## Let's build a paper calculator



To practice traditional calculation systems, a printer, scissors, glue and ruler are enough: you will instantly have a precision instrument.
To be printed in A4 format and with English instructions:

- the Lalanne's Abaque.
- metric and logarithmic nomograms;
- a paper slide rule;
- the "Easy E6-B", a full working flight computer;
- a logarithmic nautical calculator.

These calculators are great for teaching and practical even on real occasions: they can fall from the tenth floor without breaking!


## Before the nomography

In 1844 Léon Lalanne created the first logarithmic graph table, calling it 'Abaque Compteur Universelle': The product of two numbers $x$ and $y$ is found from their intersection with the $45^{\circ}$ lines.

The Abaque was adopted by the French Railways and distributed in various versions, each designed to solve specific problems. It was indispensable for the construction of bridges, which no-one today would know how to design without 8 decimal digits at hand; in its time it was widespread but today it is very rare

This interesting system had little success and was abandoned for the easier and more practical nomography.

ABAOUE of COMPTEUR DNIVERSEL,


One of the few surviving copies of the Abaque Compteur. How many would be able today to design a railway bridge with no other help?

## The Abaque Compteur of Léon Lalanne



Lalanne's Abaque allows very quick operation at the expense of a small loss of precision.

To perform $3.5 \times 4.5$ just search for the two factors on the lateral scales, look for their intersection on the diagonal and read the result. In this case the intersection is close to 16 and we can evaluate the result in ca. 15.7-8. The exact result is 15.75, within the accuracy range of $2 \%$ which was considered acceptable.

To perform 35/8 go on the diagonal value of 35 and seek for the intersection with the horizontal line of value 8: this point is close to the vertical line 4.5, and we can read a value between 4.3 \& 4.4. The exact result is 4.375, again an error of less than $2 \%$.

This is a simplified graphic, the original Abaque also allows the user to raise numbers to powers and to extract roots.

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## The Metric Nomogram

Before electronic pocket calculators this system, called nomography, was used in several fields. To add just search and connect with a ruler the two addends $A$ and $B$ in the outer scales and read the result in the central scale, to subtract reverse the process.


## The Logarithmic Nomogram

The nomogram can also operate as a modern scientific calculator: to multiply search and connect with a ruler the two factors $A$ and $B$ in the outer scales and read the result in the central scale, to divide reverse the process. In the $A^{2}$ and $B^{3}$ scales you can also square and cube a number (or do roots).


## A $A^{2}$


$A \times B$

$B^{3} B$

## Nomography: examples to practice

## Multiplication

Example: $2.3 \times 3.4$.

- connect with a ruler 2.3 of the A scale with 3.4 of the B scale;
- read the answer (ca. 7.81) in the $\mathbf{A} \times \mathbf{B}$ scale. The correct answer is 7.82 .



## Division

## Example: 4.5/7.8.

- connect with a ruler 4.5 of the $\mathbf{A} \times \mathbf{B}$ scale with 7.8 of the $\mathbf{B}$ scale;
- read the answer (ca. 5.76) on the A scale. We know that the result of $4 / 8$ is near 0.5 , so we adjust the decimal place to get 0.576 . The correct answer is 0.576 .


Let's do now an example of a division where the numerator is a square root: $\sqrt{ } 350 / 1,51$ :

- to the left of 3.5 of the $\mathbf{A}^{2}$ scale we find on the $\mathbf{A}$ scale the square root of 350 : 18.7;

- now we connect 18.7 on the $\mathbf{A} \times \mathbf{B}$ scale with 1.51 of the $\mathbf{A}$ scale: on the $\mathbf{B}$ scale we can read the answer: ca. 12.39. A calculator would have been just a little more precise, finding 12.3896.



## The paper slide rule

See how calculations were done before the electronic calculators, find an important piece of engineering history with this paper slide rule, ideal to learn without getting confused between many scales. Cut along the solid lines, fold along the dotted lines and put the slide inside the body. As cursor use a clip of ca. 5 cm .


Body


## Slide

 The simplified paper slide rule - www.nicolamarras.it


## The Paper Slide Rule: examples to practice

In the slide rules the scales are indicated by letters: the two most important are on the slide ( C ) and on the body ( D ). The others are used to simplify the calculations when you are in the presence of square roots ( $A$ and $B$ ), cubes and cube roots ( $K$ ), exponential (LL), etc. up to more than 30 . In this simple slide rule we find only the essentials: A-B-C-D. As cursor we will use just a clip, then the numbers should not be placed under it as usual, but immediately to its left side.

Multiplication (uses C and D scales)
Example: $2.3 \times 3.4$

- slide the C leftmost ' 1 ' on by side 2.3 on the D scale;
- move the cursor by side 3.4 on the C scale;
- the cursor is on the D scale just a bit over 7.8. The correct answer is 7.82 .


Division (uses C and D scales)
Example: 4.5 / 7.8

- move the cursor by side 4.5 on the D scale;
- slide 7.8 on the C scale by side the cursor;
- the $C$ rightmost ' 1 ' is now at 5.76 on the $D$ scale. We know that the result of $4 / 8$ is near 0.5 , so we adjust the decimal place to get 0.576 . The correct answer is 0.576 .


Squares and Square Roots (uses A and D or B and C scales)
Example: 2, $5^{2}$

- moving the cursor by side 2.5 on the $C$ scale; we get on the $B$ scale ca. 6.25 . The correct answer is 6,25 .


## Example: $\sqrt{ } 3.500$

- the $A$ and $B$ scales have two similar halves. The left half is used to find the square root of numbers with odd numbers of digits; the right half is for numbers with even numbers of digits. Since 3.500 has an even number of digits we'll use the right half of the scale. Moving the cursor by side 3,5 of the A/D scales we get on the C/D scales ca. 59,15. The correct answer is 59,16.


Now we can try this operation: $\sqrt{ } 350 / 1,51$

- moving the cursor by side the 350 of the A scale (odd number of digits, then the left side) we get its square root, 18.7, on the D scale;
- now we match 18.7 with 1.51 of the $C$ scale: on the $D$ scale, in correspondence with the $C$ leftmost index '1', we can read the answer: ca. 12.35.
Scale A - Radicand: 350 (

An electronic calculator would have been a little more precise, finding 12.3896. This slight approximation has not prevented von Braun to send Man on the Moon: the slide rule is in fact less difficult than it sounds, and the only secret is just to practice.

## The Easy E6-B

This is the reduced version of the aeronautical slide rule E6-B, useful in solving problems of time, speed, fuel consumption and to perform conversions between different units of measure. To assembly cut along solid lines, fold along dotted lines and on inside disc fold stars flaps down. Then place discs together, push stars flaps of inside disc through center of outside disc, fold flaps all the way backwards and flatten. To make it perfect put a tiny drop of glue on each flap, position the small circle over flaps and press down. Remember to rotate the inner disc few times in case glue oozes out and glues the two discs together.



## The Easy E6-B: examples to practice



The Easy E-6B lacks of the graphic for the correction of the routes and of some specialized functions, such as determining the Mach number, but is still useful in solving problems of time, speed, fuel and to perform conversions between different units. It's a real Flight Computer!

This easy model has just two sliding scales: in the inner one we will found the number 60 (Speed Index) marked with a red arrow to help the calculations.

Remember that, as in a standard slide rule, "0.9", "9", "90", "900", "9,000" are always read as "9" and how to locate the dot or add tenths we must find by ourselves, but it's instinctive to know if we are dealing with tens, hundreds or thousands.

## General Calculations

## 01) Multiplication

## Example: $12 \times 15$.

Align 12 on the outer scale with 10 on the inner scale. Then 15 on the inner scale corresponds to 18 on the outer scale. Take into account the position of the decimal point and add one zero to obtain 180.


## 02) Division

Example: 300/15.
Align 30 on the outer scale with 15 on the inner scale. Then 10 on the inner scale corresponds to 20 on the outer scale. Take into account the position of the decimal point to obtain 20.


## Calculations for road use

## 03) Time required

Example: obtain the time required for travel 330 kilometers driving at $55 \mathrm{~km} / \mathrm{h}$.
Align 55 on the outer scale with the Speed Index (MPH). Then 33 on the outer scale corresponds to 36 on the inner scale. Thus the time required is 360 minutes ( 6 hours). Can also be calculated in miles.


## 04) Average speed

Example: obtain the average speed needed to travel 120 kilometers in 1:30 h .
Align 12 on the outer scale with 90 (minutes) on the inner scale. Then the Speed Index (MPH) corresponds to 80 . Thus the average speed is 80 kilometers per hour.


## 05) Mileage

Example: obtain the mileage when the speed is $40 \mathrm{~km} / \mathrm{h}$ and the running time is $1: 30 \mathrm{~h}$.
Align 40 on the outer scale with the Speed Index (MPH). Then 90 (the minutes) on the inner scale corresponds to 60 on the outer scale. Thus the mileage is 60 kilometers.


## 06) Rate of fuel consumption

Example: obtain the hourly rate of fuel consumption for 5 h running time and a total consumption of 35 liters.
Align 35 on the outer scale with 30 on the inner scale ( 300 minutes $=5$ hours). Then the Speed Index (MPH) corresponds to 70 . Thus the fuel consumption rate is 7 liters per hour.


## 07) Fuel required

Example: obtain the fuel required for a trip with $6 \mathrm{l} / \mathrm{h}$ of consumption and a running time of 5 h .
Align 60 on the outer scale with the Speed Index (MPH). Then 30 on the inner scale ( 300 minutes $=5$ hours) corresponds to 30 on the outer scale. Thus the fuel required is 30 liters.


## 08) Estimated running time

Example: obtain the estimated running time with $5 \mathrm{l} / \mathrm{h}$ of fuel consumption and a tank of 30 liters.
Align 50 on the outer scale with the Speed Index (MPH). Then 30 on the outer scale corresponds to 360 on the inner scale. 360 minutes $=6$ hours thus the estimated running time is 6 hours.


## Calculations for aeronautic use

## 09) Time required

Example: obtain the time required to flight 240 nautical miles at 160 knots.
Align 16 on the outer scale with the Speed Index (MPH). Then 24 on the outer scale corresponds to 90 on the inner scale. Thus the time required for the flight is 1 hour and 30 minutes.


## 10) Average speed

Example: obtain the average air speed needed to travel 250 nautical miles in 1:40 h .
Align 25 on the outer scale with 100 (1 hours 40 minutes) on the inner scale. Then the Speed Index (MPH) corresponds to 15 . Thus the air speed for the flight is 150 knots.


## 11) Flight distance

Example: obtain the distance when the air speed is 180 knots and the flight time is 40 minutes.
Align 18 on the outer scale with the Speed Index (MPH). Then 40 on the inner scale corresponds to 12 on the outer scale. Thus the air distance of the flight is 120 nautical miles.


## 12) Rate of fuel consumption

Example: obtain the hourly rate of consumption for 40 minutes flight time and a tot. consumption of 140 gal.
Align 14 on the outer scale with 40 on the inner scale. Then the Speed Index (MPH) corresponds to 21. Thus the rate of fuel consumption is 210 gallons per hour.


## 13) Fuel required

Example: obtain the fuel required for a flight with a consumption of $240 \mathrm{gal} / \mathrm{h}$ and a flight time of 1:40 h . Align 24 on the outer scale with the Speed Index (MPH). Then 100 ( 1 hour 40 minutes) on the inner scale corresponds to 40 on the outer scale. Thus the fuel required is 400 gallons.


## 14) Estimated flight time

Example: obtain the estimated flight time with a rate of fuel consumption of $200 \mathrm{gal} / \mathrm{h}$ and a tank of $1,400 \mathrm{gal}$. Align 20 on the outer scale with the Speed Index (MPH). Then 14 on the outer scale corresponds to 42 on the inner scale. Thus the estimated flight time is 420 minutes ( 7 hours).


## Conversions

## 15) Distance

Example: convert 45 miles into nautical miles and kilometers.
Align 45 with STAT. Then NAUT corresponds to about 39 nautical miles and $K M$ to about 72 km . For feet/meters conversions align the required value under $F T$ and read the result under $K M$ and vice versa.


## 16) Fuel

Example: Convert 16.8 U.S. gallons into and IMP. gallons and liters.
Align 16.8 on the inner scale with U.S. GAL on the outer scale. Then IMP. GAL corresponds to about 14 on the inner scale and LITERS to about 63.5.


You can also make currency conversions, suppose the conversions rate for Euro is $€ 1.60$ to $\$ 1.00$ : rotate the outer scale to match 16 over 10, the outer scale becomes Euro and the middle scale becomes US Dollars. To convert $€ 4.00$ to Dollars, look at the number in the middle scale opposite the 40 on the outer scale. The number is 25 : $€ 4.00$ are $\$ 2.50$. For other useful conversions follow this table.

|  | opposite to the inner 10 | Outer scale is | Inside scale is |
| :--- | :---: | :---: | :---: |
| Centimeter / Inches | 2.54 | Centimeters | Inches |
| Kilometers / Miles | 1.61 | Kilometers | Miles |
| Acres / Hectares | 2.47 | Acres | Hectares |
| Grams / Ounces | 28.35 | Grams | Ounces |
| Pounds / Kilograms | 2.21 | Pounds | Kilograms |





Find the distance traveled by knowing the time taken and speed
Keeping the same opening, place now the right end on 60: the left end will indicate the speed

Find the speed knowing the time taken to travel a distance

## Instructions for the Speed - Time Scale

In this calculator only the numbers are given: " 0.9 ", " 9 ", " 90 ", " 900 ", are always read as " 9 " and how to locate the dot or how to add tenths or hundreds we must find by ourselves,. For convenience, the examples are in kilometers, but can also be calculated in miles or nautic miles without any difference.

## Multiplication

$32 \times 5$ :
A) place the left end of a dividers on 1 and the right end on 3,2;
B) keeping the same opening, place now the left end on 5: the right end will indicate16. Take into account the position of the decimal point and add one zero to obtain 160, the correct answer.


## Division

900/5:
A) place the right end of a dividers on 9 and the left end on 5 ;
A) keeping the same opening, place now the left end on 1 : the right end will indicate 1,8 . Take into account the position of the decimal point to obtain 180, the correct answer.


## Time required

Obtain the time required for travel 140 kilometers driving at $120 \mathrm{~km} / \mathrm{h}$ :
A) place the right end of a dividers on 60 and the left end on 12;
B) keeping the same opening, place now the left end on1,4: the right end will indicate 7 . Thus the time required is 70 minutes ( $1: 10$ hours).


## Mileage

Obtain the mileage when the speed is $120 \mathrm{~km} / \mathrm{h}$ and the running time is 1 hour and 10 minutes:
A) place the right end of a dividers on 60 and the left end on 12;
B) keeping the same opening, place now the left end on 7 ( 70 minutes): the right end will indicate 1,4 . Thus the mileage is 140 kilometers.


## Average speed

Obtain the average speed ( $\mathrm{km} / \mathrm{h}$ ) having traveled 140 kilometers in an hour and 10 minutes.
A) place the left end of a dividers on 1,4 and the right end on 7 ( 70 minutes);
B) keeping the same opening, place now the right end on 60 : the left end will indicate 12 . Thus the average speed is 120 kilometers per hour.


## Rate of fuel consumption

Obtain the rate of fuel consumption (liters/h) when the running time is 3 hours and the total fuel consumption was 60 liters.
A) place the right end of a dividers on 18 ( 180 minutes $=3$ hours) and the left end on 6 ;
B) keeping the same opening, place now the right end on 60 : the left end will indicate 20 . Thus the fuel consumption rate is 20 liters per hour.


## Fuel required

Obtain the fuel required for a trip when the rate of fuel consumption is 20 liters per hour and the estimated running time is 3 hours:
A) place the right end of a dividers on 60 and the left end on 20 ;
B) keeping the same opening, place now the right end on 18 ( 180 minutes $=3$ hours): the left end will indicate 6 . Thus the fuel required is 60 liters.


## Estimated running time

Obtain the estimated running time when the rate of fuel consumption is 20 liters per hour and the tank has 60 liters of fuel:
A) place the right end of a dividers on 60 and the left end on 20 ;
B) keeping the same opening, place now the left end on 6 ( 60 liters): the right end will indicate 18. Thus the estimated running time is 3 hours ( 180 minutes $=3$ hours).


