$	$(.5 \cdot 1.5)^{1/2} = .87$ $5/.87 = .87/1.5$
TANGENT	$\tan \alpha = K$	$\tan 30° \doteq .58$
COTANGENT	$\cot \alpha = L$	$\cot 30° = 1.75$
ARC TANGENT	$\tan^{-1} K = \alpha$	$\tan^{-1} .58 = 30°$
ARC COTANGENT	$\cot^{-1} L = \alpha$	$\cot^{-1} 1.75 = 30°$

and this reactance circle is then followed downward to the intersection with the vertical line representing the one-eighth wavelength position, where the value of the normalized resistance component is observed to be

$$\frac{R}{Z_0} = 0.6$$

And again, since $Z_0 = 275$, the resistance component

$$R = 0.6 \times 275 = 165 \text{ ohms}$$

Vector triangles representing voltages, currents, and admittances are similarly solved on the SMITH CHART.

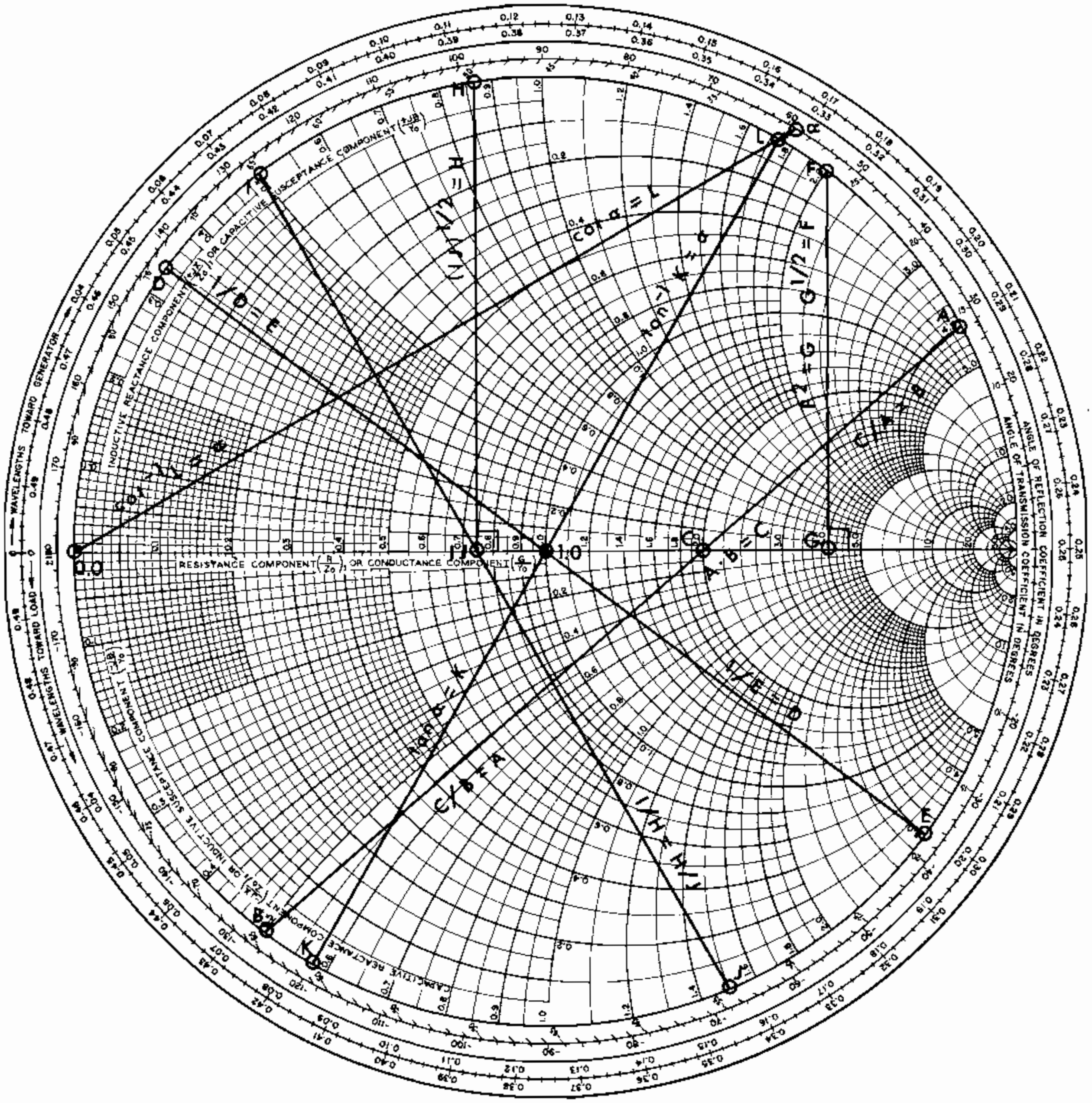


Fig. 13.7 Constructions for numerical operations on a SMITH CHART as summarized in Table 13.1.

CHAPTER 14

Smith Chart Instruments

14.1 CLASSIFICATION

One important class of instruments displays data electronically on a SMITH CHART. Some of these instruments employ a translucent SMITH CHART attached to the face of a cathode ray tube; others employ an opaque SMITH CHART on an electrically operated plotting table. In either of these the chart serves as the reference coordinate system. Such devices generally sample the amplitudes and phases of forward- and backward-traveling waves, respectively, at the output ports of a directional coupler. The samples are translated by sine and cosine function generators to control the position of the cathode ray spot, or of the pen on the plotting table. By using sweep-frequency oscillators such devices can display a large amount of data in a brief interval of time. Their detailed features and operation are best described by the individual manufacturer.

Other types of SMITH CHART "instruments" which are described herein are purely mechanical in operation, and are thus more analogous to the ordinary logarithmic-scaled slide rule.

14.2 RADIO TRANSMISSION LINE CALCULATOR

The calculator shown in Fig. 14.1, constructed in 1939, employs a pair of cardboard disks, and a radial arm pivoted from the center. This arrangement allows separate control of the zero position on the outer peripheral scales which were printed on the larger disk, and the position of the radial arm with its attenuation and SWR scale (expressed as a ratio less than unity). A slidable cursor permits the establishment of reference points on the chart coordinates. This calculator, currently out of print, was superceded in 1944 by the improved version shown in Fig. 14.2.

The following instructions are printed on the back of this early "Radio Transmission Line Calculator":

RADIO TRANSMISSION LINE CALCULATOR INSTRUCTIONS

The Calculator may be used for open-wire or coaxial transmission lines. It gives the impedance at any position along the line, standing-wave amplitude ratio, and attenuation. If the power is known, the voltage and current at any position along the line is readily calculable from the impedance information obtained.

The curved lines on the central disk are simply resistance and reactance coordinates upon which all impedances, both known and unknown, can be read. These coordinates indicate series components of the impedances and are labeled as a fraction of the characteristic impedance of the line used.

The separately rotatable scale around the rim of the calculator provides the means for measuring the distance* along the transmission line between any two points in question. (Any distance in excess of a half wavelength can be reduced to an equivalent shorter distance to bring it within the scale range of the calculator by subtracting the largest possible whole number of half wavelengths.)

Attenuation in decibel intervals and current or voltage ratio scales are plotted along a rotatable radial arm. This arm and its slider also provide a cross-hair-indicating mechanism.

EXAMPLE A. To find the impedance at any given point along a line when the impedance at any other point is known:

1. Adjust the cross hair formed by the lines on the radial arm and slider to intersect over the known impedance point.
2. Rotate only the outer "distance" scale until its 0 point lines up with the line on the radial arm.
3. Rotate only the radial arm (with its slider fixed thereon) the required distance as measured along the outer scale, either "towards load" or "towards generator" as the case may be, and read unknown impedance under the new cross-hair location.

*For coaxial lines with insulation, distance is in effect increased by the factor $\sqrt{K}$ where K = dielec. const.

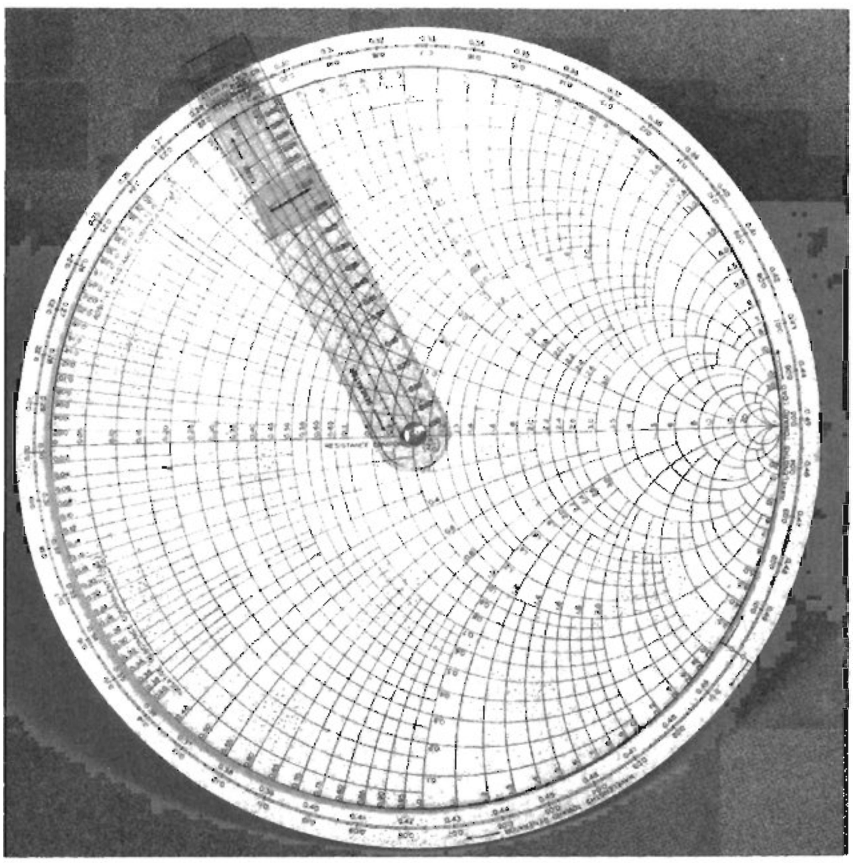


Fig. 14.1. Radio transmission line calculator [101].

4. To correct for attenuation, move the slider only along the radial arm in the direction indicated thereon the desired number of decibel intervals before reading the unknown impedance.

EXAMPLE B. To find the impedance at any given point along a line (with respect to the position of a current or voltage minimum or maximum point) when the standing-wave amplitude ratio is known:

1. Point the radial arm along the resistance axis in either direction depending upon whether measurements are to be made from a minimum or maximum current or voltage point. (A current minimum and voltage maximum occur simultaneously at a resistance maximum point, while a current maximum and voltage minimum occur simultaneously at a resistance minimum point.)
2. Rotate only the outer "distance" scale until its 0 point lines up with the line on the radial arm.
3. Move the slider along the ratio scale on the radial arm to the known standing-wave amplitude ratio.
4. Rotate only the radial arm (with its slider fixed thereon) the desired distance from the initial pure resistance point either "towards load" or "towards generator" as the case may be, and read the unknown impedance at the new cross hair location.

EXAMPLE C. To find the standing-wave amplitude ratio if the impedance at any point along the line is known:

1. Adjust the cross hair to intersect over the known impedance point and read directly the standing-wave amplitude ratio on the ratio scale along the radial arm.

EXAMPLE D. To find the attenuation of a line when the input and load impedances are known:

1. Adjust the cross hair, formed by the lines on the radial arm and slider, to intersect over the known input impedance point, noting the position of the slider along the decibel scale on the arm.
2. Readjust the cross hair to intersect over the load impedance point and again note the position of the slider along the decibel scale.
3. The attenuation of the line is obtained by counting the number of decibel intervals along the radial arm across which it was necessary to move the slider.

Refer to: "Transmission Line Calculator"
by P. H. Smith, Bell Telephone Laboratories,
January 1939, Electronics.

14.3 IMPROVED TRANSMISSION LINE CALCULATOR

The all-plastic calculator shown in Fig. 14.2 was first constructed in 1944, and has been commercially available [16] without alteration since that time. Its design is described in a magazine article [102] by the author in 1944 entitled "An Improved Transmission Line Calculator." It provided additional radial scales, shown more clearly in Fig. I.4, and its coordinate system was graduated in a more orderly way. A newer and larger combined calculator and plotting device called a "computer-plotter" is shown in Fig. 14.11 [20].

The instructions printed on the back of the calculator in Fig. 14.2 are:

RADIO TRANSMISSION LINE CALCULATOR INSTRUCTIONS

The calculator relates the series components of impedance at any position along an open-wire or coaxial transmission line to (1) the impedance at any other point, (2) the standing-wave amplitude ratio (SWR), and (3) the attenuation. It also relates the impedance to the reflection coefficient and to the admittance and, in addition, provides a means for determining the equivalent parallel components of impedance. If the power P is known the voltage and current at any position along the line are readily calculable from impedance information obtained (see Example F).

All impedances are read on the resistance and reactance coordinates on the central disk. These coordinates are designated as a fraction of the characteristic impedance Z_0 of the line in order to eliminate this parameter, which is a constant in any given problem.

The rotatable "wavelengths" scale around the rim provides the means for translating impedances at a given point along the line to another point a given distance* away. (Any distance in excess of a half wavelength should be reduced to an equivalent shorter distance to bring it within the scale range of the calculator by subtracting the largest possible whole number of half wavelengths.)

Attenuation in 1.0 dB steps and current or voltage SWR scales are plotted on the rotatable radial arm (nearest index) together with scales which are a function of SWR. These include, to the left, scales for minimum and maximum values of current or voltage at node or antinode points (relative to current or voltage on matched line), and the SWR expressed in dB; and to the right, loss coefficient due to standing waves, reflection loss, and reflection coefficient magnitude. The cross hair index on the arm provides the indicating mechanism for relating all parameters at any given point along the line.

EXAMPLE A. TO FIND SWR IF IMPEDANCE AT ANY POINT ALONG LINE IS KNOWN:

1. Adjust cross hairs to intersect over known impedance point and read SWR and related parameters on the scales on radial arm.

EXAMPLE B. TO FIND IMPEDANCE AT ANY GIVEN POINT ALONG LINE (WITH RESPECT TO POSITION OF A CURRENT OR VOLTAGE MINIMUM OR MAXIMUM POINT) WHEN SWR IS KNOWN:

1. Align index on arm with resistance axis in the direction depending upon whether reference point is a minimum or maximum current or voltage point. (A current minimum and voltage maximum are always at the same position as the resistance maximum point, i.e., along resistance axis in direction of $R/Z_0 = \infty$, while a current maximum and voltage minimum are always at the same position as resistance minimum point, i.e., along resistance axis in direction of $R/Z_0 = 0$.)
2. Move slider index along arm to known SWR.
3. Rotate only "WAVELENGTHS" scale until its 0 point is aligned with index line on arm.

*For coaxial lines with insulation, distance is effectively increased by $\sqrt{K}$ where K = dielectric const.

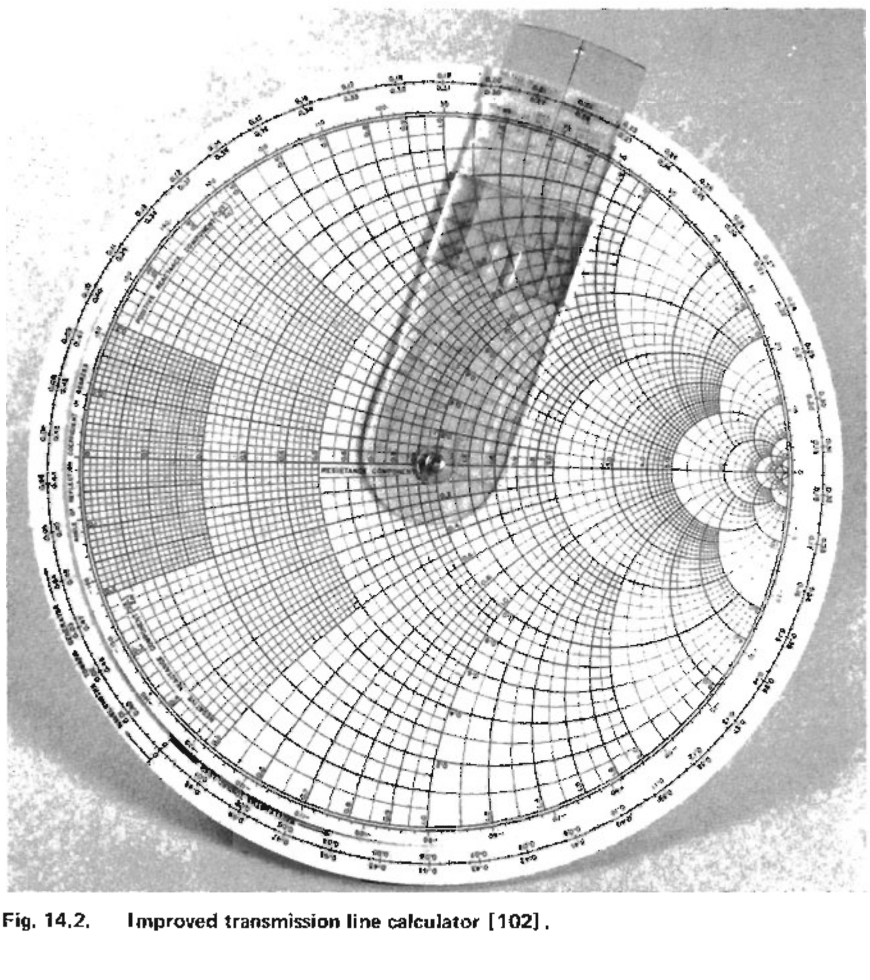


Fig. 14.2. Improved transmission line calculator [102].

4. Rotate only arm (with slider fixed thereon) the desired distance in direction indicated on "WAVELENGTH" scale and read unknown impedance at new cross hair location.
5. Correct above impedance for attenuation by moving slider only along arm the desired number of decibel intervals in the proper direction indicated thereon.

EXAMPLE C. TO FIND IMPEDANCE AT ANY GIVEN POINT ALONG LINE WHEN IMPEDANCE AT ANY OTHER POINT IS KNOWN:

1. Adjust cross hairs to intersect over known impedance point.
2. Same as B(3), B(4), and B(5).

EXAMPLE D. TO FIND ATTENUATION OF LINE WHEN INPUT AND LOAD IMPEDANCES ARE KNOWN:

1. Adjust cross hairs to intersect over input impedance point and note position of slider index on arm at scales designated "1 dB STEPS" and "LOSS (dB)"
2. Readjust cross hairs to intersect over load impedance point and again note position of slider index on above scales.
3. The number of decibel intervals between the two slider index positions above on scales designated "1-dB STEPS" gives the nominal attenuation of the line due to resistance, etc. To this add the corresponding dB difference on the "LOSS (dB)" scale, which gives the additional loss due to standing waves.

EXAMPLE E. TO CONVERT IMPEDANCES TO ADMITTANCES AND TO EQUIVALENT PARALLEL COMPONENTS OF IMPEDANCE

1. Equivalent admittances, as a fraction of the characteristic admittance, appear diametrically opposite any impedance point. To read admittances, consider resistance components to be conductance components and reactance components to be susceptance components of admittance. (Capacitance is considered to be a positive susceptance and inductance a negative susceptance.) No other change in scale designations, direction of rotation, etc., are required for using calculator to obtain admittance relations to other parameters.
2. Equivalent parallel components of impedance are the reciprocal of the equivalent admittance values. (Use reciprocal "LIMITS" scale on arm.)

EXAMPLE F. TO FIND VOLTAGE OR CURRENT AT ANY POINT ALONG LINE:

$$E = \sqrt{R_{par} \times P} \qquad E_{min} = \sqrt{\frac{P \times Z_0}{SWR}} \qquad E_{max} = \sqrt{P \times Z_0 \times SWR}$$

$$I = \sqrt{P/R_{ser}} \qquad I_{min} = \sqrt{\frac{P}{SWR \times Z_0}} \qquad I_{max} = \sqrt{\frac{P \times SWR}{Z_0}}$$

R_{par} obtained from $E(2)$.

Refer to "Transmission Line Calculator"
by P. H. Smith, Bell Telephone Laboratories
Jan. 1939 and Jan. 1944 Electronics

THE EMELOID CO., INC.
HILLSIDE 5, N. J.

14.4 CALCULATOR WITH SPIRAL CURSOR

The cursor mechanism shown in Fig. 14.3 was devised by R. S. Julian in 1944 at Bell Telephone Laboratories. Approximately 100 calculators of this type were manufactured but they were never made commerically available. However, the ingenious design of the dual logarithmic (equiangle) spiral cursor is worthy of note.

The dual spiral cursor served several purposes, one of the more important of which was to enable the attenuation scale to be moved from the radial arm to the perimeter of the chart. In this position the scale could be expanded in length by a factor of at least pi. Since the spiral cursor is adjustable with

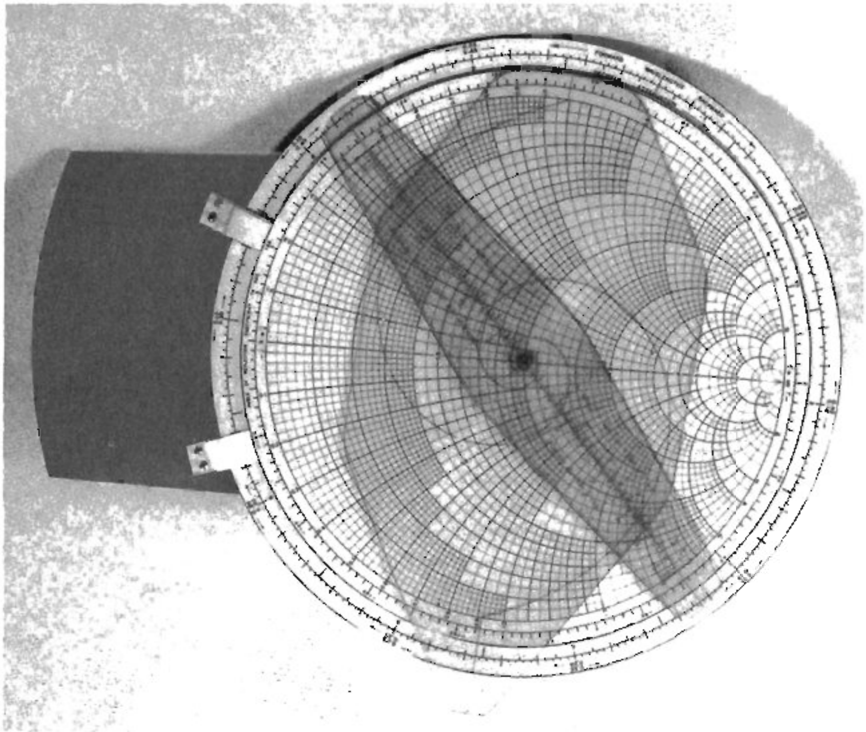


Fig. 14.3. Transmission line calculator [18].

respect to the coordinates, as is also the straight radial cursor, their intersection can be made to coincide with any point on the chart coordinates. Having thus set the two adjustable cursors the peripheral attenuation scale can be rotated until its zero position corresponds to the position of the straight line segment at the tip of the spiral. Finally, by rotating the spiral cursor only to a desired setting on the peripheral attenuation scale, the intersection of the spiral and radial cursors will move a distance corresponding to the change of the attenuation. In this way it is not necessary to count dB steps of attenuation as is required for a fixed radial attenuation scale with a "floating" zero position.

By using the dual spiral cursor, points on the chart coordinates which are diametrically opposite and at equal chart radius from any selected point can be located without swinging the single radial arm and its slidable cursor through 180°. Thus, with this device impedances can more readily be converted to admittances.

A minor difficulty with the spiral cursor is the ambiguity caused by more than one intersection of the spiral with the straight radial lines.

14.5 IMPEDANCE TRANSFER RING

A manual plotting device called an impedance transfer ring is shown in Fig. 14.4. This device [122, 213] consists of a circular protractor graduated in wavelengths, and a narrow diametrical scale (with a slidable cursor) graduated in standing wave ratio. The *impedance transfer ring* is intended for use with paper SMITH CHARTS with a coordinate radius of 9.1 cm. (Kay form 82-BSPR shown in Fig. 8.6.)

The construction and operation of the impedance transfer ring, with specific examples of its use, is contained in a 29-page U.S. Naval Electronics Laboratory Report [213]. A device which can accomplish the same objectives as the impedance transfer ring, called the *Mega-Plotter* and described in the following paragraphs, is commercially available [14].

14.6 PLOTTING BOARD

A plotting board [204] designed to plot impedances in the frequency range around 65 MHz is shown in Fig. 14.5. Such a device can as well be designed to operate around any other chosen frequency. This device depends on the fact that the impedance at a point along a transmission line is uniquely related

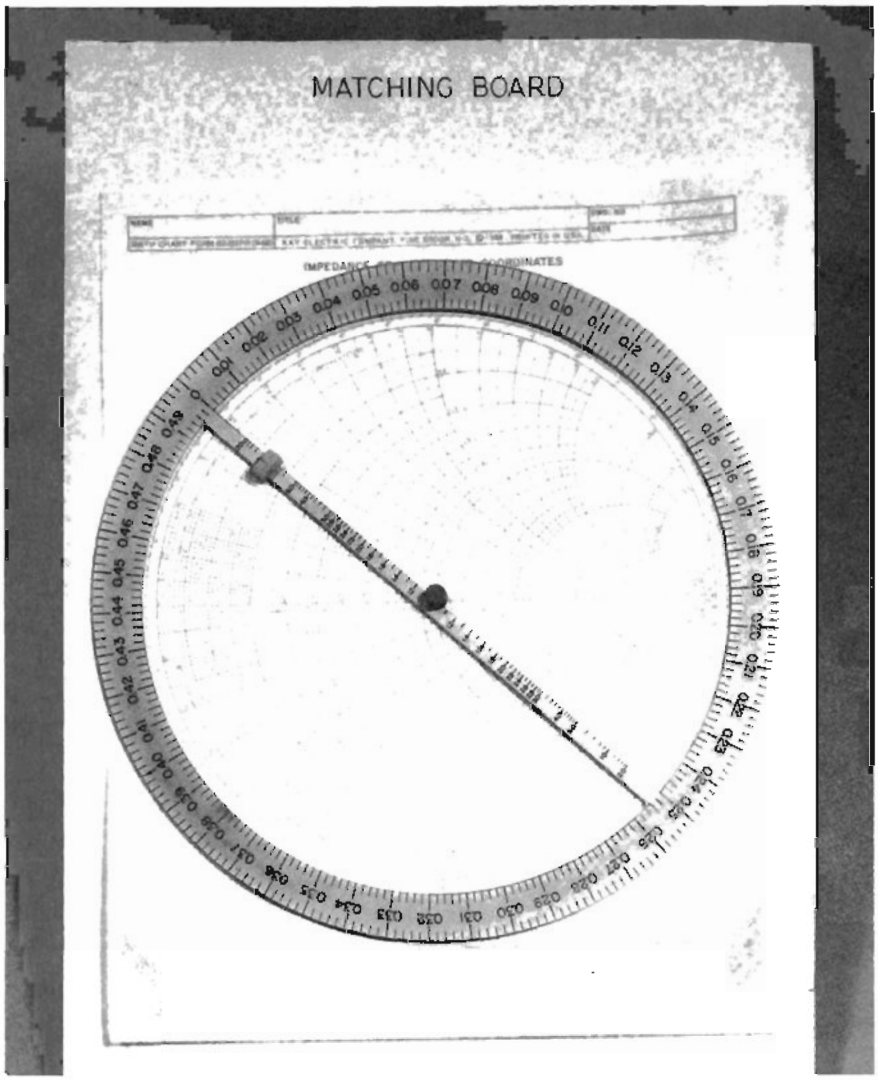


Fig. 14.4. The impedance transfer ring and SMITH CHART mounted on matching board [122].

to the magnitude and phase relationships between the voltages or currents along the line at discrete sampling points of known separation. (See Chap. 11 on probe measurements.) Probe spacings of approximately one-eighth wavelength are used for the plotting board shown, and probe voltages are measured with vacuum tube voltmeters.

The impedance at a point in question is determined from the SMITH CHART by the intersection of three arcs whose radii are proportional to the three respective probe voltages.

An expanded center SMITH CHART (Kay form 82-SPR shown in Fig. 7.2) is used with this particular plotter, which enables a determination of the impedance with an accuracy of about 10 percent. The three dials control the position of a single pointer to which they are connected through a system of cords and springs. The angle between the cords is proportional to the phase relationships between the probe voltages, and can be adjusted to compensate for the change in electrical length of the transmission line between fixed probe positions as the frequency is varied over a limited range. This adjustment is accomplished for all three cords simultaneously by varying the position of the pointer shown in the upper left corner of Fig. 14.5. The pointer is connected to all three arms on which the dials are mounted through a pantograph which controls the angular separation of the cords.

The plotting board shown in Fig. 14.5 is not commercially available.

14.7 MEGA-PLOTTER

The plotting board shown in Fig. 14.6 is commercially available [14] under the trade

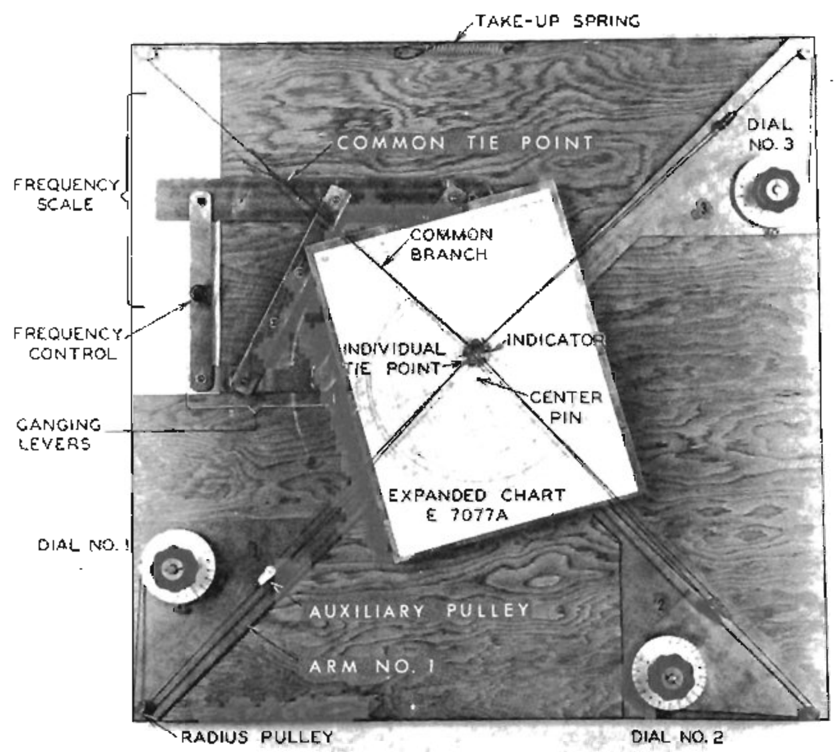


Fig. 14.5. Plotting board for SMITH CHARTS [204].

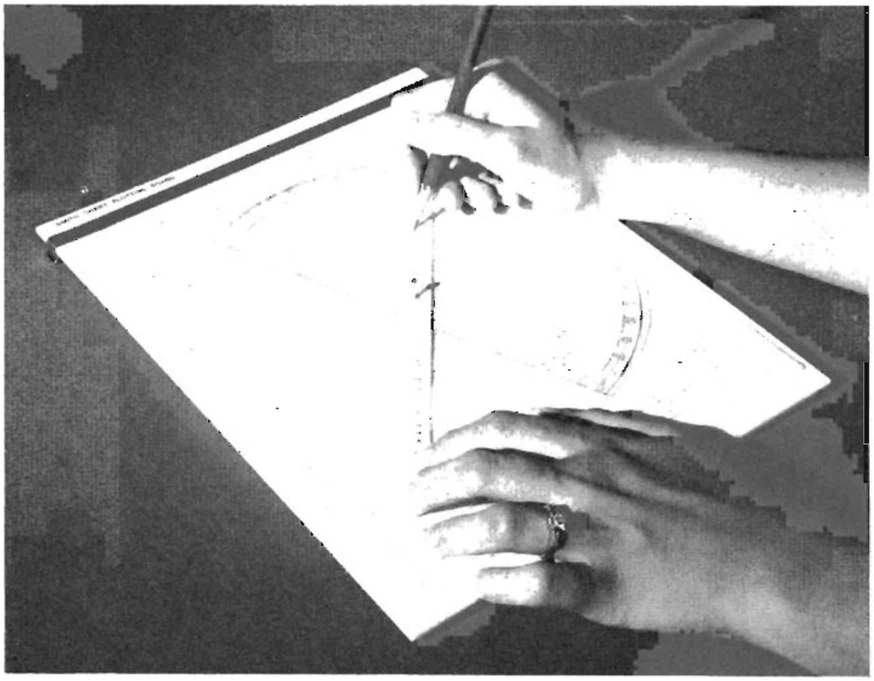


Fig. 14.6. Mega-Plotter [14].

name "Mega-Plotter." The separately rotatable scales for this device are shown in Fig. 14.7.

The Mega-Plotter is designed primarily to facilitate the plotting of electrical transmission line data on SMITH CHARTS such as Kay chart forms 82-BSPR (SWR = 1.0 to ∞) and 82-SPR (SWR = 1.0 to 1.59). However, this plotting board accommodates other commonly used SMITH CHARTS, and polar reflection charts of equal or smaller radius, if they are printed on standard size (8½ × 11 in.) sheets. It may also be used for plotting or reading out data in polar form from antenna radiation diagrams, and as a general-purpose polar plotting board.

SMITH CHARTS are held securely in place on the board with a handy snap-on nickel-silver fastener. A small hollowed point pin pierces the paper chart center and provides a pivot for a compass (not supplied) and for independently rotatable peripheral and radial scales.

A protractor provides adjustable peripheral scales. On one side these are graduated in wavelengths for measuring angular distances on the coordinates of any SMITH CHART from any initial point. On the other side, the scales are graduated in degrees for general-purpose polar plots.

Two diametrical straightedges are provided, each with two commonly used radial scales of correct length for the above SMITH CHARTS. One side of each of these straightedges is blank, with a matte surface, to permit pencil marking or pasting thereon any of the more specialized scales which are printed across the bottom of the aforementioned SMITH CHARTS, or for marking thereon radial scales for charts of other radii.

The straightedge on each of the diametrical scales, like the inside straightedge on the protractor, is intentionally offset from center by about the amount a line would be offset if drawn with a sharp pencil. When setting these edges to the desired peripheral scale value, or to coordinate points on the chart, allowance should be made for this offset by using the pencil point as a gauge.

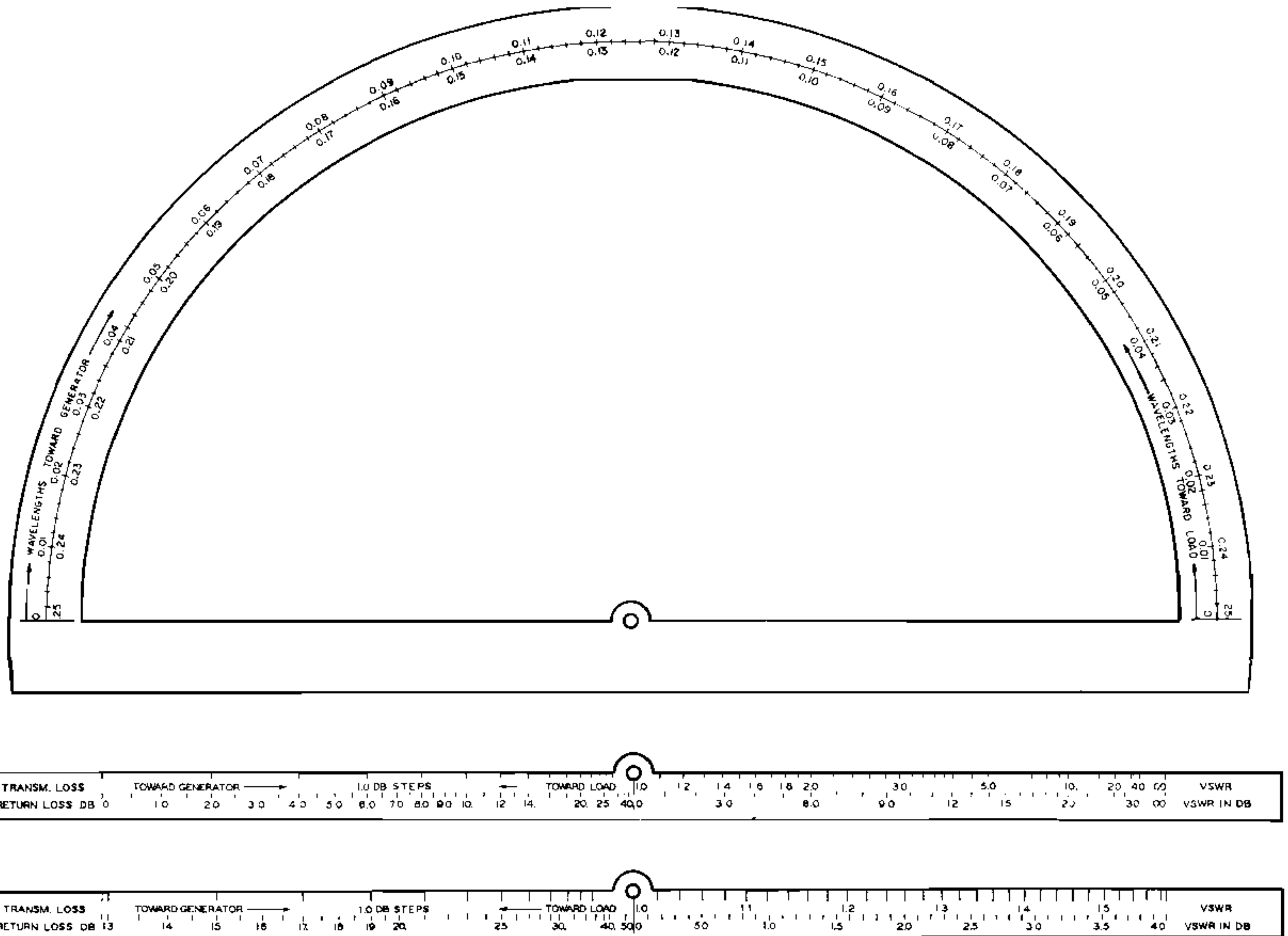


Fig. 14.7. Protractor and radial scales for Mega-Plotter in Fig. 14.6.

14.8 MEGA-RULE

The "Mega-Rule" shown in Figs. 14.8, 14.9, and 14.10 provides a convenient and rapid means for comparing and evaluating ten commonly used functions which specify transmission and reflection characteristics of electromagnetic waves on waveguides. The ten function values are plotted on a set of ten parallel interrelated scales including an attenuation scale printed in red. The voltage reflection coefficient scale thereon is linear, and therefore is directly related to the radial distance on a SMITH CHART. The relationship of individual values of these ten scales to each other, and to the attenuation of the waveguide, is evaluated by means of a slidable cursor. The most commonly used of these functions is perhaps the standing wave ratio S, to which all others are mathematically related by the formulas on the back of the Mega-Rule.

The Mega-Rule is applicable to any type of uniform waveguide, for example, uniconductor waveguide, coaxial cable, strip-line, or parallel-wire transmission line. The waveguide may have any characteristic impedance. The ten functions represented on the Mega-Rule scales apply to any mode of propagation such as the transverse electromagnetic (TEM) mode in coaxial cables and parallel wire transmission lines, or any of the many transverse electric

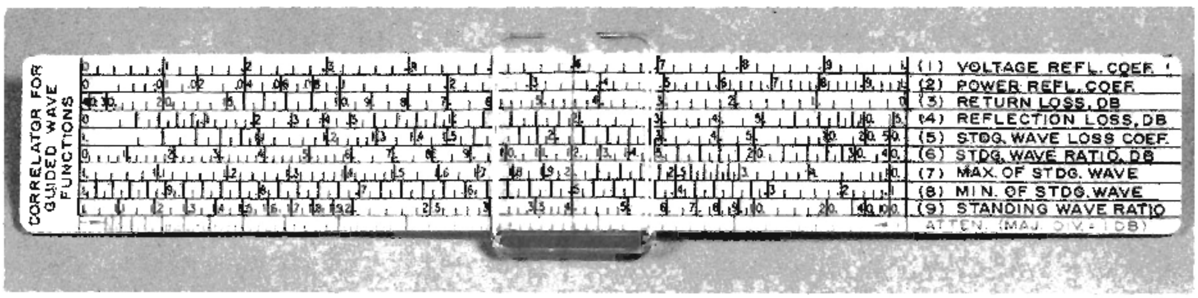


Fig. 14.8. Mega-Rule [20].

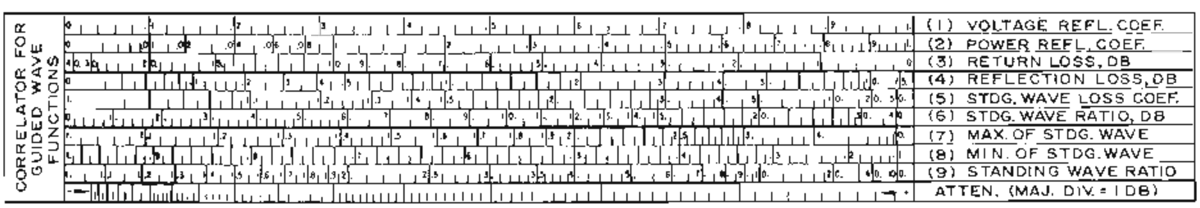


Fig. 14.9. Mega-Rule Scale—Front Side.

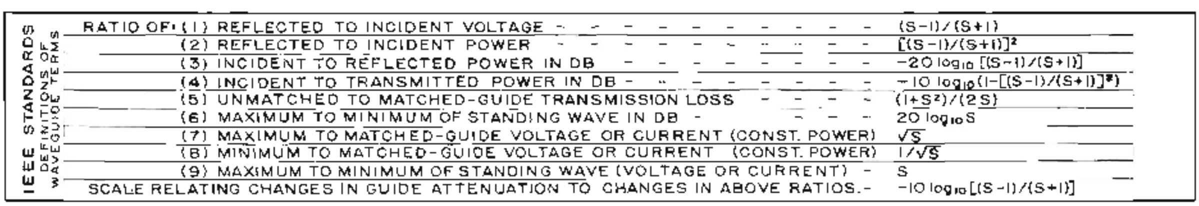


Fig. 14.10. Mega-Rule Formulas—Back Side.

(TE_{mn}) or transverse magnetic (TM_{mn}) modes in uniconductor waveguides.

When using the Mega-Rule for problems involving dissipative loss (attenuation) it is assumed that the waveguide is sufficiently long so that the loss within a quarter wavelength is a negligible fraction of the total loss.

Individual graduations on the ten scales of the Mega-Rule are plotted from data computed to seven significant figures. The values in Table 14.1 may be interpolated should it be desired to obtain more accuracy than is possible by visual settings of the cursor on the Mega-Rule scales.

The three columns in Table 8.1 express the transmission-reflection function values for the ten respective scales on the Mega-Rule in terms of:

1. Incident i and reflected r traveling waves of voltage or current.
2. Voltage or current reflection coefficient magnitude ρ.
3. Voltage or current standing wave ratio S.

The ten functions with their definitions printed on the back of the Mega-Rule and represented by the ten scales on the front are as follows:

1. *Voltage reflection coefficient*—the ratio of reflected to incident voltage.
2. *Power reflection coefficient*—the ratio of reflected to incident power.
3. *Return loss in dB*—the ratio of incident to reflected power in dB.
4. *Reflection loss in dB*—the ratio of incident power to the difference between the incident and reflected power in dB.
5. *Standing wave loss factor*—the ratio of mismatched- to matched-guide transmission loss.
6. *Standing wave ratio in dB*—twenty times the logarithm to the base 10 of the standing wave ratio (special use of dB).
7. *Standing wave maximum*—the ratio of standing-wave maximum to matched-guide voltage (or current).
8. *Standing wave minimum*—the ratio of standing-wave minimum to matched-guide voltage (or current).
9. *Standing wave ratio*—the ratio of maximum to minimum of the standing wave of voltage or current.
10. *Attenuation*—this scale relates the effect of waveguide dissipative losses on all other scales.

All above definitions are printed on the back of the Mega-Rule.

14.8.1 Examples of Use

1. A given length of waveguide connecting a radio transmitter to an antenna has an attenuation (one-way dissipative loss) of 1.3 dB at a particular operating frequency. At the antenna end of the waveguide, at this frequency, the standing wave ratio is observed to be 2.5 (8.0 dB).

It is desired to determine the total loss incurred by the load mismatch and the dissipative losses in the waveguide.

Enter the Mega-Rule by setting the cursor to 2.5 on the standing wave ratio scale (no. 9), or 8.0 dB on the standing wave ratio, dB, scale (no. 6). Observe from scale no. 4 that the reflection loss at the antenna (a nondissipative loss defined on the back of the Mega-Rule) is 0.88 dB.

Next, slide the cursor 1.3-dB intervals in the positive direction on the attenuation scale, at the bottom. Now observe from scale no. 4 that the reflection loss at the input end of the waveguide is 0.47 dB. The increase in dissipative loss, due to a reflected wave in the waveguide, over the one-way dissipative loss (attenuation) is obtainable from the Mega-Rule by subtracting the reflection loss at the input end, that is, 0.47 dB, from the reflection loss at the load end, that is, 0.88 dB, to yield 0.41 dB. The total dissipative loss is, therefore, 1.3 dB plus 0.41 dB, or 1.71 dB.

Table 14.1 Function Values at SWR Scale Divisions of Mega-Rule.

(1) VOL. REFL. COEF.	(2) PWR. REFL. COEF.	(3) RETN LOSS, DB	(4) REFL. LOSS, DB	(5) STDG. WAVE LOSS COEF.	(6) STDG. WAVE, DB	(7) STDG. WAVE MAX.	(8) STDG. WAVE MIN.	(9) STDG. WAVE RATIO	(10) ATTEN-UATION (1-WAY)
0.000000	0.000000	INF.	0.000000	1.000000	0.000000	1.000000	1.000000	1.00	INF.
0.024390	0.000595	32.255676	0.002584	1.001190	0.423786	1.024695	0.975900	1.05	16.127838
0.047619	0.002268	26.444386	0.009859	1.004545	0.827854	1.048809	0.953463	1.10	13.222193
0.069767	0.004867	23.126944	0.021191	1.009783	1.213957	1.072381	0.932505	1.15	11.563472
0.090909	0.008264	20.827854	0.036041	1.016667	1.583625	1.095445	0.912871	1.20	10.413927
0.111111	0.012346	19.084850	0.053950	1.025000	1.938200	1.118034	0.894427	1.25	9.542425
0.130435	0.017013	17.692132	0.074523	1.034615	2.278867	1.140175	0.877058	1.30	8.846066
0.148936	0.022182	16.539996	0.097420	1.045370	2.606675	1.161895	0.860663	1.35	8.269998
0.166667	0.027778	15.563025	0.122345	1.057143	2.922560	1.183216	0.845154	1.40	7.781513
0.183673	0.033736	14.719071	0.149042	1.069828	3.227360	1.204159	0.830455	1.45	7.359536
0.200000	0.040000	13.979400	0.177288	1.083333	3.521825	1.224745	0.816497	1.50	6.989700
0.215686	0.046521	13.323550	0.206887	1.097581	3.806634	1.244990	0.803219	1.55	6.661775
0.230769	0.053254	12.736442	0.237667	1.112500	4.082399	1.264911	0.790569	1.60	6.368221
0.245283	0.060164	12.206650	0.269478	1.128030	4.349679	1.284523	0.778499	1.65	6.103325
0.259259	0.067215	11.725314	0.302186	1.144118	4.608978	1.303840	0.766965	1.70	5.862657
0.272727	0.074380	11.285429	0.335673	1.160714	4.860760	1.322876	0.755929	1.75	5.642714
0.285714	0.081633	10.881361	0.369836	1.177778	5.105450	1.341641	0.745356	1.80	5.440680
0.298246	0.088950	10.508519	0.404580	1.195270	5.343434	1.360147	0.735215	1.85	5.254259
0.310345	0.096314	10.163110	0.439824	1.213158	5.575072	1.378405	0.725476	1.90	5.081555
0.322034	0.103706	9.841968	0.475494	1.231410	5.800692	1.396424	0.716115	1.95	4.920984
0.333333	0.111111	9.542425	0.511525	1.250000	6.020599	1.414214	0.707107	2.00	4.771213
0.354839	0.125911	8.999380	0.584441	1.288095	6.444386	1.449138	0.690066	2.10	4.499690
0.375000	0.140625	8.519374	0.658173	1.327273	6.848453	1.483240	0.674200	2.20	4.259687
0.393939	0.155188	8.091412	0.732401	1.367391	7.234557	1.516575	0.659380	2.30	4.045706
0.411765	0.169550	7.707018	0.806866	1.408333	7.604225	1.549193	0.645497	2.40	3.853509
0.428571	0.183673	7.359536	0.881361	1.450000	7.958800	1.581139	0.632456	2.50	3.679768
0.444444	0.197531	7.043650	0.955717	1.492308	8.299466	1.612452	0.620174	2.60	3.521825
0.459459	0.211103	6.755056	1.029797	1.535185	8.627275	1.643168	0.608581	2.70	3.377528
0.473684	0.224377	6.497222	1.103492	1.578571	8.943160	1.673320	0.597614	2.80	3.245111
0.487179	0.237344	6.246220	1.176712	1.622414	9.247960	1.702939	0.587220	2.90	3.123110
0.500000	0.250000	6.020600	1.249387	1.666667	9.542425	1.732051	0.577350	3.00	3.010300
0.512195	0.262344	5.811291	1.321460	1.711290	9.827234	1.760682	0.567962	3.10	2.905646
0.523810	0.274376	5.616532	1.392886	1.756250	10.102999	1.788854	0.559017	3.20	2.808266
0.534884	0.286101	5.434812	1.463630	1.801515	10.370278	1.816590	0.550482	3.30	2.717406
0.545455	0.297521	5.264829	1.533664	1.847059	10.629578	1.843909	0.542326	3.40	2.632414
0.555556	0.308642	5.105450	1.602970	1.892857	10.881360	1.870829	0.534522	3.50	2.552725
0.565217	0.319471	4.955690	1.671532	1.938889	11.126049	1.897367	0.527046	3.60	2.477845
0.574468	0.330014	4.814682	1.739340	1.985135	11.364034	1.923538	0.519875	3.70	2.407341
0.583333	0.340278	4.681664	1.806389	2.031579	11.595671	1.949359	0.512989	3.80	2.340832
0.591837	0.350271	4.555962	1.872676	2.078205	11.821292	1.974842	0.506370	3.90	2.277981
0.600000	0.360000	4.436975	1.938200	2.125000	12.041199	2.000000	0.500000	4.00	2.218487
0.615385	0.378698	4.217067	2.066974	2.219048	12.464985	2.049390	0.487950	4.20	2.108534
0.629630	0.396433	4.018297	2.192748	2.313636	12.869053	2.097618	0.476731	4.40	2.009148
0.642857	0.413265	3.837711	2.315582	2.408696	13.255156	2.144761	0.466252	4.60	1.918855
0.655172	0.429251	3.672888	2.435547	2.504167	13.624824	2.190890	0.456435	4.80	1.836444
0.666667	0.444444	3.521825	2.552725	2.600000	13.979400	2.236068	0.447214	5.00	1.760913
0.692308	0.479290	3.194017	2.834040	2.840909	14.807253	2.345208	0.426401	5.50	1.597008
0.714286	0.510204	2.922561	3.099848	3.083333	15.563025	2.449490	0.408248	6.00	1.461280
0.733333	0.537778	2.693972	3.351492	3.326923	16.258267	2.549510	0.392232	6.50	1.346986
0.750000	0.562500	2.498775	3.590219	3.571429	16.901960	2.645751	0.377964	7.00	1.249387
0.764706	0.584775	2.330111	3.817166	3.816667	17.501225	2.738613	0.365148	7.50	1.165056
0.777778	0.604938	2.182889	4.033350	4.062500	18.061799	2.828427	0.353553	8.00	1.091445
0.789474	0.623269	2.053247	4.239683	4.308824	18.588378	2.915476	0.342997	8.50	1.026623
0.800000	0.640000	1.938200	4.436975	4.555556	19.084850	3.000000	0.333333	9.00	0.969100
0.809524	0.655329	1.835408	4.625950	4.802632	19.554472	3.082207	0.324443	9.50	0.917704
0.818182	0.669421	1.743004	4.807254	5.050000	19.999999	3.162278	0.316228	10.00	0.871502
0.846154	0.715976	1.451013	5.466454	6.041667	21.583624	3.464102	0.288675	12.00	0.725507
0.866667	0.751111	1.242958	6.039945	7.035714	22.922559	3.741657	0.267261	14.00	0.621479
0.882353	0.778547	1.087153	6.547178	8.031250	24.082399	4.000000	0.250000	16.00	0.543577
0.894737	0.800554	0.966094	7.001747	9.027778	25.105449	4.242641	0.235702	18.00	0.483047
0.904762	0.818594	0.869314	7.413486	10.025000	26.020599	4.472136	0.223607	20.00	0.434657
0.935484	0.875130	0.579274	9.035421	15.016667	29.542424	5.477226	0.182574	30.00	0.289637
0.951220	0.904819	0.434385	10.214476	20.012500	32.041199	6.324555	0.158114	40.00	0.217192
0.960784	0.923106	0.347483	11.141102	25.010000	33.979399	7.071068	0.141421	50.00	0.173741
0.967213	0.935501	0.289556	11.904483	30.008333	35.563024	7.745967	0.129099	60.00	0.144778
0.971831	0.944455	0.248185	12.553585	35.007143	36.901959	8.366600	0.119523	70.00	0.124093
0.975309	0.951227	0.217158	13.118200	40.006250	38.061798	8.944272	0.111803	80.00	0.108579
0.978022	0.956527	0.193028	13.617801	45.005555	39.084849	9.486833	0.105409	90.00	0.096514
0.980198	0.960788	0.173724	14.065827	50.005000	39.999999	10.000000	0.100000	100.00	0.086862
1.000000	1.000000	0.000000	INF.	INF.	INF.	INF.	0.000000	INF.	0.000000

The **total** loss incurred by the nondissipative load-mismatch at the input end of the waveguide (0.47 dB), plus the attenuation of the waveguide (1.3 dB), plus the increase in dissipative loss due to the reflected wave (0.41 dB), is thus 2.19 dB. Of this total it is possible to recover only the reflection loss at the input end of the waveguide (0.47 dB), by conjugate matching the transmitter impedance to the input impedance of the waveguide rather than to its characteristic impedance, as is sometimes done.

2. It is required to measure the attenuation (one-way dissipative loss) in a section of coaxial cable 50 feet long. For this measurement the cable may be either open-circuited or short-circuited at its far end, at which point the power reflection coefficient is unity. (This corresponds to a standing-wave ratio of infinity.) The standing wave ratio at the input end of this cable is 2.8, or 9.0 dB.

Enter the Mega-Rule at the extreme right end, where the power reflection coefficient is unity. The attenuation which the cable would have if it were terminated in a matched load is now obtained by sliding the cursor from the initial setting to the position where the standing wave ratio is 2.8 on scale no. 9, or 9.0 dB on scale no. 6, and then reading the number of dB on the attenuation scale at the bottom traversed by the cursor. The result, in this case, is seen to be 3.25 dB attenuation for the 50-foot length, or 0.065 dB per foot.

Alternatively, the attenuation may be obtained from the Mega-Rule by reading the round-trip loss on the return loss, dB, scale no. 3, and taking one-half of the 6.5 dB value read thereon, namely, 3.25 dB.

14.8.2 Use of Mega-Rule with SMITH CHARTS

The voltage reflection coefficient scale (scale no. 1) on the Mega-Rule is linear from zero at its left end to unity at its right end. Since individual values on all other scales are related by the cursor to individual values on scale no. 1, any or all of these scales may be used to determine radial distance on a SMITH CHART to the point where specific scale function values apply. The left end of all Mega-Rule scales correspond to the center point of a SMITH CHART; the right end corresponds to any point around its periphery.

If the radius of the SMITH CHART is not the same as the length of the Mega-Rule, all corresponding intermediate positions on the chart and Mega-Rule can, of course, be determined by simple proportionality, or graphically by the use of proportional dividers.

The Mega-Rule is commercially available [20].

14.9 COMPUTER-PLOTTER

A combined transmission line calculator and plotting board is shown in Fig. 14.11. The relatively large (10-in.-diameter coordinates) plotting area is matte finished to permit plotting data directly thereon which can be easily erased. Ten radial scales as shown on the Mega-Rule are provided on a radial arm with a notched cursor for drawing circles with a pencil on the coordinates. This newest and largest SMITH CHART instrument is commercially available [20].

14.10 LARGE SMITH CHARTS

Large SMITH CHARTS are available commercially in several forms, individually described below. Such charts are particularly useful for classroom instruction or for group discussions around a table where viewing distances are generally too great for the conventional size (8½ × 11 in.) charts. It is also possible to achieve somewhat greater accuracy with a larger chart.

14.10.1 Paper Charts

Large paper SMITH CHARTS are commercially available from one supplier [15] in 22½ × 35 in. pads of approximately 75 sheets each. The charts are printed on a relatively heavy paper in red ink, and have an actual coordinate diameter of 18½ in. These are direct enlargements of the form shown in Fig. 3.3, and are the most economical of the large charts which are available. Eight of the radially scaled parameters, which have been individually described herein, are printed across the bottom of each chart. These pads have a clipboard backing and may conveniently be supported on an easel, or laid flat on a table top.

14.10.2 Blackboard Charts

Large cloth SMITH CHARTS (30 × 45 in.) with a black matte finished black background

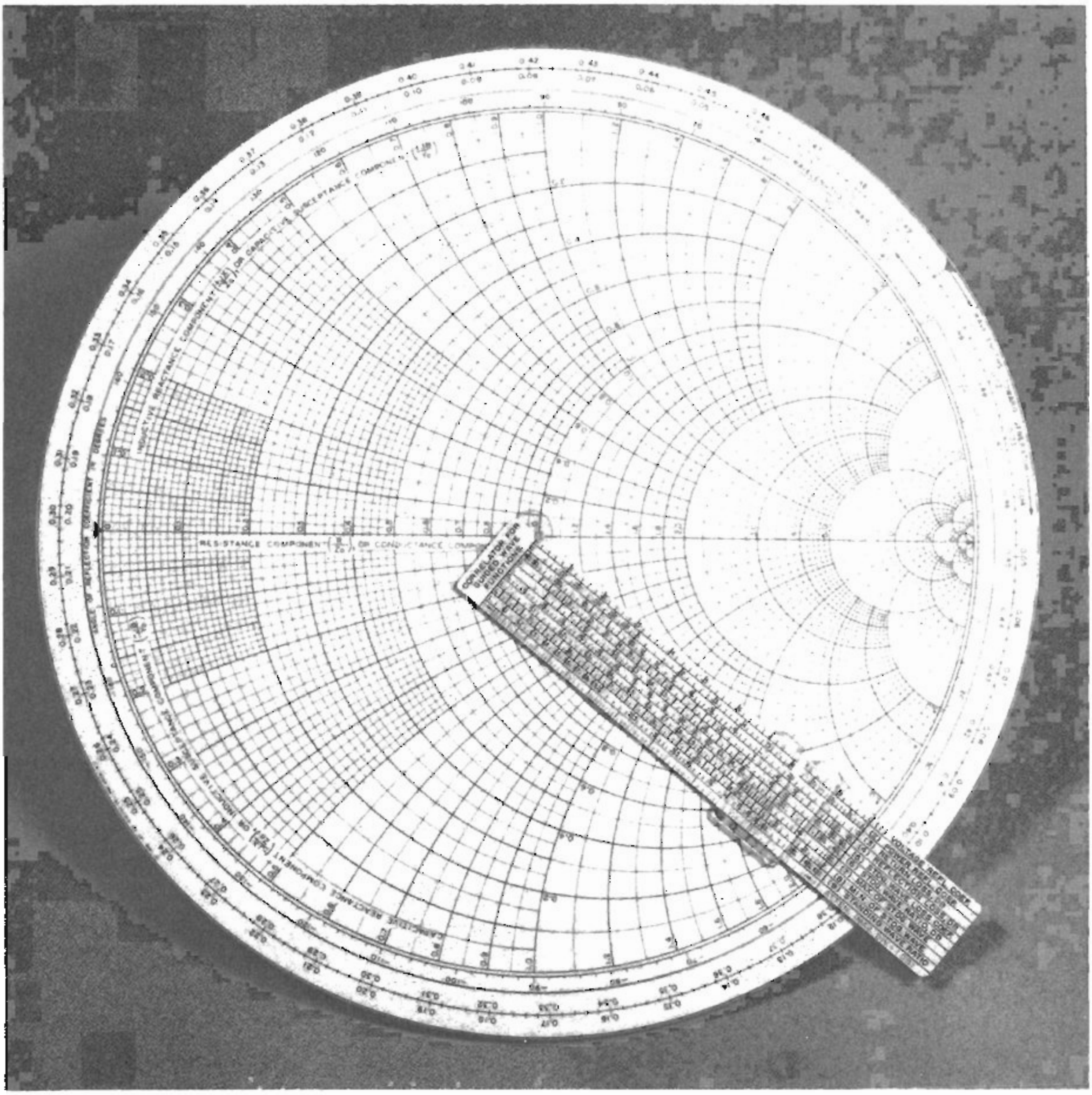


Fig. 14.11. SMITH CHART Computer-Plotter [20].

for erasable chalk marking are also commercially available [20]. (See Fig. 14.12.) These are printed with white characters and can be rolled up and down on a curtain roller. The blackboard chart is intended for basic instruction in a large classroom, and consequently designations are in a bold type and a coarse coordinate grid is employed. This chart is not suitable for accurate solution of specific problems.

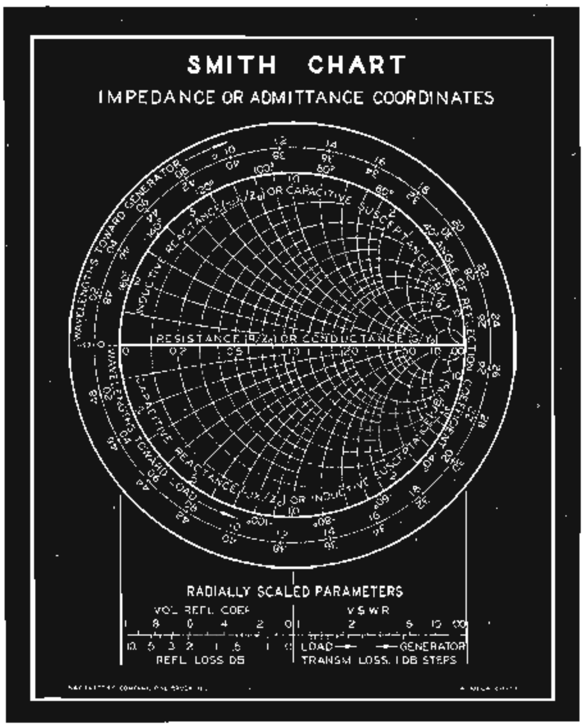


Fig. 14.12. Blackboard SMITH CHART for classroom use [20].

14.11 MEGA-CHARTS

14.11.1 Paper SMITH CHARTS

Regular size (8½ × 11 in.) SMITH CHARTS in the following several forms, each of which is described herein, are commercially available [14]. These "Mega-Chart" forms are printed in red ink on 15 lb. (approximately 7 lbs/1,000 sheets) translucent master paper, and packaged in clear plastic envelopes of 100 sheets each, either padded or loose:

1. Standard SMITH CHART–Form 82-BSPR (9-66) (see Fig. 8.6).
2. Expanded Center SMITH CHARTS–Form 82-SPR (2-49) (see Fig. 7.2).
3. Highly Expanded Center SMITH CHARTS–Form 82-ASPR (see Fig. 7.3).

Also, SMITH CHARTS with coordinates having negative real parts (negative SMITH CHARTS) are available in the same paper and packaging, printed in green ink, in the following form:

1. Negative SMITH CHART–Form 82-CSPR (see Fig. 12.3).

14.11.2 Plastic Laminated SMITH CHARTS

All of the above chart forms (except the negative SMITH CHART) are available [14] laminated to a thickness of 0.025 in., with a matte finish on the front for erasable pencil marking. Abbreviated instructions for use of SMITH CHARTS are printed on the back.

14.11.3 Instructions for SMITH CHARTS

Abbreviated sets of instructions for use of the SMITH CHART, containing an explanation of the chart coordinates and radial scales and printed on single sheets, are available commercially [14] printed on 50 lb offset paper. These may be used for classroom instruction.

Glossary — Smith Chart Terms

The terms which appear on SMITH CHARTS as coordinate designations, radially scaled parameters, peripheral scale captions, etc., are individually defined and reviewed in this glossary. A more complete discussion of these terms is found in applicable sections of the text.

Although relevant to all SMITH CHARTS, these terms are specifically associated with the basic chart forms printed in Chaps. 6, 8 and 12, and enlarged chart forms described in Chap. 7. Other SMITH CHARTS with which these terms are specifically associated include the normalized current and voltage overlay in Chap. 4, and the charts with dual (polar and rectangular) coordinate transmission and reflection coefficients in Chap. 8.

In the definitions which follow, certain qualifying words and phrases are omitted when, in the context in which the terms are used, these words and phrases will be understood to apply. For example, the phrase "at a specified frequency" will apply to many of the definitions, and the phrases "normalized input impedance of a uniform waveguide" and "normalized input admittance of a uniform waveguide" will generally be understood to be meant by the shorter terms, "waveguide impedance" and "waveguide admittance," respectively.

This glossary supplements definitions which have been formulated and published by the Institute of Radio Engineers (IRE) [11] (presently the Institute of Electrical and Electronics Engineers, IEEE), and by the American Standards Association (ASA-C42.65-1957) (presently the United States of America Standards Institute, USASI), and upon which usage of such terms in this text is based.

Angle of Reflection Coefficient, Degrees

At a specified point in a waveguide, the phase angle of the reflected voltage or current wave relative to that of the corresponding incident wave. The relative phase angle of the reflection coefficient, i.e., the total angle reduced to a value less than $\pm 180°$, is generally indicated by this term. This relative phase angle has a fixed relationship to a specific combination of waveguide impedances or admittances and, accordingly, to a specific locus on the impedance or admittance coordinates of a SMITH CHART, this locus being a radial line.

Note: The angles of both the voltage and the current reflection coefficients are represented on SMITH CHARTS by a single linear peripheral scale, with designated values ranging between 0 and ±180°. The angle of the *voltage* reflection coefficient is directly obtainable for any point on the *impedance* coordinates by projecting the point radially outward to this peripheral scale, labeled "Angle of Reflection Coefficient, Degrees." Similarly, the angle of the *current* reflection coefficient is directly obtainable for any point on the *admittance* coordinates. At any specified point along a waveguide the angle of the current reflection coefficient always lags that of the voltage reflection coefficient by 180°.

Angle of Transmission Coefficient, Degrees

At a specified point along a waveguide, the phase angle of the transmitted wave relative to that of the corresponding incident wave. The "transmitted" wave is the complex ratio of the resultant of the incident and reflected wave to the incident wave. The angle of the transmission coefficient has a fixed relationship to a specific combination of waveguide impedances or admittances and, accordingly, to a specific locus on the impedance or admittance coordinates of a SMITH CHART, this locus being a straight line stemming from the origin of the coordinates.

Note: The angles of both the voltage and the current transmission coefficients are represented on SMITH CHARTS by a single linear angle scale at the periphery, referenced to the origin of the impedance or admittance coordinates, and ranging between 0 and ±90°. The angle of the *voltage* transmission coefficient is directly obtainable for any point on the *impedance* coordinates by projecting the point along a straight line stemming from the origin of the impedance coordinates to the intersection of the peripheral scale labeled "Angle of Transmission Coefficient, Degrees." Similarly, the angle of the *current* transmission coefficient is directly obtainable for any point on the *admittance* coordinates.

Attenuation (1 dB Maj. Div.)

The losses due to dissipation of power within a waveguide and/or the radiation of power therefrom when the waveguide is match-terminated. On SMITH CHARTS attenuation is expressed as a ratio, in dB, of the relative powers in the forward-traveling waves at two separated reference points along the waveguide.

Note: On a SMITH CHART, "attenuation" is a radially scaled parameter. The attenuation scale is divided into dB (or fraction of dB) divisions which are not designated with specific values, with an arbitrarily assignable (floating) zero point. The number of attenuation scale units (dB) radially separating any two impedance or admittance points on the impedance or admittance coordinates of a SMITH CHART is a measure of the attenuation in the length of waveguide which separates the two reference points.

Coordinate Components

The normalized rectangular components of the equivalent series or parallel input impedance or admittance of a waveguide or circuit, which are represented on SMITH CHARTS by two captioned families of mutually orthogonal circular curves comprising the coordinates of the chart.

Note 1: Coordinate components on the three SMITH CHARTS printed in red on translucent sheets in the back cover envelope are:

Chart A

1. the equivalent series circuit impedance coordinates: resistance component R/Z_0 and inductive (or capacitive) reactance component $\pm jX/Z_0$.

2. the equivalent parallel circuit admittance coordinates: conductance component G/Y_0 and inductive (or capacitive) susceptance component $\mp jB/Y_0$.

Chart B

1. the equivalent parallel circuit impedance coordinates: parallel resistance component R/Z_0 and parallel inductive (or capacitive) reactance component $\pm jX/Z_0$.

2. the equivalent series circuit admittance coordinates: series conductance components G/Y_0 and series inductive (or capacitive) component $\mp jB/Y_0$.

Chart C

1. the equivalent series circuit impedance or shunt circuit admittance coordinates *with negative real parts:* negative resistance component $-R/Z_0$ and negative conductance component $-G/Y_0$.

Note 2: The inductive reactance and inductive susceptance coordinate components represent equivalent primary circuit elements which are capable of storing magnetic field energy only. The resistance component and the conductance component of the coordinates represent equivalent primary circuit elements which are capable of dissipating electromagnetic field energy. The negative resistance component and the negative conductance component of the coordinates represent equivalent circuit elements which are capable of releasing electromagnetic field energy, as would be represented by the equivalent circuit of a generator.

Impedance or Admittance Coordinates

The families of orthogonal circular curves representing the real and imaginary components of the waveguide or circuit impedance and/or admittance, and comprising the main body of a SMITH CHART. The designated values of the curves are normalized with respect to the characteristic impedance and/or the characteristic admittance of the waveguide, and the entire range of possible values lies within a circle. Enlarged portions of SMITH CHART coordinates are sometimes used to represent or display a portion of the total area of the coordinate system, thereby providing improved accuracy or readability.

Note: Most commonly, SMITH CHART impedance or admittance coordinates express components of the equivalent series circuit impedance or parallel circuit admittance. However, a modified form of SMITH CHART expresses components of the equivalent parallel circuit impedance or series circuit admittance. A coordinate characteristic which is common to all SMITH CHARTS is that a complex impedance point on the impedance coordinates and a complex admittance point on the admittance coordinates which is diametrically opposite, and at equal chart radius, always represent equivalent circuits.

Negative Real Parts

On the SMITH CHART form in Fig. 12.5, a designation of the sign of the normalized resistance component of the impedance, or the normalized conductance component of the admittance coordinates.

Note 1: See "Coordinate Components (Chart C)."

Note 2: A SMITH CHART whose impedance or admittance coordinates are designated with negative real parts is useful in portraying conditions along a waveguide only when the returned power is greater than the incident power.

Normalized Current $i/\sqrt{P/Z_0}$ or $i/\sqrt{PY_0}$

The rms current which would exist at a specified point along a hypothetical waveguide having a characteristic impedance of one ohm (or a characteristic admittance of one mho) and transmitting one watt of power to a load. This current is the vector sum of the incident and reflected currents at the point.

Note 1: The actual current at any specified power level in a waveguide is obtainable from the normalized current by multiplying it by the square root of the ratio of the power and characteristic impedance (or by the square root of the product of the power and characteristic admittance).

Note 2: A plot of normalized current and/or normalized voltage is provided as an overlay for SMITH CHART impedance or admittance coordinates in Fig. 4.2.

Normalized Voltage $e/\sqrt{PZ_0}$ or $e/\sqrt{P/Y_0}$

The rms voltage which would exist at a specified point along a hypothetical waveguide having a characteristic impedance of one ohm (or a characteristic admittance of one mho) and transmitting one watt of power to a load. This voltage is the vector sum of the incident and reflected voltages at the point.

Note 1: The actual voltage at any specified power level in a waveguide is obtainable from the normalized voltage by multiplying it by the square root of the product of the power and characteristic impedance (or by the square root of the ratio of the power and characteristic admittance).

Note 2: See Note 2 in definition for normalized current.

Percent Off Midband Frequency $n \cdot \Delta f$

Captions for peripheral scales near the pole regions on expanded SMITH CHARTS, which relate specific values of the frequency deviation, from the resonant or antiresonant frequency, to the impedance or admittance characteristics of open- and short-circuited stub transmission lines n quarter wavelengths long. Δf is the deviation from the midband frequency in percent.

Peripheral Scales

The four scales encircling the impedance or admittance coordinates of the SMITH CHART, individual graduations on each of which are applicable to a straight line locus of points on the impedance or admittance coordinates.

Note: Each graduation on each of the three outermost of these scales is applicable to all points on the impedance or admittance coordinates which are radially aligned therewith; each graduation on the innermost of these is applicable to all points on the coordinates which are in line with the graduations and the point of origin of the impedance or admittance coordinates.

Radially Scaled Parameters

A set of guided wave parameters represented by a corresponding number of scales whose overall lengths equal the radius of a SMITH CHART, and which are used to measure the radial distance between the center and the perimeter of the impedance or admittance coordinates, at which point a specific value of the parameter exists.

Note 1: Radially scaled parameter values are mutually related to each other as well as to a circular locus of normalized impedances or admittances centered on these coordinates (see Chap. 14, Par. 14.8).

Note 2: The use of radial scales to represent radially scaled parameter values avoids the need to superimpose families of concentric circles on the impedance or admittance coordinates which (if all parameters were thus represented) would completely obscure the coordinates.

Reflection Coefficients *E* **or** *I*

At a specified point in a waveguide, the ratio of the amplitudes of the reflected and incident voltage or current waves. If the waveguide is lossless the magnitude of the "Reflection Coefficients *E* or *I*" is independent of the reference position. If it is lossy the magnitude will diminish as the reference position is moved toward the generator.

Note 1: At any specified reference position along any uniform waveguide the magnitude of the voltage reflection coefficient is equal to that of the current reflection coefficient.

Note 2: On a SMITH CHART the "Reflection Coefficients *E* or *I*" is a radially scaled parameter.

Reflection Coefficient *P*

At a specified point in a waveguide the ratio of reflected to incident power.

Note 1: In a uniform lossless waveguide the "Reflection Coefficient *P*" is independent of the reference position.

Note 2: When expressed in dB the "Power Reflection Coefficient *P*" is equivalent to the "Return Loss, dB."

Note 3: On a SMITH CHART the "Reflection Coefficient, *P*" is a radially scaled parameter.

Reflection Coefficient, *X* **or** *Y* **Component**

In a waveguide, the in-phase or quadrature-phase rectangular component, respectively, of the "Reflection Coefficients *E* or *I*" represented on a SMITH CHART as a rectangular-coordinate overlay. (See Chap. 8.)

Reflection Loss, dB

A nondissipative loss introduced at a discontinuity along a uniform waveguide, such as at a mismatched termination. "Reflection Loss, dB" can be expressed as a ratio, in dB, of the reflected to the absorbed power at the discontinuity and/or at all other points along a uniform waveguide toward the generator therefrom.

Note 1: If the input impedance of a lossless waveguide is matched to the internal impedance of the generator, a compensating gain will occur at the generator end of the waveguide. Any difference between the "Reflection Loss, dB" at each end of a waveguide corresponds to the increase in attenuation in a waveguide due to reflected power from the load.

Note 2: On a SMITH CHART "Reflection Loss, dB" is a radially scaled parameter.

Return Gain, dB

In a waveguide terminated in an impedance or admittance with a negative real part, the ratio in dB of the power in the reflected and incident waves.

Note 1: On a SMITH CHART whose impedance or admittance coordinates are designated with negative real parts this is a radially scaled parameter.

Return Loss, dB

In a waveguide, the ratio in dB of the power in the incident and reflected waves. The term "Return Loss, dB" is synonymous with "Power Reflection Coefficient" when the latter is expressed in dB.

Note: On a SMITH CHART this is a radially scaled parameter.

SMITH CHART

A circular reflection chart composed of two families of mutually orthogonal circular coordinate curves representing rectangular components of impedance or admittance, normalized with respect to the characteristic impedance and/or characteristic admittance of a waveguide. Peripheral scales completely surrounding the coordinates include a set of linear waveguide position and phase angle reference scales. The SMITH CHART also includes a set of radial scales representing mutually related radially scaled parameters.

Note: The SMITH CHART is commonly used for the graphical representation and analysis of the electrical properties of waveguides or circuits [25].

Standing Wave Loss Coefficient (Factor)

The ratio of combined dissipation and radiation losses in a waveguide when mismatch-terminated and when match-terminated.

Note 1: A specific value of this coefficient applies to the transmission losses integrated over plus or minus one-half wavelengths from the point of observation, as compared to the attenuation in the same length of waveguide. Thus, spatially repetitive variations in transmission loss within each standing half wavelength are smoothed.

Note 2: On a SMITH CHART this is a radially scaled parameter.

Standing Wave Peak, Const. *P*

The ratio of the maximum amplitude of the standing voltage or current wave along a mismatch-terminated waveguide to the amplitude of the corresponding wave along a match-terminated waveguide when conducting the same power to the load.

Note: On a SMITH CHART this is a radially scaled parameter.

Standing Wave Ratio (dBS)

In a waveguide, twenty times the logarithm to the base 10 of the standing wave ratio (S).

Note: On a SMITH CHART this is a radially scaled parameter.

Standing Wave Ratio (SWR)

The ratio of the maximum to the minimum amplitudes of the voltage (or current) along a waveguide.

Note 1: For a given termination, and in a given region along a waveguide the SWR is identical for voltage or current. If the waveguide is lossy the SWR will diminish as the point of observation is moved toward the generator.

Note 2: On a SMITH CHART the SWR is a radially scaled parameter.

Transmission Coefficient *E* or *I*

At a specified point along a waveguide the ratio of the amplitude of the transmitted voltage (or current) wave to the amplitude of the corresponding incident wave.

Note 1: The "transmitted voltage (or current) wave" is the complex resultant of the incident and reflected voltage (or current) wave at the point.

Note 2: On the SMITH CHART the "Transmission Coefficient *E* or *I*" is a linear scale equal in length to the diameter of the impedance or admittance coordinates and pivoted from their origin. As so plotted, this scale applies to voltage in relation to impedance coordinates or to current in relation to admittance coordinates.

Transmission Coefficient *P*

In a waveguide, the ratio of the transmitted to the incident power. The "Transmission Coefficient *P*" is equal to unity minus the "Reflection Coefficient *P*."

Note: On a SMITH CHART the "transmission Coefficient *P*" is a radially scaled parameter.

Transmission Coefficient, *X* and *Y* Components

In a waveguide, the in-phase and quadrature-phase components of the "Transmission Coefficient *E* or *I*" represented on a SMITH CHART as a rectangular coordinate overlay. (See Chap. 8.)

Transmission Loss Coefficient

In a waveguide, the "Standing Wave Loss Coefficient (Factor)."

Note: On SMITH CHARTS this is a radially scaled parameter.

Transmission Loss, 1-dB Steps

A term used on SMITH CHARTS to indicate the total losses due to dissipation of power within a waveguide and/or the radiation of power therefrom when the waveguide is match-terminated. "Transmission Loss" is expressed as a ratio in dB of the relative powers in the forward-traveling waves at two separated reference points along the waveguide.

Note: As above defined and used on earlier SMITH CHARTS, the term is synonymous with the term "Attenuation (1 dB Maj. Div.)" which is used on more recent SMITH CHARTS. The change in designation was made to avoid possible misunderstanding of the foregoing SMITH CHART usage of the term which in other usage frequently means the total losses when the waveguide is mismatch-terminated. (See "Standing Wave Loss Coefficient (Factor).")

Wavelengths Toward Generator (or Toward Load)

In a waveguide, the relative distances and directions between any two reference points, represented on SMITH CHARTS by the two outermost peripheral scales expressing electrical lengths in wavelengths from an arbitrarily selected radial reference locus on the impedance or admittance coordinates.

Note: On SMITH CHARTS with fixed scales, the zero points of these scales are arbitrarily referenced to the position of a voltage standing wave minimum on the impedance coordinates and/or a current standing wave minimum or admittance coordinates. On a SMITH CHART instrument, in which these scales are rotatable with respect to the impedance or admittance coordinates, the zero point may be aligned with any other reference position on the coordinates.

APPENDIX A

Transmission Line Formulas

The mathematical relationships of the various parameters involved in guided wave propagation are basic to the construction of the SMITH CHART, or any other chart which portrays these relationships. Accordingly, the applicable transmission line equations are included herein for those who may desire this background information without the necessity of referring to other sources. Also, these relationships serve to provide more exact solutions to specific problems in cases where graphical solutions cannot provide the desired accuracy or when, for example, a computer is available.

In these formulas the sending-end impedance Z_s is the input impedance of a transmission line looking toward the load. Except for its mismatch effect on the transfer of power from generator to line (and/or to load), the value of Z_s is completely independent of the impedance looking toward the generator from this position, and is thus the impedance which would be seen if the line (or other connections to the generator) were cut off at this point. Z_s is thus the impedance at any specified point along the line.

There are an infinite number of input or sending-end impedances along any finite length of transmission line, and the position of the specified sending-end impedance with respect to the receiving-end or load impedance Z_r must be known in order to evaluate it. The receiving-end impedance, like the sending-end impedance, can be referenced to any position along the line provided that it is on the load side of the specified input impedance position. The length l is the distance between Z_s and Z_r, and must be expressed in the same units as the wavelength λ, for example, meters.

E_s and I_s are the voltage and current, respectively, at the position corresponding to Z_s, and represent root-mean-square values of the complex sinusoid at this point. Similarly, E_r and I_r are the voltage and current, respectively, at the position assigned to Z_r. P is the propagation constant and Z_0 is the characteristic impedance, as defined in Chap. 2.

A computer program for plotting impedance or admittance data on a SMITH CHART has been written [146,150] in FORTRAN IV to which the reader who may be interested is referred. An example is given in the referenced article.

The formulas given herein are extracted, with some editing from J. A. Flemming [2] and other sources.

1. General Relationships for Any Finite-length Transmission Line

$$E_s = E_r \cosh Pl + I_r Z_0 \sinh Pl \tag{A-1}$$

$$I_s = I_r \cosh Pl + \frac{E_r}{Z_0} \sinh Pl \tag{A-2}$$

and

$$Z_s = \frac{E_s}{I_s} = \frac{E_r \cosh Pl + I_r Z_0 \sinh Pl}{I_r \cosh Pl + (E_r/Z_0) \sinh Pl} \tag{A-3}$$

$$= \frac{Z_0 \cosh Pl + (Z_0{}^2/Z_r) \sinh Pl}{(Z_0/Z_r) \cosh Pl + \sinh Pl} \tag{A-4}$$

Two special cases of finite line lengths are presented by lines which are terminated in (a) open circuits and (b) short circuits, viz.,

(a) Open-circuited lines

If the receiving end of any uniform transmission line is open circuited, $I_r = 0$, and

$$E_s = E_r \cosh Pl \tag{A-5}$$

$$I_s = \frac{E_r}{Z_0} \sinh Pl \tag{A-6}$$

and

$$Z_s = \frac{E_s}{I_s} = Z_0 \coth Pl \tag{A-7}$$

(b) Short-circuited lines

If the receiving end of any uniform transmission line is short-circuited, $E_r = 0$, and

$$E_s = I_r Z_0 \sinh Pl \tag{A-8}$$

$$I_s = I_r \cosh Pl \tag{A-9}$$

and

$$Z_s = \frac{E_s}{I_s} = Z_0 \tanh Pl \tag{A-10}$$

2. Relationships for any Finite-length Lossless Transmission Line

The following equations for lossless lines are obtained from Eqs. (A-1) through (A-10) by setting the attenuation constant α equal to zero, in which case $P = \alpha + j\beta = +j\beta$ (see Eq. (2-7)), and by employing the following relationships between hyperbolic and circular trigonometric functions:

$$\sinh j\beta = j \sin\beta$$

$$\cosh j\beta = \cos\beta$$

and

$$\tanh j\beta = j \tan\beta$$

(a) Lines terminated in an impedance

If the receiving end of any uniform lossless transmission line is terminated in an impedance Z_r

$$E_s = E_r \cos\beta l + jZ_0 I_r \sin\beta l \tag{A-11}$$

$$I_s = I_r \cos\beta l + j\frac{E_r}{Z_0} \sin\beta l \tag{A-12}$$

and

$$Z_s = \frac{E_s}{I_s} = \frac{E_r \cos\beta l + jZ_0 I_r \sin\beta l}{I_r \cos\beta l + j(E_r/Z_0)\sin\beta l} \quad \text{(A-13)}$$

or

$$Z_s = Z_0 \frac{Z_r + jZ_0 \tan\beta l}{Z_0 + jZ_r \tan\beta l} \quad \text{(A-14)}$$

Rationalizing Eq. (A-14)

$$\frac{Z_s}{Z_0} = \frac{(Z_r/Z_0)(1 + \tan^2\beta l)}{1 + (Z_r^2/Z_0^2)\tan^2\beta l} + j\frac{(1 - Z_r^2/Z_0^2)\tan^2\beta l}{1 + Z_r^2/Z_0^2 \tan^2\beta l} \quad \text{(A-15)}$$

By substituting real values for Z_r/Z_0 in Eq. (A-15), the loci of the normalized input impedance components along a transmission line, as represented graphically along a circle of constant standing wave ratio on a SMITH CHART, can readily be obtained.

(b) Open-circuited lines

If the receiving end of any uniform lossless transmission line is open circuited, $I_r = 0$, and

$$E_s = \frac{E_r}{\sec\beta l} \quad \text{(A-16)}$$

$$I_s = j\frac{E_r}{Z_0}\sin\beta l \quad \text{(A-17)}$$

and

$$Z_s = \frac{E_s}{I_s} = -jZ_0 \cot\beta l \quad \text{(A-18)}$$

or

$$Y_s = \frac{I_s}{E_s} = jY_0 \tan\beta l \quad \text{(A-19)}$$

where Y_s and Y_0 are, respectively, the sending-end admittance and the characteristic admittance of the line.

From Eq. (A-16) it can be seen that in lossless open-circuited lines the receiving-end voltage E_r varies between the sending-end voltage E_s and infinity as the length of line is varied. The receiving-end voltage is equal to the sending-end voltage when l is $1/2\lambda$, $3/2\lambda$, etc., and is infinite when l is $1/4\lambda$, $3/4\lambda$, $5/4\lambda$, etc.

(c) Short-circuited lines

If the receiving end of any uniform lossless transmission line is short-circuited, $E_r = 0$, and

$$E_s = jZ_0 I_r \sin\beta l \quad \text{(A-20)}$$

$$I_s = I_r \cos\beta l \quad \text{(A-21)}$$

and

$$Z_s = \frac{E_s}{I_s} = jZ_0 \tan\beta l \quad \text{(A-22)}$$

or

$$Y_s = \frac{I_s}{E_s} = -jY_0 \cot\beta l \quad \text{(A-23)}$$

Since the tangent and the cotangent of a given angle always have the same sign, it is possible to change the input impedance of a line which is capacitive when open circuited at its receiving end into an inductance by short-circuiting its receiving end. It is also possible to change an inductive open-circuited line into a capacitive line by short-circuiting its receiving end.

APPENDIX B

Coordinate Transformation

BILINEAR TRANSFORMATION

As will be shown herein, a conformal transformation can be applied to the curves on the rectangular transmission line chart in Fig. I.2 in order to obtain the more convenient circular form shown in Fig. I.3. When the latter figure is rotated 90° counterclockwise from the orientation shown (see Sec. I.4), the transformation whose general form is

$$w = \frac{aZ + b}{cZ + d} \tag{B-1}$$

will be found to give the desired result. By assigning the proper values to the constants a, b, c, and d, the axis of X/Z_0 may be transformed into a circle of any convenient radius, and the entire chart will then lie within this circle. Each of the circles corresponding to a particular value of D will become a diameter of the new boundary circle and all of the other circles or straight lines in the rectangular chart will become circles or arcs of circles in the circular chart.

In order to perform such a transformation let each point on the rectangular chart be denoted by a complex number

$$w = u + jv$$

where

$$u = \frac{X}{Z_0}$$

and

$$v = \frac{R}{Z_0}$$

Similarly let each point on the circular chart be denoted by

$$Z = x + jy$$

Then the following conditions may be set up: The u axis is to be transformed into a circle of radius A whose center lies on the y axis a distance A above the origin. At the same time, the point ($u = 0$, $v = 1$) is to be transformed into the center of this circle. These conditions fix the transformation of the following points: when

$$\begin{aligned} w &= 0 + j0 & Z &= 0 + j0 \\ w &= 0 + j1.0 & Z &= 0 + jA \\ w &= +\infty + j0 & Z &= 0 + j2A \end{aligned} \tag{B-2}$$

Substituting in Eq. (B-1), the transformation becomes

$$w = \frac{-jZ}{Z - j2A} \tag{B-3}$$

This may also be written

$$Z = \frac{j2Aw}{w + j} \tag{B-4}$$

Either one of these equations makes it possible to determine the location of the point in one chart which corresponds to any given point in the other. However, it would be more convenient to have the equations of the curves in the circular chart which correspond to the circles and the coordinate system in the rectangular chart. These equations will therefore be derived.

a. The Lines v = a Constant

The lines v = a constant comprise those straight lines parallel to the u axis in the original chart. To find what these lines become in the circular chart it will be necessary to substitute for w and Z in Eq. (B-3) the complex expression $(u + iv)$ and $(x + iy)$. Then the reals and imaginaries may be separated to give

$$ux - vy - y + 2Av + j(uy + x + vx - 2Au) = 0$$

To satisfy this equation, the real and imaginary parts must be separately zero. Thus,

$$\begin{aligned} xu - (y - 2A)v &= y \\ (y - 2A)u + xv &= -x \end{aligned} \tag{B-5}$$

Eliminating u,

$$x^2 + y^2 - \frac{2Ay(1 + 2v)}{(1 + v)} = -4A^2 \frac{v}{(1 + v)}$$

When v is constant, this is the equation of a circle in the Z plane. By adding the quantity $(1 + 2v)^2A^2/(1 + v)^2$ to each side, the following more useful form is obtained:

$$x^2 + y - \frac{(1 + 2v)A^2}{(1 + v)} = \frac{A^2}{(1 + v)^2} \tag{B-6}$$

From Eq. (B-6) it is evident that the radius of the circle is $A/(1 + v)$ while its center is located on the y axis at a distance $[(1 + 2v)/(1 + v)]A$ from the origin. Since $v = R/Z_0$, the circles corresponding to various values of R/Z_0 can now be drawn. It will be noticed that the circle for $R/Z_0 = 0$ has radius A and center at $(x = 0, y = A)$. For higher values of R/Z_0, the radius of the circle becomes smaller and its center rises until for $R/Z_0 = \infty$ the circle is of zero radius and its center is at $(x = 0, y = 2A)$.

b. The Lines u = a Constant

The ordinates of the rectangular coordinate system correspond to various constant values of u. Consequently, the equation of the curves to which they are transformed can be obtained by eliminating v from Eqs. (B-5). When this is done, there results

$$x^2 + (y - 2A)^2 = 2\frac{A}{w}x \tag{B-7}$$

For each value of u this represents a circle of radius $2A/u$, whose center is at $(x = A/u, y = 2A)$. Thus these circles are all tangent to the y axis and their centers are all on the line $y = 2A$ which is parallel to the x axis and tangent to the circle for $v = 0$. Since $u = X/Z_0$ the circle for each value of X/Z_0 may readily be drawn, its radius and center now being determined.

c. The Circles of Constant Electrical Line Angle

Each of the circles, on the rectangular chart, corresponding to a particular value of D will transform into a diameter of the circle whose center is at ($x = 0$, $y = A$). These straight lines may be represented by

$$y = mx + A \tag{B-8}$$

where m is the slope. Substituting Eq. (B-8) in Eqs. (B-5):

$$xu - (mx - A)v = mx + A$$

$$(mx - A)u + xv = -x$$

or

$$x(u - mv - m) = A(1 - v)$$

$$x(mu + v + 1) = Au$$

Dividing,

$$(u - mv - m)u = (mu + v + 1)(1 - v)$$

or

$$(u - m)^2 + v^2 = 1 + m^2 \tag{B-9}$$

which, when m is a constant, represents a circle in the rectangular chart. The radius is $\sqrt{1 + m^2}$ and the center is on the u axis a distance m from the origin. When $u = 0$, this equation gives $v = \pm 1$, which shows that all the circles pass through the required point $(u = 0, v = +1)$.

Now the slope is the tangent of the angle with the horizontal or is the $\cot\varphi$ where φ is the angle between the y axis and the diameter in question. Thus Eq. (B-9) becomes

$$(u - \cot\varphi)^2 + v^2 = 1 + \cot^2\varphi$$

This equation is identical to the equation for the lines of force about two equal and opposite parallel line changes. Consequently φ is proportional to the parameter D and thus is directly proportional to the electrical angle of the line.

d. The Circles of Constant Standing Wave Ratio

Since the transformation is conformal, and since the circles of constant standing wave ratio r are orthogonal to those of constant D, these two systems must be orthogonal in the circular chart. But the circles of constant D turned out to be straight lines, all passing through one point. Hence the circles of constant r must transform into circles having a common center at the intersection of the set of straight lines. Also, the value of r may be determined by determining the scale of R/Z_0 which is to be plotted along the y axis. This is obtained directly from Eq. (B-4) by putting $x = 0$ and $u = 0$. Then

$$y = \frac{2Av}{v + 1}$$

or, putting $v = R/Z_0$

$$y = \frac{2A(R/Z_0)}{(R/Z_0) + 1}$$

Thus for

$$\frac{R}{Z_0} = 1 \qquad y = A$$

$$\frac{R}{Z_0} = 0 \qquad y = 0$$

$$\frac{R}{Z_0} = \infty \qquad y = 2A$$

and so on. Also, the point $X/Z_0 = 0$, $R/Z_0 = 1/2$ and becomes $y = 2/3$ and the point $X/Z_0 = 0$, $R/Z_0 = 2$ becomes $y = 4/3$.

Hence, these two points lie on a circle of radius 1/3 on the circular chart. On the rectangular chart they were also on the same circle, that for $r = 0.5$. Thus having determined the scale for R/Z_0 in the circular chart, the radii of the circles of constant r are automatically determined.

Symbols

The following is a list of symbols used throughout this book. Where more than a single definition is given for the same symbol the proper choice will be evident from its usage in the text. Voltage and current symbols, except where specifically stated, indicate the root mean square value of the alternating sinusoid.

B	Susceptance, mhos
B_p	Susceptance of an equivalent parallel circuit element
B_s	Susceptance of an equivalent series circuit element
$+B_p$	Capacitive susceptance of an equivalent parallel circuit element
$-B_p$	Inductive susceptance of an equivalent parallel circuit element
$+B_s$	Capacitive susceptance of an equivalent series circuit element
$-B_s$	Inductive susceptance of an equivalent series circuit element
B/Y_0	Normalized susceptance component of admittance
C	Capacitance, farads
D	Distance from a voltage maximum or minimum
dB	Decibel = $\log_{10}(P_1/P_2)$
dBS	Standing wave ratio in decibels = $20 \log_{10} S$
E	Voltage (rms sinusoid) in volts
E_p	Voltage at a point along a standing wave
E_r	Voltage at receiving end of a waveguide
E_s	Voltage at sending end of a waveguide
E_{max}	Voltage at a standing wave maximum
E_{min}	Voltage at a standing wave minimum
e	Potential at any distance x from the sending end of a waveguide at time t
e	Base of natural logarithms
ϵ	Base of natural logarithms
$e/(PZ_0)^{1/2}$	Normalized waveguide voltage (rms)
$e/(P/Y_0)^{1/2}$	Normalized waveguide voltage (rms)
f	Frequency, hertz

f_0	Midband frequency of a resonant (or antiresonant) circuit or waveguide
G	Conductance, mhos
G_{max}	Conductance at a voltage standing wave minimum
G_{max}	Conductance of a resonant waveguide
G_{min}	Conductance at a voltage standing wave maximum
G_{min}	Conductance of an antiresonant waveguide
G_p	Conductance of an equivalent parallel circuit element
G_s	Conductance of an equivalent series circuit element
g	Equivalent circuit conductance of an active device
g_{co}	Equivalent circuit conductance of an active device at resistance cutoff frequency
g_0	Equivalent circuit conductance of an active device at zero operating frequency
g_r	Equivalent circuit conductance of an active device at its self-resonant frequency
G/Y_0	Normalized conductance component of admittance
Hz	Cycles/second
I	Current (rms sinusoid), amperes
I_r	Current at receiving end of a waveguide
I_s	Current at sending end of a waveguide
I_{max}	Current at a standing wave maximum
I_{min}	Current at a standing wave minimum
I_p	Current at a point along a waveguide
I_s'	Current at a point one-quarter wavelength removed from the voltage along a waveguide
i	Current at any distance x from the sending end of a waveguide at time t
i	Amplitude of incident traveling wave (voltage or current)
$i/(P/Z_0)^{1/2}$	Normalized waveguide current (rms)
$i/(PY_0)^{1/2}$	Normalized waveguide current (rms)
L	Inductance, henries
L	Electrical length of a waveguide matching stub of characteristic impedance Z_0
L'	Electrical length of a waveguide matching stub of characteristic impedance Z_{os}
L_s	Electrical length of an individual waveguide slug
L_t	Total electrical length of a multiple waveguide slug transformer
l or ℓ	Electrical length of a section of waveguide
n	Number of integral quarter wavelengths in a resonant (or antiresonant) waveguide
P	Complex propagation constant of a waveguide $\alpha + j\beta$
P	Power, watts
P_1	Relative power, watts
P_2	Reference power, watts
P_g	In a pair of voltage (or current) sampling probes along a waveguide, the probe nearest the generator

P_ℓ	In a pair of voltage (or current) sampling probes along a waveguide, the probe nearest the load
Q	Ratio of midband frequency and half-power bandwidth
R	Resistance, ohms
R	Resultant amplitude of incident and reflected traveling waves (voltage or current)
R_l or R_ℓ	Resistive component of load impedance
R_{max}	Resistance at a voltage standing wave maximum
R_{max}	Resistance of an antiresonant waveguide
R_{min}	Resistance at a voltage standing wave minimum
R_{min}	Resistance of a resonant waveguide
R_0	Resistive component of characteristic impedance
R_p	Resistance of an equivalent parallel circuit element
R_r	Resistive component of receiving-end impedance
R_s	Resistance of an equivalent series circuit element
R/Z_0	Normalized resistance component of impedance
r	Amplitude of reflected traveling wave (voltage or current)
r	Standing wave ratio when less than unity
r_s	Series resistance of small-signal equivalent circuit of an active device
S	Electrical spacing between two voltage (or current) sampling probes along a waveguide
S	Standing wave ratio (voltage or current)
SWR	Standing wave ratio (voltage or current)
SWR_{max}	Maximum value of the standing wave ratio
t	Time, seconds
V	Voltage (rms of sinusoid)
V	DC bias voltage on an active device
V_1	Relative voltage
V_2	Reference voltage
V_{max}	Maximum voltage along a waveguide
V_{min}	Minimum voltage along a waveguide
VSWR	Voltage standing wave ratio
v	Phase velocity of propagation in a uniform waveguide, meters/second
W	Power, watts
X	Reactance, ohms
X	In-phase component of angle of power factor
X	In-phase component of complex voltage (or current) transmission or reflection coefficient
X	Input reactance of waveguide stub of characteristic impedance Z_0
X'	Input reactance of waveguide stub of characteristic impedance Z_{os}
X_C	Capacitive reactance, ohms
X_L	Inductive reactance, ohms
X_l or X_ℓ	Reactive component of load impedance
X_0	Reactive component of characteristic impedance

X_p	Reactive component of an equivalent parallel circuit
$+X_p$	Inductive reactance of an equivalent parallel circuit element
$-X_p$	Capacitive reactance of an equivalent parallel circuit element
X_r	Reactive component of receiving-end impedance
X_s	Reactive component of sending-end impedance
X_s	Reactance of an equivalent series circuit element
$+X_s$	Inductive reactance of an equivalent series circuit element
$-X_s$	Capacitive reactance of an equivalent series circuit element
X/Z_0	Normalized reactance component of impedance
x	Distance along a waveguide from the sending end
Y	Admittance, mhos
Y	Quadrature-phase component of complex voltage (or current) transmission or reflection coefficient
Y	Quadrature-phase component of angle of power factor
Y_l or Y_ℓ	Load admittance of waveguide
Y_{max}	Maximum admittance along a waveguide
Y_{min}	Minimum admittance along a waveguide
Y_p	Admittance of an equivalent parallel circuit
Y_s	Admittance of an equivalent series circuit
Y_0	Characteristic admittance of a uniform waveguide, mhos
Y/Y_0	Normalized admittance of a uniform waveguide
Z	Impedance, ohms
Z_{11}	Image impedance at the input port of a two-port symmetrical passive network
Z_{12}	Image impedance at the output port of a two-port symmetrical passive network
Z_l or Z_ℓ	Load impedance of waveguide
Z_{max}	Maximum impedance along a waveguide
Z_{min}	Minimum impedance along a waveguide
Z_0	Characteristic impedance of a uniform waveguide, ohms
Z_{oc}	Input impedance of a two-port symmetrical passive network with output port open-circuited
Z_{os}	Characteristic impedance of a waveguide stub
Z_p	Impedance of an equivalent parallel circuit
Z_r	Receiving-end impedance of a waveguide
Z_s	Sending-end impedance of a waveguide
Z_{sc}	Input impedance of a two-port symmetrical passive network with output port short-circuited
Z_t	Characteristic impedance of a transforming section of waveguide (slug)
Z/Z_0	Normalized impedance of a uniform waveguide

α Attenuation constant of a waveguide, nepers/unit length

α Angle of voltage (or current) reflection coefficient

β Phase constant of a waveguide, radians/unit length

β Angle of voltage (or current) transmission coefficient

ϵ Dielectric constant, relative to air

λ Wavelength, units of electrical length

ϕ Phase of voltage (or current) at a point along a standing wave relative to that at nearest minimum

φ_E Voltage insertion phase

φ_I Current insertion phase

ρ Voltage (or current) reflection coefficient

ρ_E Voltage reflection coefficient

ρ_I Voltage reflection coefficient vector

ρ_X In-phase component of voltage (or current) reflection coefficient

ρ_Y Quadrature-phase component of voltage (or current) reflection coefficient

τ Voltage (or current) transmission coefficient

τ_I Current transmission coefficient

τ_E Voltage transmission coefficient

τ_X In-phase component of voltage (or current) transmission coefficient

τ_Y Quadrature-phase component of voltage (or current) transmission coefficient

θ Image transfer constant in a two-port passive symmetrical network

θ Angle of the impedance vector (angle of the power factor)

ω 2π times the frequency f

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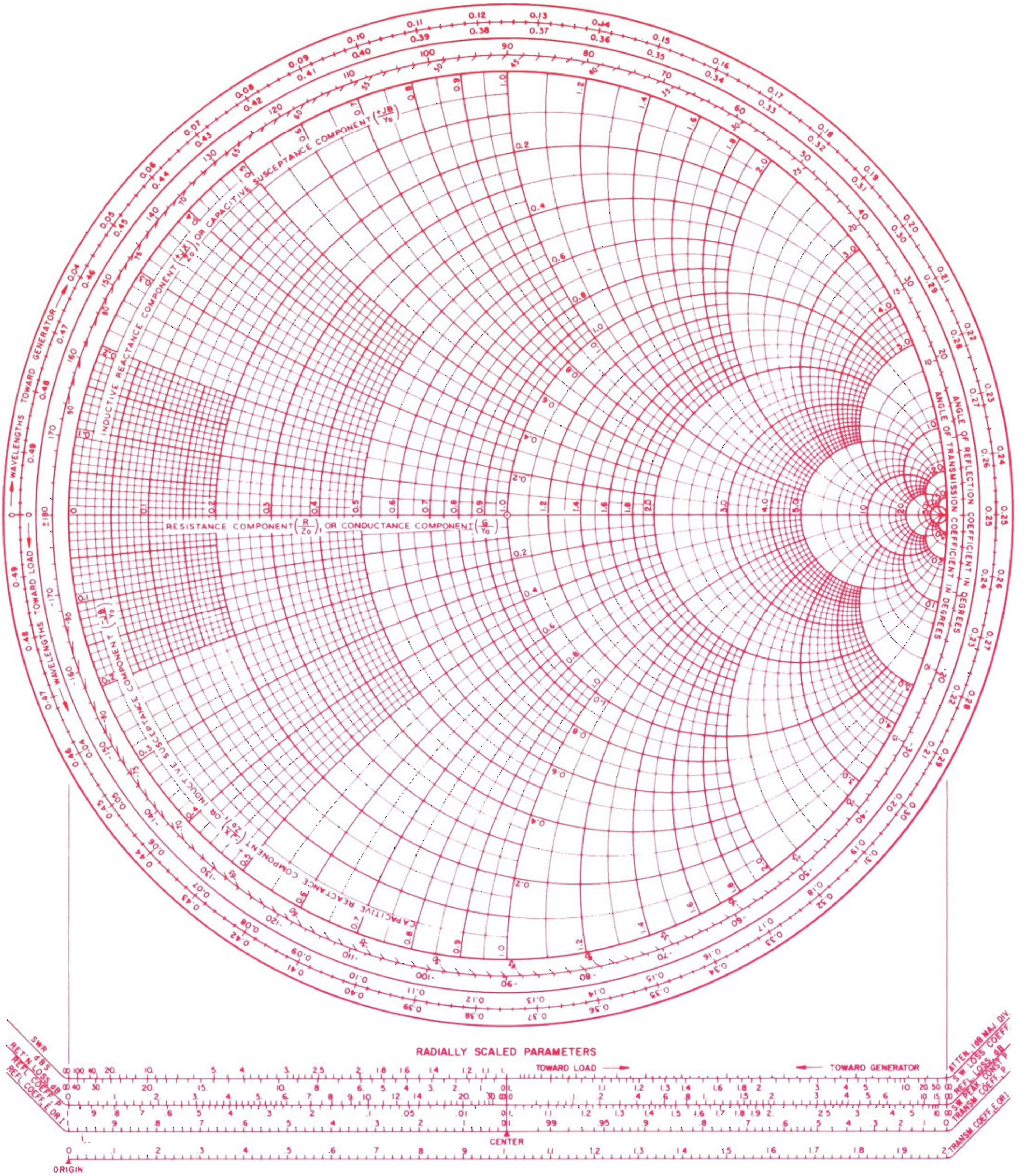

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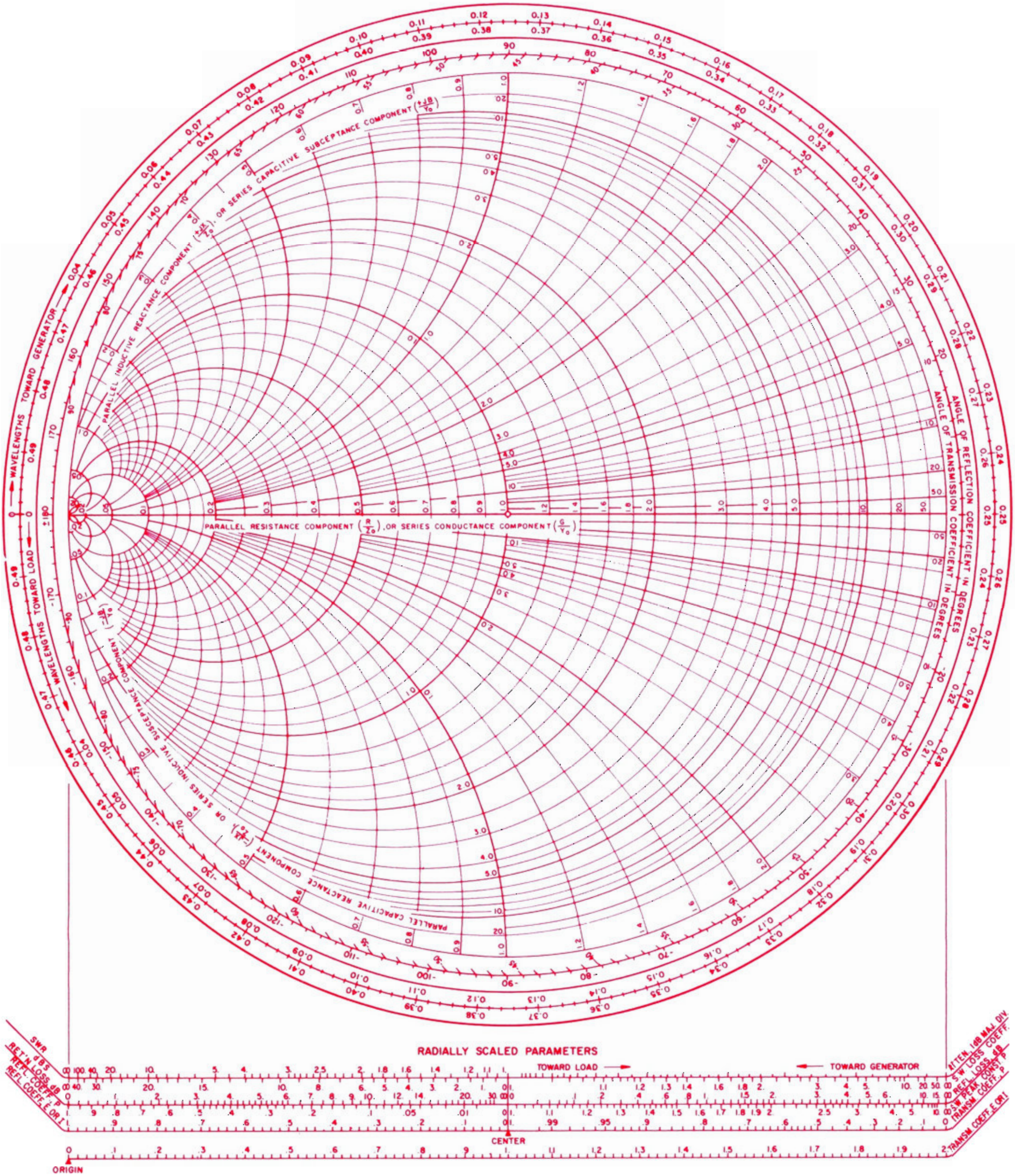

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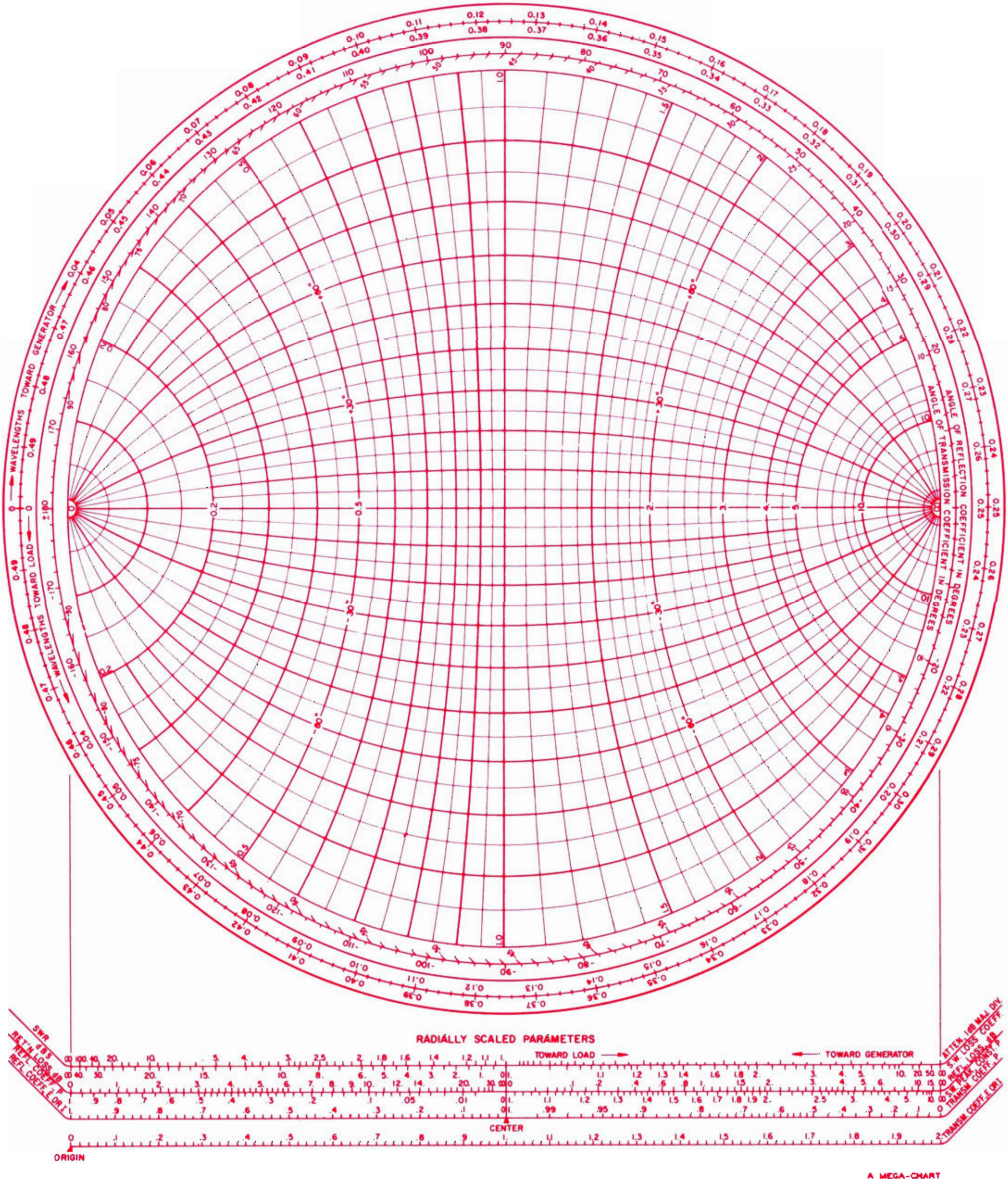

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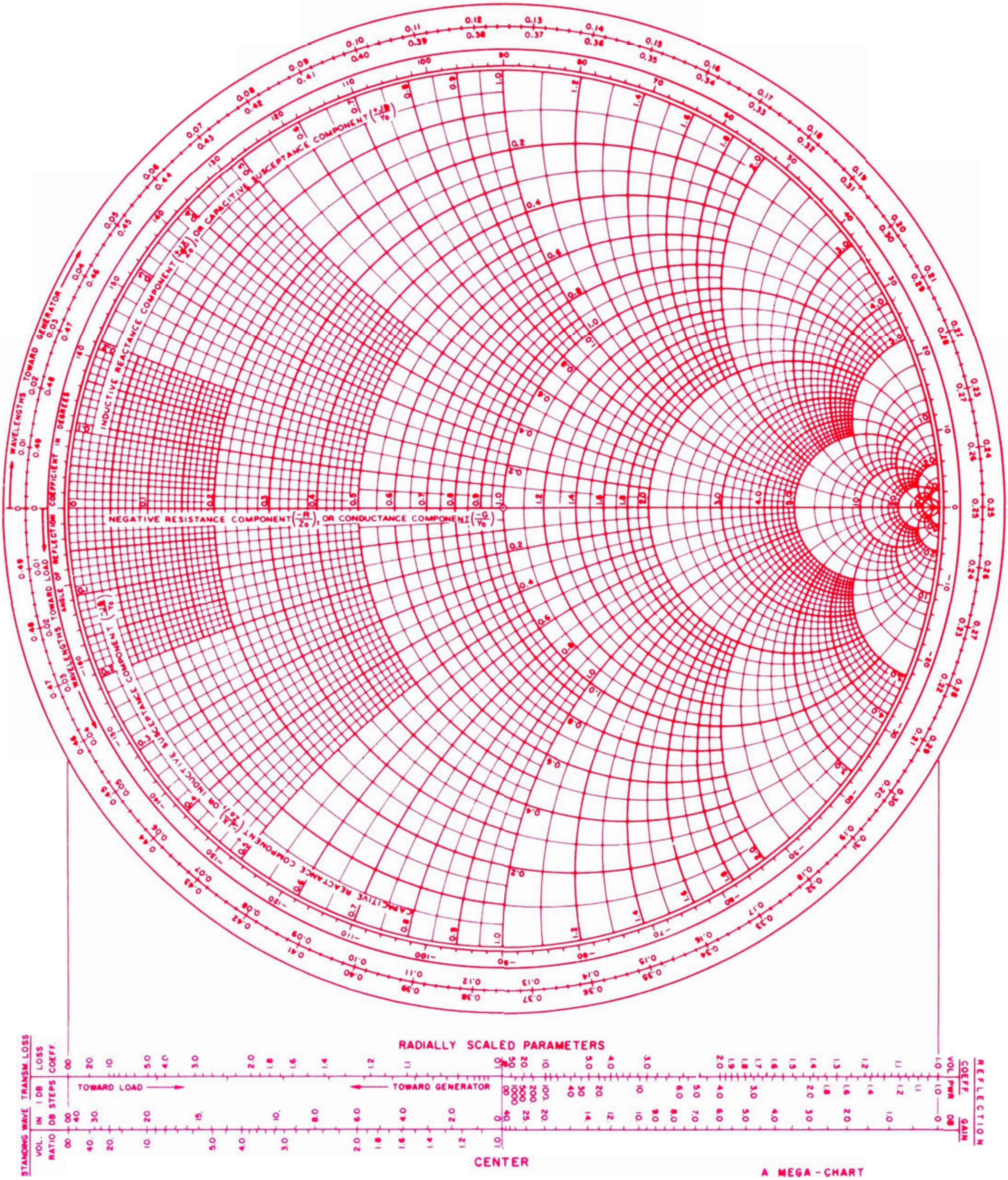

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